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Nonparametric Rotational Motion Compensation Technique for High-Resolution ISAR Imaging via Golden Section Search

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Abstract—A novel rotational motion compensation algorithm for high-resolution inverse synthetic aperture radar (ISAR) imaging based on golden section search (GSS) method is presented. This paper focuses on the migration through cross-range resolution cells (MTCRRC) compensation, which requires rotation angle and center as priori information. The method performs in a nonparametric way and uses entropy criterion to estimate rotation angle and rotation center, which are used for rotational motion compensation. Experimental results show that the rotational motion in ISAR imaging can be effectively compensated. Moreover, the proposed method is robust and computationally more efficient compared to the parametric methods.

1. INTRODUCTION

Inverse synthetic aperture radar (ISAR) can generate 2-D image for moving targets. The high range resolution is proportional to the bandwidth of transmitted signal, while the high cross-range resolution is obtained by using a synthetic antenna aperture generated by rotational motion of the target. With the 2-D resolution of the ISAR increasing, migration through resolution cells (MTRC) will be inevitably produced because of the rotational motion of the target. As a result, MTRC may lead to image defocusing in both the range and cross-range domain [1, 2].

The rotational motion compensation is constituted of two steps: the migration through range resolution cells (MTRRC) compensation and migration through cross-range resolution cells (MTCRRC) compensation. In the first step, fast Keystone transform [3] is used, which does not need rotation angle. Since good performance can be achieved in MTRRC compensation step, this paper mainly focuses on the MTCRRC compensation. For MTCRRC compensation, the rotation angle and center are required [4]. In general, the estimation of rotation parameter can be implemented in parametric and nonparametric ways, and most of algorithms belong to the former ones [5–9]. They are, however, computationally inefficient and sensitive to noise. Once the phase of signal is not correctly estimated, the rotation parameters will be deviated from the real ones.

This paper proposes a novel rotational motion compensation algorithm implemented by iteratively minimizing the entropy of the ISAR image in a nonparametric way. To be specific, the golden section search (GSS) method [10, 11] is used iteratively to estimate the rotation angle and center. The optimal parameters can be obtained when the entropy of the compensated ISAR image is minimized. When a rigid-body target stably flies, the rotation angle of target in the coherent processing interval (CPI) is close to the rotation of radar line of sight (RLOS) [12]. The initial rotation angle can be obtained from the RLOS. Moreover, the rotation center is restricted to the target region of high range resolution profile (HRRP). With the prior knowledge of the rotation angle and center, the algorithm can be performed more efficiently. The framework of the proposed algorithm is shown in Figure 1. The major contribution

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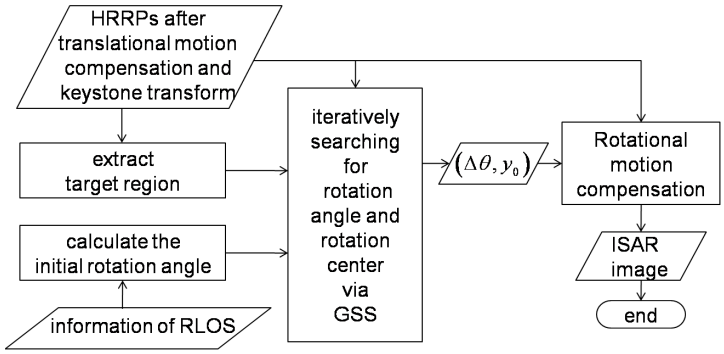

Figure 1. Schematic diagram of algorithm.

of the proposed algorithm is the iterative search for rotation angle $\Delta\theta$ and center y_0. The effectiveness and robustness of the proposed algorithm are demonstrated by synthetic and real data in Section 5.

The remainder of this paper is organized as follows. The signal model of MTCRRC is introduced in Section 2. Section 3 presents the proposed algorithm. The iterative search method is detailed in Section 4. Experimental results and analysis are reported in Section 5, followed by conclusions in Section 6.

2. SIGNAL MODEL OF MTCRRC

The point scatterer model is frequently used in radar imaging. Assuming that the translational motion has been fully removed, the geometry of radar imaging is depicted in Figure 2(a). The instantaneous distance from the scattering point p with position (x_k, y_k) to the radar can be approximated as:

$$r_p(t) \approx r_a + x_k \sin\theta(t) + y_k \cos\theta(t) \tag{1}$$

where r_a is the range from target rotation center to the radar and $\theta(t)$ the rotation angle at arbitrary time.

