

## Microstrip Bandpass Filters Based on Inductive-Coupled Stepped-Impedance Quarter-Wavelength Resonators

Fei Cheng, Xian Qi Lin\*, Yuan Jiang, Kai Jun Song, and Yong Fan

**Abstract**—Inductive-coupling scheme for microstrip bandpass filters with quarter-wavelength stepped-impedance resonators is proposed. This is realized by a short-end stub which behaves as a  $K$ -inverter. It is investigated that the coupling coefficient of the resonators can be easily controlled by the length of the short-end stub. The filter has a compact size and good stopband rejection by employing the quarter-wavelength stepped-impedance resonators. The design procedure of this kind of filter is provided. Two filters working at 2.4 GHz are designed and fabricated to demonstrate the proposed method.

### 1. INTRODUCTION

Microstrip filters based on centrally loaded resonators have attracted more and more attention in recent years [1–5]. Filters using open stub-loaded resonators or the dual-mode resonators were first reported in [1]. The open stub works as a  $K$ -inverter between the quarter-wavelength resonators. Due to direct coupling, the designed filters have low insertion loss. Transmission zeros created by the open stubs can be located near the passband by adjusting the stub length. However, the filter has a first spurious response at about  $2f_0$  and the open stub will occupy size. Afterwards, many researches have been carried out to improve the performance of this kind of filter. Among them, the source-load coupling is a promising method used to create more transmission zeros [2, 3]. Nevertheless, the source-load coupling structure would occupy more dimensions than the tapped-line feeding structure, due to the long feeding lines. Other researchers found that the coupling strength of the two quarter-wavelength resonators can be adjusted by the length of open stub. This characteristic is used to design multiband filters [4, 5]. In 2004, a shunt inductance inverter achieved by a metallized via was added in the middle of the conventional half-wavelength resonator to construct a filter [2]. The proposed filter exhibits better stopband rejection with the first harmonic passband at about  $3f_0$ . However, this kind of filter cannot have a wide passband due to a narrow range of coupling produced by the small inductance.

Stepped-impedance resonators (SIRs) used are desirable to reduce the circuit's dimension [6]. With a certain length ratio, the smaller the impedance ratio is, the shorter the total length of the SIR is. At the same time, they have the capability to push the spurious responses to higher frequencies by changing the impedance ratio and the length ratio [7].

In this paper, the new coupling scheme for realizing microstrip filters is proposed. By loading a short-end stub in the middle of two  $\lambda/4$  SIRs and then suitably designing the length of the short-end stub, one may adjust the coupling coefficient between two resonators so that a wide range of coupling may be realized on the planar filter structures. Based on the well-known coupled-resonator approach [8] and the designable coupling coefficients, filters using inductive-coupled  $\lambda/4$  SIRs may be designed with simple design procedure. Specifically, a second-order filter is carefully designed and implemented to demonstrate the proposed concept. Furthermore, a fourth-order filter is also designed and implemented to achieve a better stopband rejection. The implemented filters exhibit compact size and good stopband rejection with the first spurious passband centered at about  $4f_0$ .

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\* Corresponding author: Xian Qi Lin (xianqilin@gmail.com).

The authors are with the EHF Key Lab of Fundamental Science, School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China.

## 2. ANALYSIS OF THE INDUCTIVE-COUPLED QUARTER-WAVELENGTH RESONATORS

Figure 1(a) shows the inductive-coupled  $\lambda/4$  SIRs, where  $Z_1$ ,  $l_1$ ,  $Z_2$ ,  $l_2$ ,  $Z_3$  and  $l_3$  denote the characteristic impedances and physical lengths of the  $\lambda/4$  SIRs and the short-end stub, respectively. The short-end stub can be equivalent to an inductor of value  $L_m$  and the inductor connected transmission lines of length  $\phi$  can be equivalent to a  $K$ -inverter [8], as shown in Fig. 1(b). The exact value of the inductor can be extracted by a full-wave simulator as HFSS. For a narrow band application,  $L_m$  is assumed to be constant.

