

Generalization Propagator Method for DOA Estimation

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Abstract—A generalization propagator method (GPM) is presented. It is the extension of traditional propagator method (PM). In order to make full use of the received data, many propagators are structured according to different block structures of array manifold. By these propagators, a high order matrix is obtained in a symmetric mode, and it is orthogonal with array manifold. Based on this matrix, a generalization spectral function is obtained to solve the problem of direction-of-arrival (DOA) estimation by spectral peak searching. Moreover, in order to avoid spectral peak searching, a generalization root-propagator method (GRPM) also is proposed, and shows excellent estimation precision. Numerical simulations demonstrate the performance of the proposed method.

1. INTRODUCTION

DOA estimation by sensor arrays has been an active research area for decades. It plays an important role in many applications [1, 2] such as acoustic source localization, radar imaging, mobile communication and so on.

Most of the DOA estimation methods have been proposed by scholars and performed well in super resolution and DOA estimation precision. The subspace-based methods like MUSIC [3, 4], root-MUSIC [5], ESPRIT [6–9], and stochastic assumption-based methods like Maximum Likelihood (ML) [10, 11] are the most successful methods of numerous DOA estimation techniques. All these methods base eigenvalue decomposition (EVD) of the cross-correlation matrix or singular value decomposition (SVD) of the received data. By these techniques, the computational complexity is costly and high, especially when the number of sources and sensors are large.

It is known that computational complexity of PM [12] is smaller than those of MUSIC and ESPRIT, because the PM does not require any EVD of the cross-correlation matrix and SVD of the received data. Hence it has been widely used for DOA estimation in recent years. In [13, 14], PM was applied into the L shaped array, and the two-dimensional PM based on L shaped array has been proposed. In [15], authors employed the PM to cylindrical conformal array and presented a fast estimation of frequency and two-dimensional DOA for cylindrical conformal array antenna. In [16], the two-dimensional DOA estimation of distributed sources based on PM has been proposed, and shown good estimation effect.

In this paper, by the different block structures of array manifold, we get the GPM. And it has excellent estimation performance even for the scenario of small snapshots and low SNR. We can't obtain the desired estimation effect by root-propagator method (RPM) which is extracted directly from PM as [5], so we get GRPM from GPM. Although this algorithm need to calculate more than one propagators, it has much better accuracy than RPM. Compared with ESPRIT, root-MUSIC, GRPM has the advantage: can maintain a good estimation performance without EVD.

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2. MATHEMATICAL FORMULATION

Consider a uniform linear array (ULA) constituted by M sensors like Fig. 1. The first sensor is common for referencing purpose. There are K ($K < M$) far-field uncorrelated narrowband sources impinging on the array from distinct directions $\theta_1, \theta_2, \dots, \theta_K$. The array output can be presented as

$$\mathbf{X}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{S}(t) + \mathbf{N}(t), \quad t = 1, 2, \dots, T \quad (1)$$

where $\mathbf{X}(t) = [x_1(t), \dots, x_M(t)]^T$, $\mathbf{S}(t) = [s_1(t), \dots, s_K(t)]^T$, $\mathbf{N}(t) = [n_1(t), \dots, n_M(t)]^T$ are the observed vectors, the incident signals and the additive noise, respectively. $\mathbf{S}(t)$ is modeled as Gaussian process, and $\mathbf{N}(t)$ is white Gaussian process with mean zero and covariance $\delta_n^2 \mathbf{I}_M$. Moreover, the noise is uncorrelated with the signal.

$\mathbf{A}(\boldsymbol{\theta})$ is the array manifold which can be presented as

$$\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_K)]^T \quad (2)$$

$$\mathbf{a}(\theta_k) = \left[1, \exp\left(-i\frac{2\pi d \sin(\theta_k)}{\lambda}\right), \dots, \exp\left(-i\frac{2\pi d(M-1) \sin(\theta_k)}{\lambda}\right) \right]^T \quad k = 1, 2, \dots, K \quad (3)$$

where $\boldsymbol{\theta} = (\theta_1, \theta_2, \dots, \theta_K)$, λ is the wavelength of signals, and d ($d \leq \frac{\lambda}{2}$) is the distance between adjacent sensor.