In ISAR imaging, a linear frequency modulation (LFM) signal is often transmitted, and then the echo is dechirped. After the range compression and shifting to the reference point by neglecting the constant phase term, the received signal can be expressed as a sum of point scatterer responses

$$s(r,t) = \sum_{k=1}^{K} \sigma_k \mathrm{sinc}\left\{T_p\left(r - 2\gamma\frac{\Delta R_k(t)}{c}\right)\right\} \exp\left\{-j\frac{4\pi}{\lambda}\Delta R_k(t)\right\} \tag{2}$$

where σ_k, c, λ and t are the backward scattering coefficient, light speed, wavelength, and dwell time, respectively. T_p and γ denote the pulse width and chirp rate of the transmitted signal. $\Delta R_k(t)$ is the instantaneous range between radar and the scatterer.

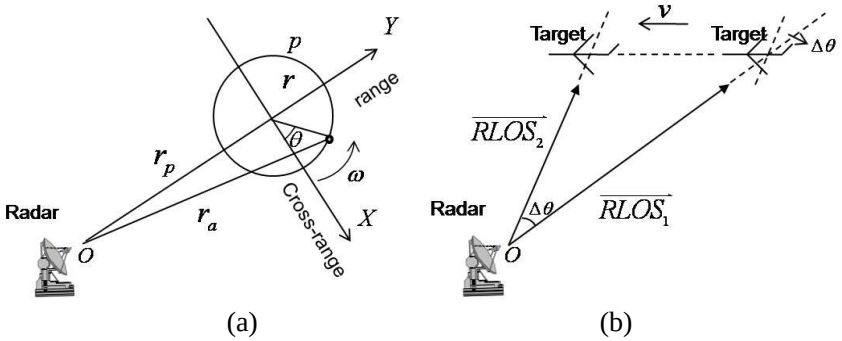

Figure 2. Geometry of ISAR imaging. (a) Geometry of the scattering point. (b) Model of target motion.

Let us assume that the translational motion has been fully compensated, and one has $\Delta R_k(t) = x_k \sin\theta(t) + y_k \cos\theta(t)$. The rotation angle $\theta(t)$ at arbitrary time t can be calculated by $\theta(t) = \omega t$, where ω is the rotation rate of the target with respect to the radar. If the total rotation angle of the target is not too large during the imaging observation interval, $\cos\theta(t)$ and $\sin\theta(t)$ can be approximated by the second-order Taylor expansion. Neglecting the second-order terms in the envelop, the radar echo can be rewritten as

$$s(r,t) = \sum_{k=1}^{K} \sigma_k \text{sinc}\left\{T_p\left[r - 2\gamma\frac{y_k + x_k\omega t}{c}\right]\right\} \exp\left\{-j\frac{4\pi}{\lambda}[y_k + x_k\omega t + y_k\omega^2 t^2/2]\right\} \tag{3}$$

The time-varying term in the envelope and the quadratic error term in the phase are the causes of the range migration and cross-range migration, respectively. The rotational motion compensation can be realized by two steps: the MTRRC compensation and the MTCRRC compensation. For MTRRC, we can utilize fast Keystone transform which does not need to know the rotation angle as a priori. In order to compensate the cross-range migration, the quadratic phase error should be estimated, which is given by

$$\phi_c = \frac{4\pi}{\lambda}\frac{1}{2}y_k\omega^2 t^2 = \frac{4\pi}{\lambda}\frac{1}{2}y_k\left(\theta\left(t\right)\right)^2 \tag{4}$$

where y_k is the distance from the scattering point to rotation center in slant range. When the total rotation angle $\Delta\theta$ in CPI is estimated, the instantaneous rotation angle $\theta(t)$ at t can be obtained under the assumption of uniform rotational motion.