The  $K$ -inverter value  $K$  is given by [8]

$$K = Z_2 \tan \left| \frac{\phi}{2} \right|, \quad (1)$$

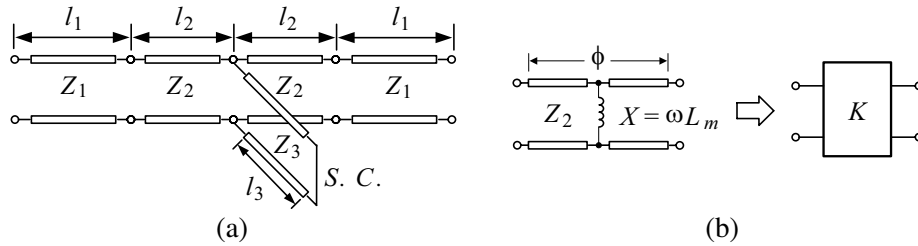
$$\phi = -\tan^{-1} \left| \frac{2\omega L_m}{Z_2} \right|, \quad (2)$$

$$\left| \frac{\omega L_m}{Z_2} \right| = \frac{K/Z_2}{1 - (K/Z_2)^2}, \quad (3)$$

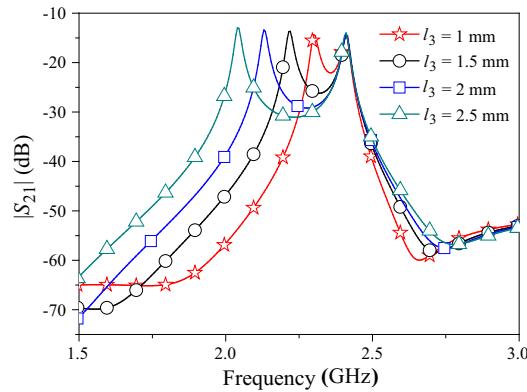
where  $\omega$  is operating angular frequency. When  $K/Z_2 \ll 1$ , Equation (3) converts to

$$K = \omega L_m \quad (4)$$

Based on Equation (4), coupling generated by the short-end stub can be evaluated. Fig. 2 presents typical simulated resonant frequency responses of the inductive-coupled  $\lambda/4$  SIRs with different  $l_3$ , where  $S_{21}$  denotes the  $S$ -parameter between the two ports which are very weakly coupled to the coupled resonator structure. Here,  $Z_1$ ,  $Z_2$ ,  $l_1$  and  $l_2$  are set to be  $41.3 \Omega$ ,  $87.9 \Omega$ ,  $9.2 \text{ mm}$ , and  $9.2 \text{ mm}$  respectively. A substrate with  $\epsilon_r = 2.45$  and  $h = 0.8 \text{ mm}$  is used in the simulation. The result shows that when the length of the short-end stub increases, the two resonant frequencies remote from each other farther. It



**Figure 1.** (a) Schematic of the inductive-coupled quarter-wavelength stepped-impedance resonators, (b) equivalent circuit model of the  $K$ -inverter.



**Figure 2.** Simulated  $S_{21}$ -magnitude of the inductive-coupled quarter-wavelength resonators with different  $l_3$  under weak coupling.

denotes a larger  $K$  can be got by increasing the length of the short-end stub. This agrees well with Equation (4).

### 3. DESIGN PROCEDURE AND REALIZATION OF A SECOND-ORDER BANDPASS FILTER

The design procedure of the filter using inductive-coupled  $\lambda/4$  SIRs is summarized as follows. Firstly, according to the specified center frequency  $f_0$  and 3-dB bandwidth FBW, the external quality factors of the resonators at the input and output  $Q_{e1}$  and  $Q_{en}$  as well as the coupling coefficients between the adjacent resonators  $K_{i,i+1}$  are obtained [8]

$$Q_e = Q_{e1} = Q_{en} = \frac{g_0 g_1}{\text{FBW}},$$

$$K_{i,i+1} = \frac{\text{FBW}}{\sqrt{g_i g_{i+1}}}, \quad i = 1, \dots, n-1$$
(5)

where,  $g_i$  is the lowpass prototype parameters and  $n$  the order of the filter. Secondly, decide the impedance ratio  $R_z = Z_1/Z_2$  and lengths ratio  $\alpha = \theta_1/(\theta_1 + \theta_2)$  for the utilized  $\lambda/4$  SIRs. These parameters determine the resonator's fundamental resonant frequency, spurious frequencies and the occupied size [6]. For size reduction and pushing the first spurious frequency higher,  $R_z$  is chosen smaller than 1. The total electric length of a  $\lambda/4$  SIR might be minimized when the electric lengths of high impedance line and low impedance line satisfy [9]