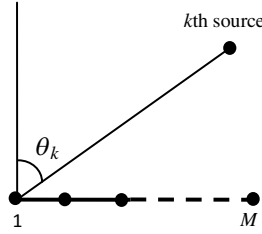


Figure 1. Array configuration.

The covariance matrix of $\mathbf{X}(t)$ is expressed as

$$\mathbf{R} = E[\mathbf{X}(t)\mathbf{X}(t)^H] = \mathbf{A}(\boldsymbol{\theta})\mathbf{R}_s\mathbf{A}^H(\boldsymbol{\theta}) + \delta_n^2 \mathbf{I}_M \quad (4)$$

where $\mathbf{R}_s = E[\mathbf{S}(t)\mathbf{S}^H(t)]$, $E[\cdot]$ stands for the mathematical expectation and $[\cdot]^H$ represents conjugate transpose.

3. GENERALIZATION PROPAGATOR METHOD

We decompose the array manifold $\mathbf{A}(\boldsymbol{\theta})$ as

$$\mathbf{A}(\boldsymbol{\theta}) = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_0 \\ \mathbf{A}_2 \end{bmatrix} \quad (5)$$

where $\mathbf{A}_1 \in C^{L \times K}$, $\mathbf{A}_0 \in C^{K \times K}$, $\mathbf{A}_2 \in C^{(M-K-L) \times K}$, $L = 0, \dots, M - K$.

It's easy for us to know that \mathbf{A}_0 is a nonsingular matrix, so there must exist two matrices (except $L = 0$, $L = M - K$) $\mathbf{P}_{1L}^H \in C^{L \times K}$, $\mathbf{P}_{2L}^H \in C^{(M-K-L) \times K}$ called as propagator. And they meet that

$$\mathbf{A}_1 = \mathbf{P}_{1L}^H \mathbf{A}_0 \quad (6)$$

$$\mathbf{A}_2 = \mathbf{P}_{2L}^H \mathbf{A}_0 \quad (7)$$

We can define a new block matrix $\mathbf{Q}_L^H \in C^{(M-K) \times M}$ as

$$\mathbf{Q}_L^H = \begin{bmatrix} -\mathbf{I}_L & \mathbf{P}_{1L}^H & 0 \\ 0 & \mathbf{P}_{2L}^H & -\mathbf{I}_{M-K-L} \end{bmatrix} \quad (8)$$

where \mathbf{I}_L , $-\mathbf{I}_{M-K-L}$ are the $L \times L$ and $(M-K-L) \times (M-K-L)$ identity matrix respectively. For arbitrary $L = 0, \dots, M-K$, from (6), (7), (8) we can get

$$\mathbf{Q}_L^H \mathbf{A} = 0 \quad (9)$$

Then, let $\mathbf{X}(t)$ divided as

$$\mathbf{X}(t) = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_0 \\ \mathbf{X}_2 \end{bmatrix} \quad (10)$$

On the assumption that there is no noise, from (1), (6), (7), (10) we have

$$\begin{cases} \mathbf{X}_1 = \mathbf{A}_1 \mathbf{S}(t) = \mathbf{P}_{1L}^H \mathbf{A}_0 \mathbf{S}(t) = \mathbf{P}_{1L}^H \mathbf{X}_0 \\ \mathbf{X}_2 = \mathbf{A}_2 \mathbf{S}(t) = \mathbf{P}_{2L}^H \mathbf{A}_0 \mathbf{S}(t) = \mathbf{P}_{2L}^H \mathbf{X}_0 \end{cases} \quad (11)$$

We decompose the covariance matrix \mathbf{R} as

$$\mathbf{R} = [\mathbf{G}_L \quad \mathbf{H}_L \quad \mathbf{T}_L] = E \left(\begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_0 \\ \mathbf{X}_2 \end{bmatrix} \begin{bmatrix} \mathbf{X}_1^H & \mathbf{X}_0^H & \mathbf{X}_2^H \end{bmatrix} \right) \quad (12)$$

where $\mathbf{G}_L \in C^{M \times L}$, $\mathbf{H}_L \in C^{M \times K}$, $\mathbf{T}_L \in C^{M \times (M-K-L)}$.