3. ALGORITHM DESCRIPTION

According to the theoretical analysis of MTCRRC compensation model, once the rotation angle and center are known, the MTCRRC during ISAR imaging can be fully compensated. As a result, the 2-D ISAR image I reconstructed from compensated HRRPS can be best focused. Assume that the sharpness of the ISAR image is measured by entropy, which is defined by:

$$H\left(I\right) = -\sum_{m=1}^{M}\sum_{n=1}^{N}\frac{\left|I\left(m,n\right)\right|^2}{S}\ln\frac{\left|I\left(m,n\right)\right|^2}{S} \tag{5}$$

where $S = \sum_{m=1}^{M}\sum_{n=1}^{N}|I(m,n)|^2$, M denotes the number of HRRPs and N is the number of range bins in HRRP. $I(m,n)$ is the complex value of the pixel (m,n) of I. Entropy can be used to measure the sharpness of a distribution function, such as a 2-D image [13, 14]. The more focused the image is, the smaller the entropy is. When the image after MTCRRC compensation becomes best focused, its entropy will be minimized. Therefore, the MTCRRC compensation can be performed by searching the rotation angle and center based on minimizing entropy of the compensated ISAR imagery. In order to increase the efficiency of the search processing, the initial rotation angle $\Delta\theta$ is used, which can be approximated as the rotation of RLOS shown in Figure 2(b). The RLOS can be computed from the range, azimuth and pitching of the target from radar. On the other hand, the rotation center is restricted to target region, which can be extracted from HRRP [15]. The target region $[y_1, y_2]$ is set as the search region of rotation center in the proposed algorithm.

The procedure of the rotational motion compensation is summarized as follows. First, the translational motion compensation should be done. Second, the MTRRC compensation is applied to correct the migration in slant range. Thirdly, the rotation angle $\Delta\theta$ in CPI and center y_0 are estimated by iteratively search utilizing GSS method to achieve the minimum entropy of the compensated ISAR imagery. The initial value of rotation angle $\Delta\theta$ and target region $[y_1, y_2]$ are obtained from RLOS and HRRP, respectively. Finally, the MTCRRC compensation can be realized as follows

$$s_{\text{COMP}}(r,t) = s(r,t)\exp\left\{j\phi_c\right\} \tag{6}$$

4. ITERATIVE SEARCH FOR ROTATION ANGLE AND CENTER VIA GOLDEN SECTION SEARCH METHOD

The major contribution of the proposed algorithm is the iterative search for $\Delta\theta$ and y_0. In this paper, the 2-D search is divided into two 1-D search in a particular dimension. The schematic diagram of the iterative searching process is shown in Figure 3. The 1-D iteratively search utilizing GSS method based on minimum entropy is presented in Figure 4.

4.1. 1-D Golden Section Search by Entropy Evaluation

Golden section search is a very efficient algorithm for finding out the extreme of an objective function with unimodal [11]. Figure 4 shows the procedure of the 1-D GSS by entropy evaluation. After iterative search, the value of compensation parameter can be obtained when the variety of the entropy under a given threshold. Before the GSS, the search region should be determined, which is depicted as $[a, b]$.

In Figure 4(b), two points x_1 and x_2 are given and satisfy $a < x_1 < x_2 < b$. Let us denote x_1 be the "left point" and x_2 be the "right point", x^* be the minimized point. $H(x)$ is the entropy of the ISAR image after rotational compensation under the compensated parameter x. If $H(x_1) < H(x_2)$, as shown in Figure 4(b), $x^* \in [a, x_2]$. Then the search can be performed in the interval $[a, x_2]$. Otherwise, if $H(x_1) > H(x_2)$, as shown in Figure 4(c), the new search will be performed in the interval $[x_1, b]$. The search stops when the interval is smaller than the pre-determined threshold ε. The median of the final search region is set as the value of the searched parameter. As a result, the error between the estimated value and true extreme value is under $\varepsilon/2$.

In order to improve the efficiency, the "left point" x_1 and "right point" x_2 should be set appropriately. Therefore, x_1 and x_2 are set as symmetrical by the median of current search region.

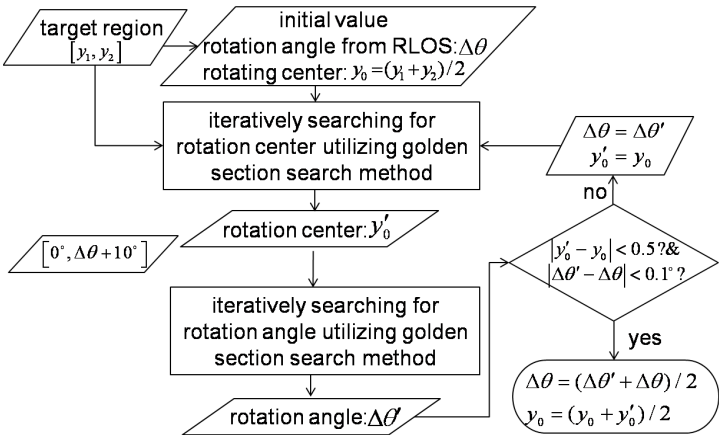

Figure 3. Schematic diagram of iteratively search.