$$\theta_1 = \theta_2 = \arctan \sqrt{R_z}. \quad (6)$$

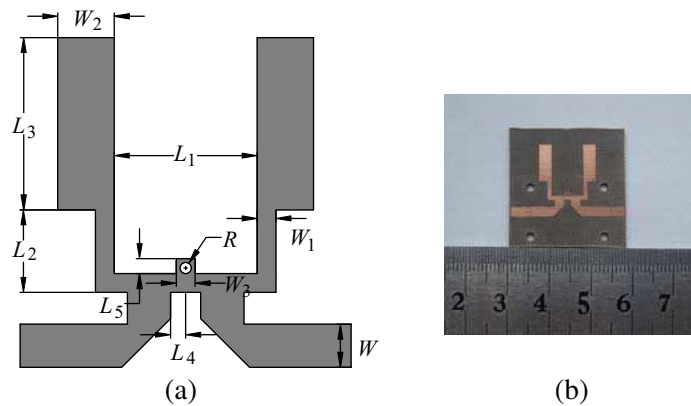
The first spurious frequency can be calculated as

$$f_{sp1} = \frac{180 - \theta_1}{\theta_1} f_0. \quad (7)$$

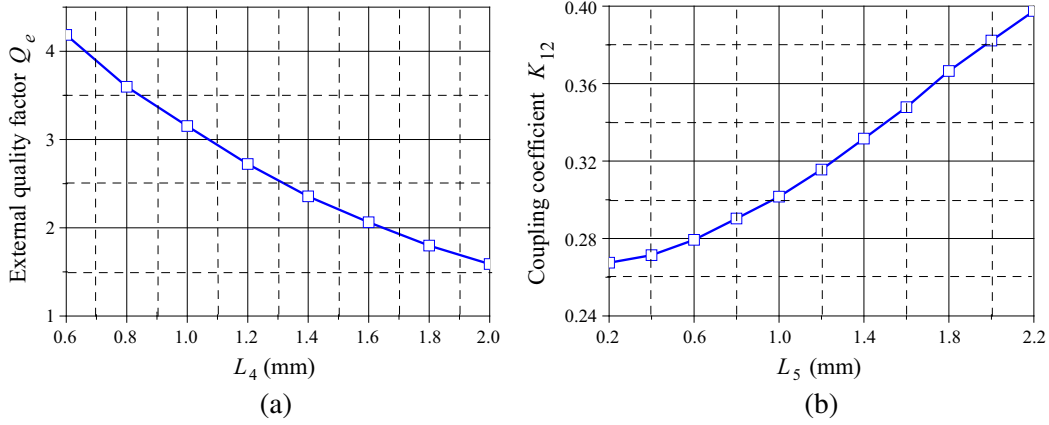
Thirdly, external quality factor  $Q_e$  for the utilized tapped-line feeding structure and the coupling coefficient  $K_{i,i+1}$  should be extracted and drawn as design curves with respect to the related positions before the filter facilitation. The input and output tapping point is got by the external quality factor while the length of the short-end stub is obtained by the coupling coefficients between the resonators.

To demonstrate, a second-order filter based on the above mentioned procedure is fabricated as shown in Fig. 3. The specified center frequency and 3-dB fractional bandwidth are 2.4 GHz and 12.5%, respectively. The element values of the low-pass Chebyshev prototype filter with a 0.1 dB ripple level are found to be

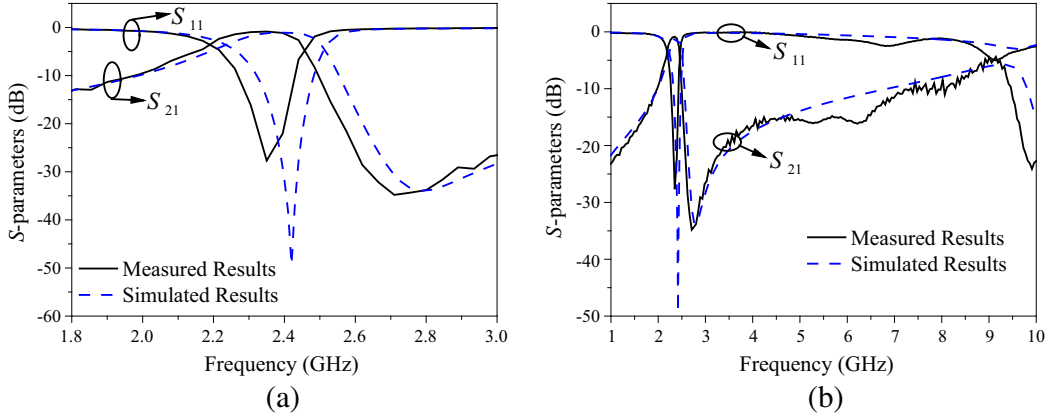
$$g_0 = 1, \quad g_1 = 0.4489, \quad g_2 = 0.4078, \quad g_3 = 1.1008. \quad (8)$$