Substituting (11) into (12) yields

$$\mathbf{G}_L = E \left(\begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_0 \\ \mathbf{X}_2 \end{bmatrix} \mathbf{X}_1^H \right) = E \left(\begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_0 \\ \mathbf{X}_2 \end{bmatrix} \mathbf{X}_0^H \right) \mathbf{P}_{1L} = \mathbf{H}_L \mathbf{P}_{1L} \quad (13)$$

$$\mathbf{T}_L = E \left(\begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_0 \\ \mathbf{X}_2 \end{bmatrix} \mathbf{X}_2^H \right) = E \left(\begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_0 \\ \mathbf{X}_2 \end{bmatrix} \mathbf{X}_0^H \right) \mathbf{P}_{2L} = \mathbf{H}_L \mathbf{P}_{2L} \quad (14)$$

As [12], according to (13) and (14), we can get that the estimation of the \mathbf{P}_{1L}^H , \mathbf{P}_{2L}^H is equivalent to the solution of the optimization problems

$$\min_{\mathbf{P}_{1L}} \|\mathbf{G}_L - \mathbf{H}_L \mathbf{P}_{1L}\|_F^2 \quad (15)$$

$$\min_{\mathbf{P}_{2L}} \|\mathbf{T}_L - \mathbf{H}_L \mathbf{P}_{2L}\|_F^2 \quad (16)$$

The solution of (15) and (16) can be easily obtained as

$$\mathbf{P}_{1L} = (\mathbf{H}_L^H \mathbf{H}_L)^{-1} \mathbf{H}_L^H \mathbf{G}_L \quad (17)$$

$$\mathbf{P}_{2L} = (\mathbf{H}_L^H \mathbf{H}_L)^{-1} \mathbf{H}_L^H \mathbf{T}_L \quad (18)$$

Because of $\mathbf{Q}_L^H \mathbf{A} = 0$ we can know $\mathbf{Q}_L^H \mathbf{a}(\theta_k) = 0$, $k = 1, \dots, K$. And PM can be described as the process that research for the spectral peak of the spectral function:

$$f_{PML}(\boldsymbol{\theta}) = \frac{1}{\mathbf{a}^H(\boldsymbol{\theta}) \mathbf{Q}_L \mathbf{Q}_L^H \mathbf{a}(\boldsymbol{\theta})} \quad (19)$$

where $L = 0$ corresponds to the traditional propagator method [12].

In order to make full use of the received data, we reconstruct a new spectral function:

$$f_{PM}(\theta) = \frac{1}{\mathbf{a}^H(\theta) \mathbf{Q} \mathbf{Q}^H \mathbf{a}(\theta)} \quad (20)$$

where $\mathbf{Q} = [\mathbf{Q}_0, \dots, \mathbf{Q}_{j-1}, \mathbf{Q}_{M-K-j+1}, \dots, \mathbf{Q}_{M-K}]^T$, $1 \leq j \leq \lfloor \frac{M-K+1}{2} \rfloor$ is an adjustable value according to the actual need. On the condition of large snapshots or high SNR, the advantage of this method is not obvious. But, this algorithm is much better than PM when small snapshots and low SNR.

4. GENERALIZED ROOTING PROPAGATOR ALGORITHM

Being similar with [5], the spectral peak searching for $f_{PM}(\boldsymbol{\theta})$ is equal to the searching for the solutions on unit circle of the equation:

$$z^{M-1} p^H(z^{-1}) \mathbf{Q} \mathbf{Q}^H p(z) = 0 \quad (21)$$

where $p(z) = [1, z, \dots, z^{M-1}]^T$.

Find K roots closest to unit circle and denote them as z_1, \dots, z_K . We can say that

$$z_k \approx \exp\left(-i \frac{2\pi d \sin(\theta_k)}{\lambda}\right) \quad k = 1, \dots, K \quad (22)$$

From (22) the estimation of θ_k can be obtained as:

$$\hat{\theta}_k = \arcsin\left(\frac{-\lambda \text{angle}(z_k)}{2\pi d}\right) \quad (23)$$

For PM, the use of received data is insufficiency, so RPM can't have desired estimation result. Hence, we use \mathbf{Q} instead of \mathbf{Q}_L .

In theory, the larger the j is, the more precise the estimation is. But when j is close to the maximum, the effect of estimation will be very close, which can be demonstrated in Experiment 1. Another fact we must admit is that the calculation will increase with the increase of j . Hence we tend to take a value of j being close to the maximum but not equal to the maximum.