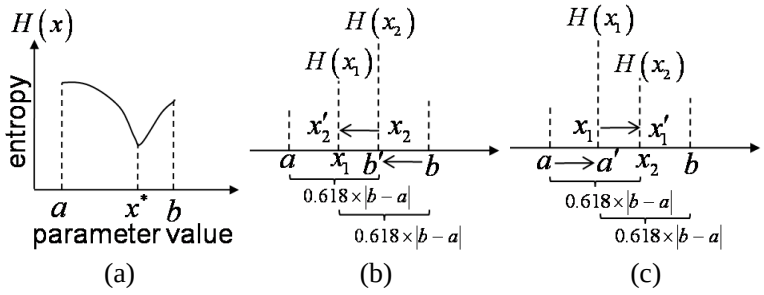

Figure 4. Schematic diagram of GSS method based on minimum entropy.

Let us suppose that the x_2 satisfies

$$x_2 = a + \tau(b - a) \tag{7}$$

where τ is the ratio which will be given later. Furthermore, x_1 is given by

$$x_1 = b - \tau(b - a) \tag{8}$$

Meanwhile, it is expected that the "left point" x_1 of the next search is updated by the former "right point" x_2. So that it can reduce one calculation in each search. Finally, one have

$$a + \tau(b - a) = b - \tau(b - a) \tag{9}$$

From (9), it can be calculated that $\tau = 0.618$, which is famous as the golden cut ratio. Additionally, that is why the search was called as golden section search. Assume that the length of search region is L and the stopping threshold is ε, the iterative search times can be given as

$$N_{IS} = \log_{0.618} \frac{\varepsilon}{L} = -4.78 \lg \frac{\varepsilon}{L} \tag{10}$$

Take iterative search for rotation center in Section 5 as example, one have the length of search region $L = \text{length}([16, 54]) = 39$ and $\varepsilon = 0.5$. The iterative search times is $N_{IS} = 9$. Consequently, GSS has very high efficiency and wide practicality.

4.2. Iterative Search for Rotation Angle and Center by GSS

Based on the 1-D GSS algorithm given above, the iterative 2-D search for rotation angle and center based on GSS method is described as follows:

Step 1: Get the initial coarse value of rotation angle and center. The initial value of rotation angle $\Delta\theta$ can be calculated from the change of RLOS. Meanwhile, the initial rotation center is set as $y_0 = (y_1 + y_2)/2$, where $[y_1, y_2]$ is the target region in HRRP.

Step 2: Search the new rotation center y_0' iteratively utilizing GSS method, as depicted in Figure 4.

Step 3: Search the new rotation center $\Delta\theta'$ iteratively utilizing GSS method.

Step 4: Check the convergence conditions. Compare the initial values and updated values of the rotation angle and center. If the difference between y_0 and y_0' is below 0.5 and the difference between $\Delta\theta$ and $\Delta\theta'$ is less than 0.1°, the iteratively searching terminates. Otherwise, the algorithm goes back to Steps 2 and 3.

Step 5: Search stops and the rotation angle and center are determined by $\Delta\theta = (\Delta\theta + \Delta\theta')/2$ and $y_0 = (y_0 + y_0')/2$.

5. EXPERIMENT AND PERFORMANCE ANALYSIS

All the algorithms in this section run on Matlab 2010a. The operating system is Microsoft Windows XP Professional SP3 and processor is Pentium Dual-Core 2.8 GHz $*$ 2 CPU. The computer memory is 3 GB of system RAM.

In the first experiment, simulated data on aircraft (MIG-25) ISAR data [1] were used to test the proposed method. The aircraft is composed of 120 point scatterers with equal reflectivity. A total 512 samples of the time history series were used. The radar transmits LFM signals with carried frequency of 9 GHz and bandwidth of 512 MHz. It was assumed that the aircraft only has rotational motion with the translational motion fully compensated. The initial values were set as $\Delta\theta = 10°$ and $[y_1, y_2] = [16, 50]$. The estimated rotation center y_0 and angle $\Delta\theta$ were 18.67 and 18.44°, with the computational time 3.19 seconds.