**Figure 3.** Configuration and photograph of the proposed second-order filter. (a) Configuration, (b) photograph.



**Figure 4.** Design curves for (a) external quality factor  $Q_e$  and (b) coupling coefficients  $K_{12}$ .



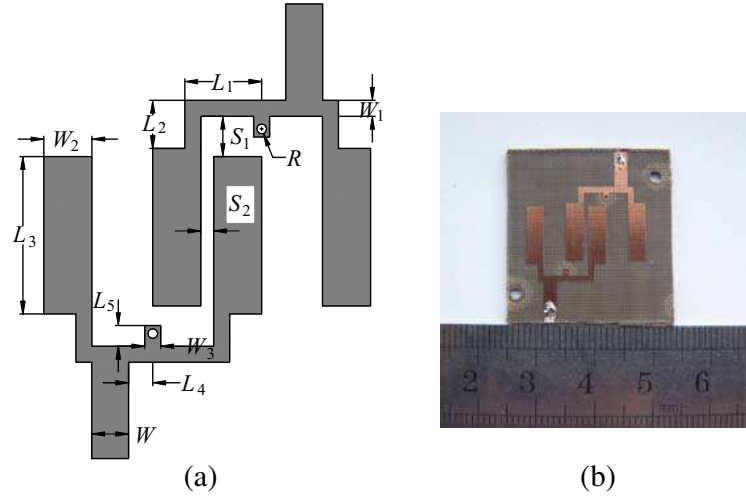
**Figure 5.** Simulated and measured results of the second-order filter. (a) Insertion loss and return loss, (b) wide-band response.

The calculated coupling coefficient  $K_{12}$  is 0.2922, and the calculated external quality factors  $Q_{e1}$  and  $Q_{e2}$  are 3.5912.  $Z_1$  and  $Z_2$  are chosen as  $41.3\Omega$  and  $80.9\Omega$  to provide the impedance ratio  $R_z = 0.51$ . According to Equation (5), the low impedance line and the high impedance line have the same electric length  $35.53^\circ$ . The first spurious frequency is evaluated to be 9.76 GHz based on Equation (6). Thus the physical dimension of the SIRs can be determined. Afterwards, the simulated external quality factor against the tapped line position  $L_4$  is drawn in Fig. 4(a) and the simulated coupling coefficients versus the distances  $L_5$  is drawn in Fig. 4(b). Then,  $L_4$  is chosen to be 0.8 mm to satisfy  $Q_{e1} = Q_{e2} = 3.5912$ , and  $L_5$  is chosen to be 0.8 mm to satisfy  $K_{12} = 0.2922$ .

A F4B substrate with a relative dielectric constant of 2.45, a thickness of 0.8 mm, and a loss tangent of 0.001 is chosen for the filter design. The geometric parameters of the filter are chosen as follows:  $W = 2.3$  mm,  $W_1 = 1$  mm,  $W_2 = 3$  mm,  $W_3 = 1$  mm,  $L_1 = 7.6$  mm,  $L_2 = 4.4$  mm,  $L_2 = 3$  mm,  $L_3 = 9.2$  mm,  $L_4 = 0.8$  mm,  $L_5 = 0.8$  mm,  $R = 0.3$  mm. Fig. 5 shows the simulated and measured responses of the proposed second-order filter. The measured center frequency is at 2.35 GHz and the measured 3-dB fractional bandwidth is 11.5%. The implemented second-order filter has an insertion loss of 0.85 dB and a maximum return loss of 27.7 dB at the center frequency. By the effect of the open stubs associated with the first and second resonators, a transmission zero around 2.71 GHz is obtained.