5. SIMULATIONS EXPERIMENT

In this section, we carry out simulation to demonstrate the performance of the proposed method. An ULA with $M = 16$ and $d = \frac{\lambda}{2}$ receives three uncorrelated equal-power signals with additive white Gaussian noise.

Let $\boldsymbol{\theta} = [5^\circ \quad 25^\circ \quad 35^\circ]$ in Experiment 1 3 4, and $\boldsymbol{\theta} = [-10^\circ \quad 5^\circ \quad 15^\circ]$ in Experiment 2. For Experiment 2 3 4, we take $j = 6$, which is reasonable via the simulation results of Experiment 1. The

root mean square error (RMSE) of estimation is defined as $RMSE = \sqrt{\frac{1}{KP} \sum_{p=1}^P \sum_{k=1}^K (\hat{\theta}_{kp} - \theta_k)^2}$, where

$P = 500$ is the number of independent experiment, $\hat{\theta}_{kp}$ is the angle estimation of the p th experiment for the k th signal.

5.1. Experiment 1

In this experiment, we take GRPM for example to compare the performance of different value of j . First, we fix the number of snapshots at 200, and compare the estimation performance of different j varying with SNR. Second, we fix the SNR at 10 dB, and compare the estimation performance of different j varying with the number of snapshots. From Figs. 2(a), (b) we can see that the estimation accuracy of $j = 7$ is the best. But the estimation accuracy of $j = 5$ and $j = 6$ are very close to $j = 7$. Take the computational complexity into account, we think that the best choice of j is closed to the maximum but not equal to the maximum. So we choice $j = 6$ for next three experiments.

5.2. Experiment 2

For the scenario of high SNR or large snapshots, both PM and GPM have excellent estimation effect. So we only consider the case of small snapshots and low SNR in this experiment. Fix the snapshots at 10, and SNR is given at 0 dB, -5 dB, -10 dB. From Fig. 3(a) we can see that when the SNR is 0 dB, the three highest peaks of PM are not obvious, and a false peak at -60° is close to the right peak at -10° and 5° . For GPM the three spectrum peaks are obvious and point to true DOA accurately. From Figs. 3(b), (c), we can find that when the SNR is less than 0 dB, the spectral peak searching picture of PM has seriously degraded. Lots of false peaks have appeared, furthermore, some of false peaks were higher than right peaks. For GPM although there are some false peaks, three highest peaks are still higher than the other peaks, and show good stability.

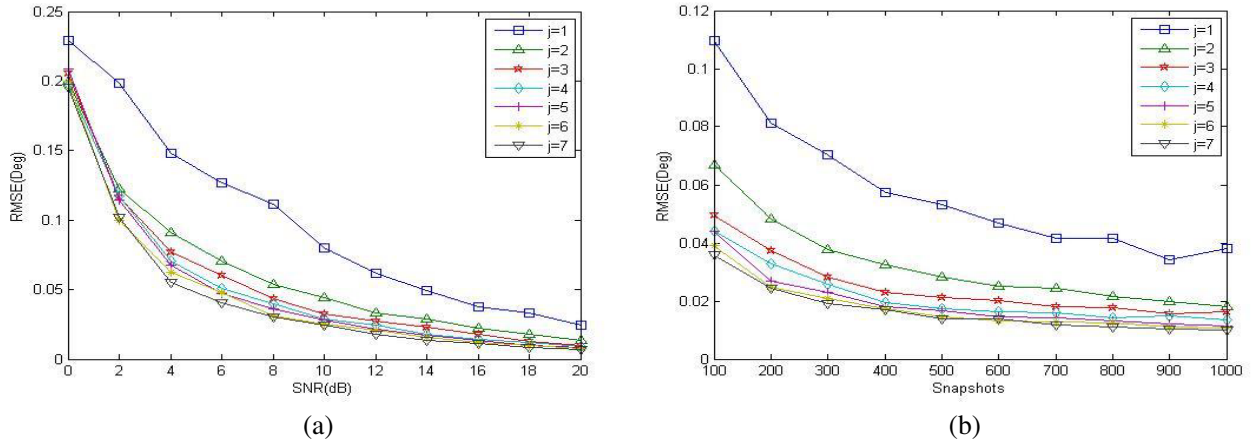


Figure 2. Performance comparison of GRPM for different j ((a) Snapshots = 200, (b) $SNR = 10$ dB).