Figure 5(a) shows the ISAR image obtained by using range-Doppler algorithm (RDA) without any compensation, where the blurring effects caused by the MTRC are obvious. Figure 5(b) presents the MTRRC compensated ISAR image, where the slant range migration has been corrected. Figure 5(c) shows the output after MTCRRC compensation. A well focused ISAR image was obtained. Based on ISAR imaging theory, the range resolution ρ_r should satisfy $\rho_r = c/(2B) = 0.59$ m, where c is the light velocity, B is the signal bandwidth, and the cross-range resolution ρ_a should satisfy $\rho_a = \lambda/(2\Delta\theta) = 0.10$ m, where λ denotes the signal wavelength, $\Delta\theta$ denotes the target rotation angle. The cross range scaling (CRS) of the image is shown in Figure 5(d), which illustrates the size of

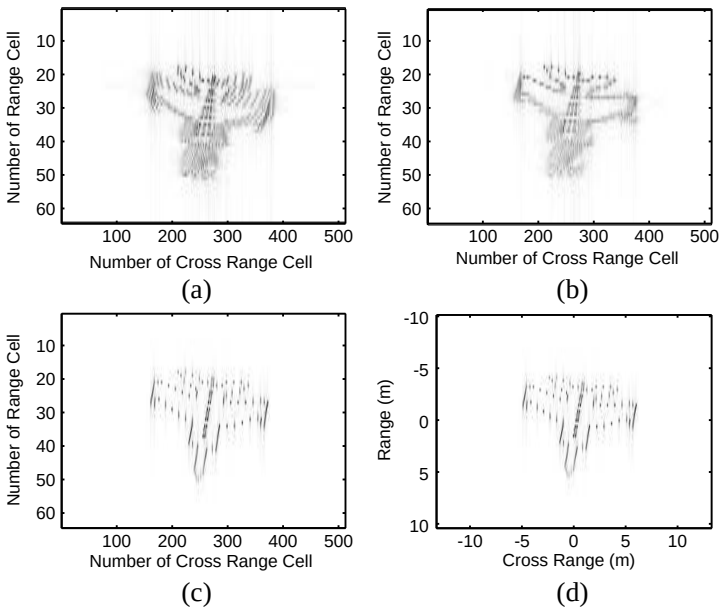

Figure 5. ISAR images of Mig-25. (a) ISAR image with MTRC. (b) ISAR image after MTRRC compensation. (c) ISAR image after MTCRRC compensation by proposed method. (d) ISAR image after CRS.

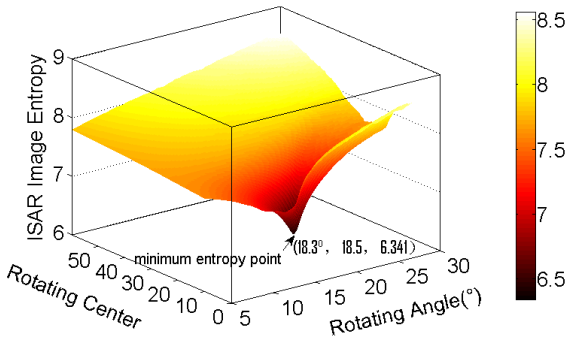

Figure 6. ISAR image entropy after MTCRRC compensation with different rotation parameters.

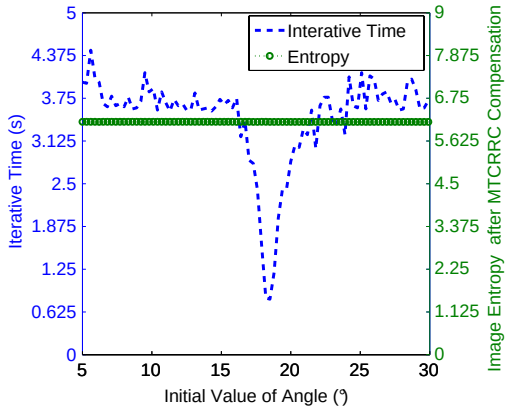

Figure 7. Performance of the proposed method along initial angle.

the simulated airplane. The image after CRS shows the accuracy of the estimated rotation angle. Figure 6 depicts the ISAR image entropy after MTCRRC compensation with different rotation angles and centers. The minimum entropy is achieved at the position (18.3°, 18.5, 6.341), where 18.3° and 18.5 are the rotation angle and center, respectively. They are very close to the estimated values by the iterative search.

Figure 7 shows the operation time and entropy of ISAR image after MTCRRC compensation along the initial angle for iterative search. The figure indicates that the proposed algorithm is robust to different initial angle $\Delta\theta$.

