

**Comments on “FRINGE WAVES IN AN IMPEDANCE  
HALF-PLANE”** by H. D. Basdemir,  
in *Progress In Electromagnetics Research*, Vol. 138, 571–584, 2013

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This paper (i.e., [1]) extends PTD to an impedance half-plane problem.

## 1. Terminology

First of all, the author in [1] uses some phrases/terms like “*exact Geometric Optics (GO) waves*”, “*uniform and non-uniform fringe fields*”, “*asymptotic exact*”, “*uniform exact*” in his introduction which are unique but obviously incorrect in wave terminology:

- In wave terminology, “*exact solution*” refers to the solution of the initial differential equation(s) (e.g., Maxwell equations or Helmholtz equation, etc.) under given boundary/initial conditions (either in 3D, 2D or 1D) without any assumption (such as  $kR \gg 1$ , high frequency, etc.), without making any simplification, without neglecting anything, without restricting any parameters, etc..
- In wave terminology, “*asymptotic solution*” refers to the solution when a parameter goes to infinity, such as when the distance or the frequency goes to infinity (which we show both with  $kR \gg 1$ ). A solution can not be “*asymptotic*” and “*exact*” at the same time.
- The term “*exact Geometric Optics (GO) waves*” is not accurate! The GO itself is an approximation.
- The word “*fringe waves*” represent diffracted waves caused by these non-uniform (i.e., fringe) currents. There are no such notions as “*uniform and non-uniform fringe fields*” in PTD. The terms “*uniform and non-uniform asymptotics*” for fringe waves would be more accurate in this context.

## 2. Content

Some not rigorous/not founded mathematical manipulations are used in [1]:

- The problem is constructed inaccurately by defining the total field in Equation (2) as the summation of the incident and reflected fields. Diffracted field is ignored. Although this big mistake is enough to write a comment on [1], we will give a few others below.
- In Equation (13), the Hankel function is replaced by its asymptotics in Equation (14) which is not valid close to the half-plane surface where  $R_1 \rightarrow 0$ . Because of that the basic PO expressions (15)–(17) are not the rigorous PO equations.
- Then from these not rigorous equations, the non-uniform asymptotics in Equation (25) is derived which is not valid in vicinity of the GO boundaries  $\phi = \pi \pm \phi_0$ .
- By the use of Equation (26), the author formally introduces the *uniform* approximations in terms of the Fresnel functions. In fact, he constructs the uniform asymptotics on the basis of the non-uniform ones. In general such a procedure is not unique. In the paper it is utilized without a rigorous foundation.

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- Similar not rigorous manipulations are performed to obtain the so-called exact solution in Equation (47) for the field diffracted at the impedance half-plane.
- On the top of page 572 in [1] it is written that “Umul showed in a study by making comparison that the values used by Ufimtsev were quite large and not giving the correct results for the fringe fields.” The related Umul’s paper [3] is referenced many times in [1]. However, as demonstrated in [2], this Umul’s paper [3] does not predict correct results for the fringe waves. See also Examples below.
- In the conclusion of [1] the author states:  
*“In Chapter 4, Figure 4.2 of [4], amplitude values of asymptotic fringe field components take equal value with GO fields [4]. However, it is noticeable that diffracted fields amplitude values have to be half of the GO amplitude values in the shadow and refraction regions. The more rigorous expressions were presented and numerically analyzed in this work”.*

The author was not enough careful analyzing this figure. This figure is correct. It represents the backscattered field ( $\phi = \phi_0$ ). Within the graphical accuracy this field equals 1/2 in the reflection directions  $\phi = \pi/2$  and  $\phi = 3\pi/2$ , that is in the total agreement with Equation (4.16) of [4]. Besides, this figure demonstrates only the directivity patterns of the fringe waves (4.12), (4.13) in [4] — not the total field (!) which is of the order  $(kr)^{(-1/2)}$  while the GO field is of the order of  $(kr)^0$ . Therefore, the fringe field in principle cannot be one half of GO field at the boundaries of the incident and reflected plane waves. Also as shown below, these “more rigorous expressions” presented in [1] are incorrect. We prove this statement considering the particular case of the impedance half-plane when its impedance equals zero. In acoustic terminology it is called a soft half-plane.