#### 4. REALIZATION OF A FOURTH-ORDER BANDPASS FILTER

It can be seen that the second-order filter only have a stopband rejection of 20 dB from 2.57 GHz to 3.43 GHz. In order to improve this performance, another fourth-order bandpass filter using inductive-



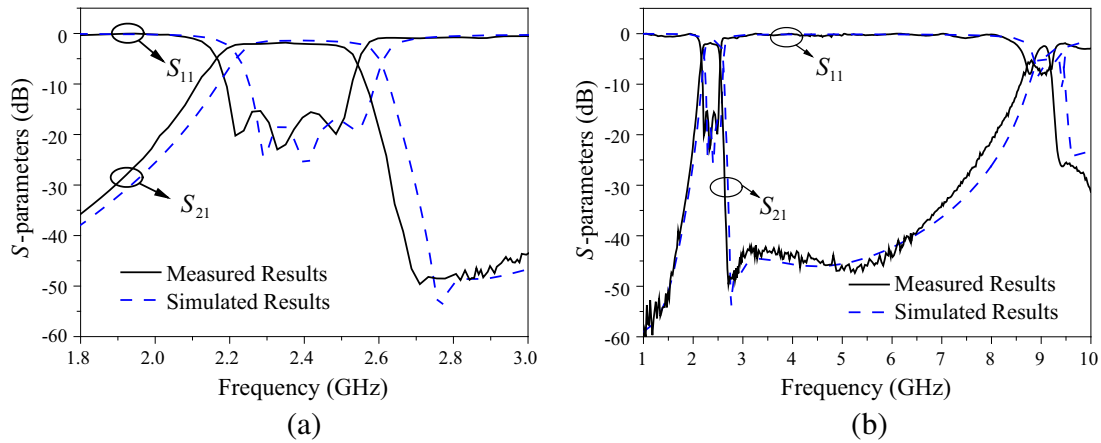
**Figure 6.** Configuration and photograph of the proposed forth-order filter. (a) Configuration, (b) photograph.

coupled  $\lambda/4$  SIRs is proposed as shown in Fig. 6. The specified center frequency is at 2.4 GHz with a fraction bandwidth of 16.7%. The design parameters for the specifications are

$$Q_{e1} = Q_{e4} = 4.269, \quad K_{12} = K_{34} = 0.1805, \quad K_{23} = 0.1326. \quad (9)$$

The design method is similar to the previous one and is not repeated here. The same F4B substrate is used for the filter design. The geometric parameters of the filter are chosen as follows:  $W = 2.3$  mm,  $W_1 = 1$  mm,  $W_2 = 3$  mm,  $W_3 = 1$  mm,  $L_1 = 4.8$  mm,  $L_2 = 4.8$  mm,  $L_2 = 3$  mm,  $L_3 = 9.8$  mm,  $L_4 = 1.51$  mm,  $L_5 = 1.28$  mm,  $S_1 = 1.25$  mm,  $S_2 = 0.8$  mm,  $R = 0.3$  mm. The implemented filter is compact and has a dimension of  $0.2\lambda_g$  by  $0.16\lambda_g$ .

Figure 7 illustrates the measured and simulated results of the filter. The measured center frequency is at 2.35 GHz with a fractional bandwidth of 16.2% while the measured minimum insertion loss in the passband is 1.4 dB. A transmission zero is located at about 2.71 GHz. The fourth-order bandpass filter exhibits a superior stopband rejection performance compared with the former second-order bandpass filter. The first harmonic passband is presented at about  $4f_0$ . The filter has a stopband rejection of 20 dB from 2.62 GHz to 8.09 GHz which is much wider than the second-order filter.



**Figure 7.** Simulated and measured results of the forth-order filter. (a) Insertion loss and return loss, (b) wide-band response.

## 5. CONCLUSION

Bandpass filters using inductive-coupled  $\lambda/4$  SIRs are investigated. The equivalent circuit of the inductive-coupled  $\lambda/4$  SIRs is analyzed. And it is found that the coupling coefficient  $K$  can be adjusted by the length of the short-end stub. Based on the analysis, a second-order bandpass filter is fabricated to demonstrate the design procedure. One transmission zero above the passband is achieved by the effect of the open stubs. In order to improve the stopband rejection, a fourth-order bandpass filter is fabricated. The design concept is validated as the measured results agree well with the simulation.

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