The ISAR images after MTCRRC compensation with SNR of 0 dB and -5 dB are shown in Figures 8(a) and (b), respectively. The Gauss noise was added to the signals. The compensated ISAR images were well focused. It is indicated that the proposed algorithm is robust to noise.

For comparison, methods presented in [6, 9] were used to compensate the simulated data with SNR of 0 dB. The methods presented in [6, 9] are parametric algorithms. The MTCRRC compensated results

are shown in Figure 9. From the results in Figure 9, it can be seen that the ISAR images after MTCRRC by methods proposed in [6, 9] is less focused than the image in Figure 8(a). That indicates that the noise in signals have influence to the accuracy of the parametric algorithms indeed. Meanwhile, the process time of the two methods are 131 seconds and 109 seconds, respectively.

Table 1 presents the comparison of computational load between the proposed method and the algorithm in [9]. The computational load can be expressed in terms of number of addition and multiplication. In the table, M denotes the number of HRRPs and N the number of range bins in HRRP. For the algorithm in [9], the major cost is in calculating the integrated cubic phase function (ICPF). Therefore, we consider only the computation load of ICPF. N_{RA} denotes the sampling number of the rotation angle, usually hundreds. The main processes in the proposed method are entropy

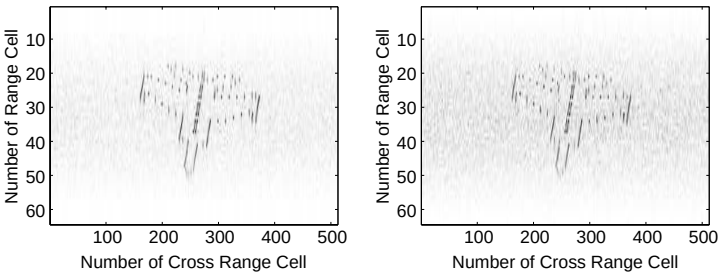

Figure 8. ISAR images after MTCRRC compensation by proposed method with SNR of 0 dB and −5 dB. (a) SNR of 0 dB. (b) SNR of −5 dB.

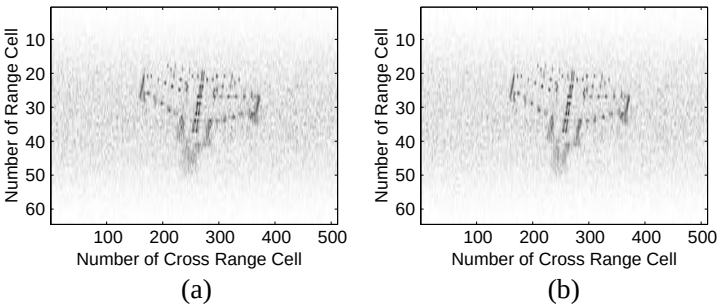

Figure 9. Results after MTCRRC compensation of simulated data by methods proposed in Refs. [6, 9] with SNR of 0 dB. (a) Method proposed in Ref. [6]. (b) Method proposed in Ref. [9].

Table 1. The computational load comparison between the proposed method and the algorithm in Ref. [9].

	addition number	multiplication number	total number	time (s)
method in [9]	ICPF: $\frac{M}{2} \cdot (M+1) \cdot N \cdot N_{RA}$	ICPF: $2M \cdot (M+1) \cdot N \cdot N_{RA}$	$\frac{5M}{2} \cdot (M+1) \cdot N \cdot N_{RA}$	109
proposed method	entropy: $M \cdot N \cdot N_{IS}$ + FFT: $M \cdot \log_2 M \cdot N \cdot N_{IS}$ ⇓ $M \cdot (N + \log_2 M) \cdot N \cdot N_{IS}$	compensation: $M \cdot N \cdot N_{IS}$ + FFT: $M/2 \cdot \log_2 M \cdot N \cdot N_{IS}$ ⇓ $M \cdot \left(N + \frac{\log_2 M}{2}\right) \cdot N \cdot N_{IS}$	$M \cdot \frac{(4N+3\log_2 M)}{4} \cdot N \cdot N_{IS}$	3.19
comparison & experimental results	$\frac{(M+1) \cdot N_{RA}}{2(N+\log_2 M) \cdot N_{IS}}$ 11.49	$\frac{4(M+1) \cdot N_{RA}}{(2N+\log_2 M) \cdot N_{IS}}$ 54.54	$\frac{5/2 \cdot (M+1) \cdot N_{RA}}{(2N+3(\log_2 M)/2) \cdot N_{IS}}$ 31.19	34.17