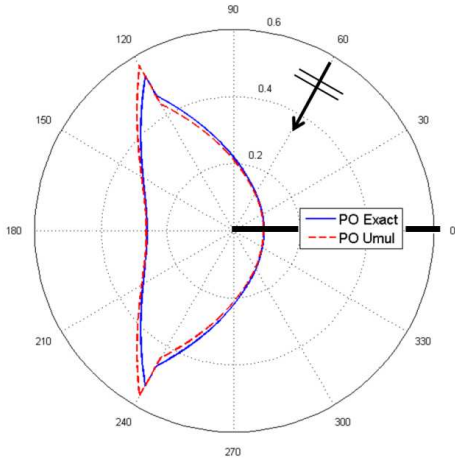
### 3. Examples

This part is for demonstration the inaccuracy of [1, 3] and related papers which are referenced many times in [1]: One of major defects of the MTPO in [3] is its PO expression which is invalid near critical angles. On the example of the soft half-plane, below in Figure 1 this PO approximation is denoted as “PO Umul” and given as [6]

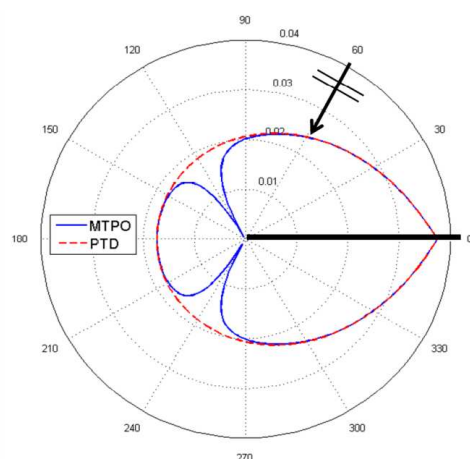
$$u_d^{PO} = \frac{\cos(\phi_0/2)}{\sin(\phi/2)} \left\{ e^{ik\rho \cos(\phi-\phi_0)} \text{sign}(\xi_-) F[|\xi_-|] - e^{ik\rho \cos(\phi+\phi_0)} \text{sign}(\xi_+) F[|\xi_+|] \right\} \quad (1)$$

where  $\xi_{\pm} = -(2k\rho)^{1/2} \cos[(\phi \pm \phi_0)/2]$ .

It is compared with the “PO Exact” calculated according to [4]. The deviations of “PO Umul” from “PO exact” in vicinity of the GO reflected and incident boundaries are clearly observed.



**Figure 1.** PO fields ( $\alpha = 360^\circ$ ,  $\phi_0 = 60^\circ$ ,  $kr = 2\pi$ ).



**Figure 2.** Fringe fields ( $\alpha = 360^\circ$ ,  $\phi_0 = 60^\circ$ ,  $kr = 12\pi$ ).

In Figure 2, the fringe waves vs. angle calculated as the difference between the exact Sommerfeld [5]

$$u_d = e^{ik\rho \cos(\phi-\phi_0)} \text{sign}(\xi_-) F[|\xi_-|] - e^{ik\rho \cos(\phi+\phi_0)} \text{sign}(\xi_+) F[|\xi_+|] \quad (2)$$

and PO solutions given in Equation (1) is plotted and compared with the exact PTD fringe waves calculated as the difference between the exact Sommerfeld in Equation (2) and the exact PO solutions according to [4].

Figure 2 clearly demonstrates that the MTPO fringe waves in [1] and [3] are incorrect in vicinity of the GO boundaries. Figures 1 and 2 also show that “the uniform representations” of the MTPO field in terms of the Fresnel functions do not describe the fine features of the diffracted field in the transition regions and lead to incorrect results for the fringe waves.

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