calculating, fast Fourier transform (FFT) and rotational compensation. As shown in Table 1, N_{IS} denotes the times of iteration search. Take the data used in the experiment 1 for example, the data dimensions were $M = 512$ and $N = y_2 - y_1 + 1 = 35$. The rotation sampling number N_{RA} was 400, increasing 16° to 20° by step of 0.01. The iteration search times $N_{IS} = 175$. Finally, we can get the cost comparison as follow

$$\frac{(M+1)\cdot N_{RA}}{2\,(N+\log_2 M)\cdot N_{IS}} = 11.49, \quad \frac{4\,(M+1)\cdot N_{RA}}{(2N+\log_2 M)\cdot N_{IS}} = 54.54 \tag{11}$$

Through test, the time cost of multiplication is nearly the same with addition in Matlab 2010a. Therefore, the full cost comparison can be expressed as

$$\frac{5/2\cdot(M+1)\cdot N_{RA}}{(2N+3\,(\log_2 M)/2)\cdot N_{IS}} = 31.19 \tag{12}$$

Formula (12) and Table 1 indicate that the mathematical analysis agrees with the process time results, as well as confirms that the proposed method is much more efficient than that in [9].

In the second experiment, the real data provided by Science and Technology on Automatic Target Recognition Laboratory (ATR) of China was used. The data were collected by ATR's experimental ground-based imaging radar. The target was a flying airplane of Boeing737. The carrier frequency was $f_c = 10\,\text{GHz}$, and the bandwidth was $B = 1\,\text{GHz}$. The initial values were set to $\Delta\theta = 5°$ and $[y_1, y_2] = [450, 750]$. The estimated rotation center and angle were $y_0 = 471.11$ and 8.03°, respectively. Figure 10(a) shows the obtained ISAR image based on RDA without any compensation. Figure 10(b) presents the ISAR image after MTRRC compensation. Figure 10(c) shows the output after MTCRRC compensation. The entropy of the images corresponding to Figures 10(a), (b) and (c) are 9.23, 8.89 and 8.37, respectively. A well focused ISAR image was obtained after rotational compensation. The CRS of the image result is shown in Figure 10(d).

The Boeing737 model is shown in Figure 11. The length of Boeing737 is 33.6 m and the wingspan 34.3 m. The estimated length and wingspan of the airplane from Figure 10(d) are 32.7 m and 35.2 m, respectively. These results confirm that the rotation angle was estimated accurately, and the proposed algorithm is effective for the real data.

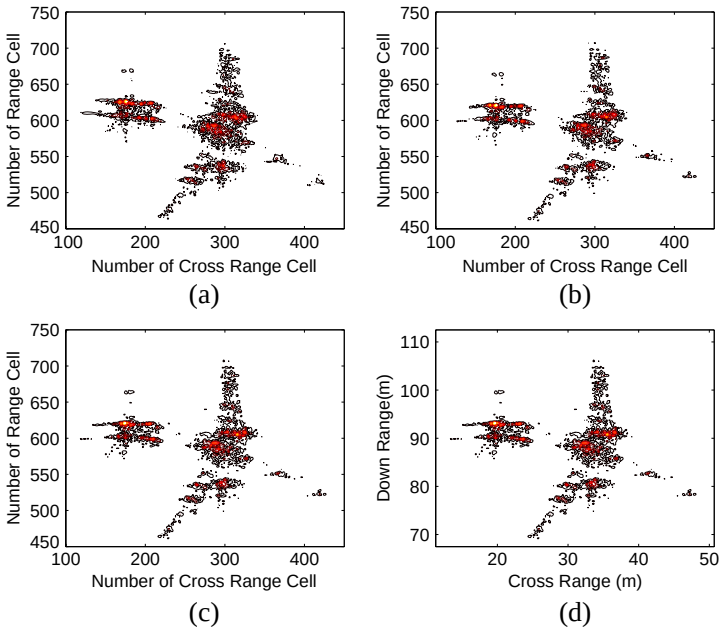

Figure 10. ISAR images of Boeing737. (a) ISAR image with MTRC. (b) ISAR image after MTRRC compensation. (c) ISAR image after MTCRRC compensation by proposed method. (d) ISAR image after CRS.

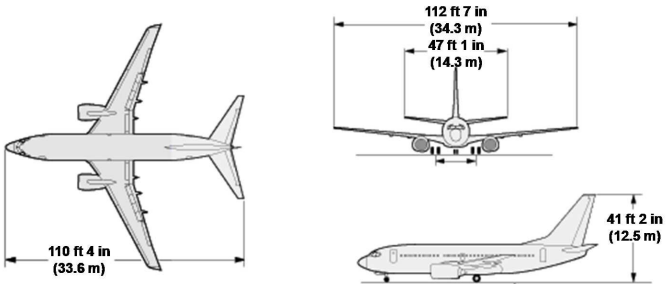

Figure 11. Three-view drawing of Boeing737.

6. CONCLUSION

The rotation angle and center is a precondition for the rotational motion compensation and cross-range scaling in ISAR imaging. This paper presents a novel nonparametric rotational motion compensation algorithm by iteratively utilizing GSS method based on minimizing the entropy of the compensated ISAR imagery. The effectiveness and robustness of the proposed algorithm are demonstrated by simulated and real ISAR images.

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REFERENCES

1. Munoz-Ferreras, J. M. and F. Pérez-Martínez, "Uniform rotational motion compensation for inverse synthetic aperture radar with non-cooperative targets," *IET Radar Sonar Navig.*, Vol. 2, No. 1, 25–34, Jan. 2008.
2. Liu, J. H., X. Li, S. K. Xu, and Z. W. Zhuang, "ISAR imaging of non-uniform rotation targets with limited pulse via compressed sensing," *Progress In Electromagnetics Research B*, Vol. 41, 285–305, 2012.
3. Xing, M., R. Wu, J. Lan, and Z. Bao, "Migration through resolution cell compensation in ISAR imaging," *IEEE Geosci. Remote Sens. Lett.*, Vol. 1, No. 2, 141–144, Apr. 2004.
4. Lu, G. and Z. Zhou, "Compensation of scatterer migration through resolution cell in inverse synthetic aperture radar imaging," *IEE Proc., Radar, Sonar Navig.*, Vol. 147, No. 2, 80–85, Apr. 2000.
5. Hu, J. M., W. Zhou, Y. W. Fu, L. Xiang, and J. Ning, "Uniform rotational motion compensation for ISAR based on phase cancellation," *IEEE Geosci. Remote Sens. Lett.*, Vol. 8, No. 4, 636–641, Jul. 2011.
6. Wang, Y. and Y. C. Jiang, "A novel algorithm for estimating the rotation angle in ISAR imaging," *IEEE Geosci. Remote Sens. Lett.*, Vol. 5, No. 4, 608–609, Oct. 2008.
7. Yeh, C. M., J. Xu, Y. N. Peng, and X. T. Wang, "Cross-range scaling for ISAR based on image rotation correlation," *IEEE Geosci. Remote Sens. Lett.*, Vol. 6, No. 3, 597–601, Jul. 2009.
8. Li, X., H. Gu, and G. S. Liu, "A method for estimating the rotation angle of the ISAR image," *Acta Electron. Sin.*, Vol. 28, No. 6, 44–47, Jun. 2000.
9. Zhang, W. C., Z. P. Chen, and B. Yuan, "Rotational motion compensation for wide-angle ISAR imaging based on integrated cubic phase function," *IET International Radar Conference 2013*, 14–16, Apr. 2013.

10. Tsai, C. H., J. Kolibal, and M. Li, "The golden section search algorithm for finding a good shape parameter for meshless collocation methods," *Engineering Analysis with Boundary Elements*, Vol. 34, No. 8, 738–746, Aug. 2010.
11. Chang, Y. C., "N-dimension golden section search: Its variants and limitations," *2nd International Conference on Biomedical Engineering and Informatics, BMEI' 09*, 17–19, Oct. 2009.
12. Mensa, D. L., *High Resolution Radar Imaging*, Artech House, MA, 1981.
13. Wang, J., X. Liu, and Z. Zhou, "Minimum-entropy phase adjustment for ISAR," *IEE Proc. Radar Sonar Navig.*, Vol. 151, No. 4, 203–209, Aug. 2004.
14. Pincus, S. M., "Approximate entropy as a measure of system complexity," *Proc. Natl. Acad. Sci. USA Mathematics*, Vol. 88, No. 4, 2297–2301, Mar. 1991.
15. Kim, K. T. and H. T. Kim "One-dimensional scattering centre extraction for efficient radar target classification," *IEE Proc. Radar Sonar Navig.*, Vol. 146, No. 3, 147–158, Jun. 1999.