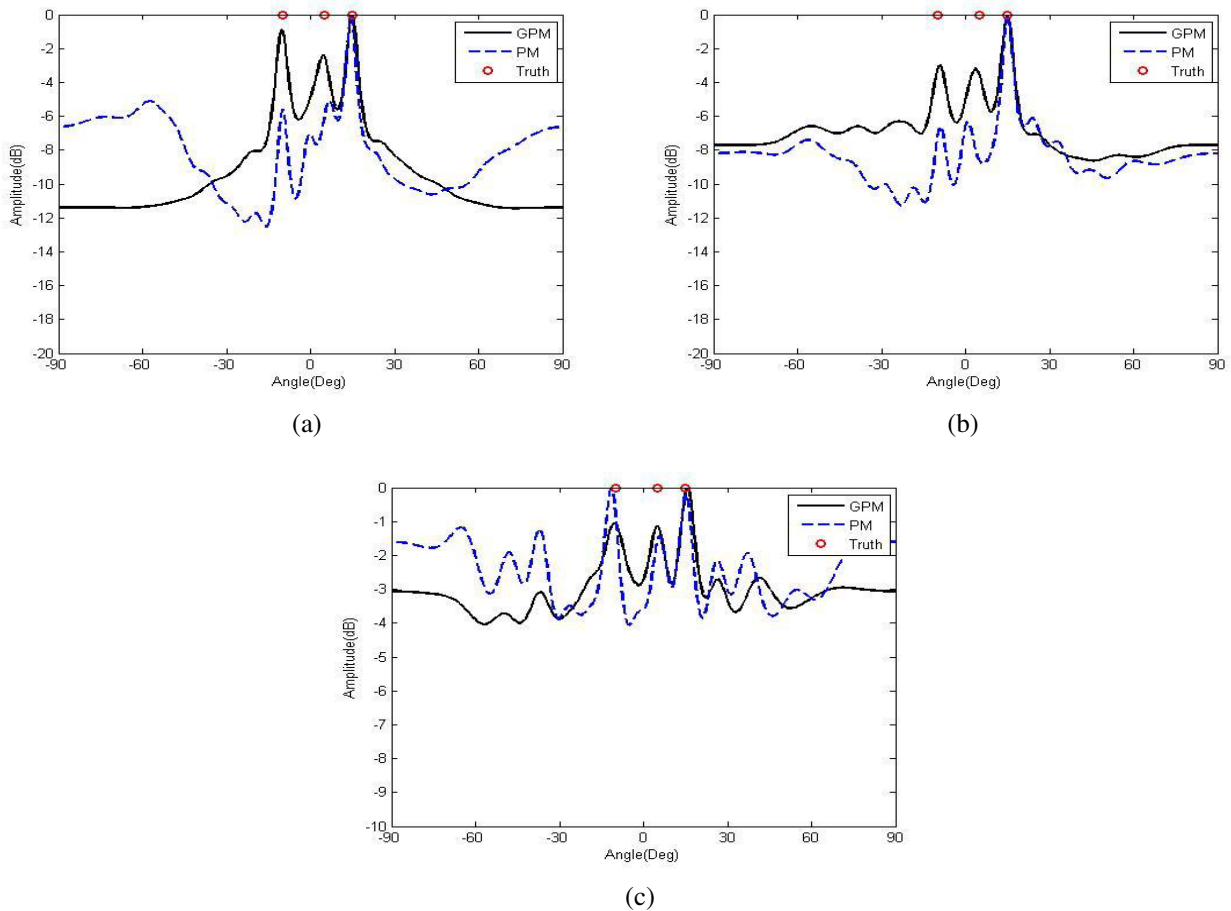


Figure 3. Spectral peak searching picture (Snapshots = 10; (a) $SNR = 0$ dB, (b) $SNR = -5$ dB, (c) $SNR = -10$ dB).

5.3. Experiment 3

The number of snapshots is fixed at 200 with SNR changing from 0 dB to 20 dB. Compare the estimation performance of root-MUSIC algorithm, ESPRIT and RPM varying with SNR, and describe

the estimation accuracy with RMSE. From Fig. 4, we can see that the estimation effect of GRPM algorithm is much better than RPM and ESPRIT in the different SNR, especially at lower than 10 dB. Compared with RPM, the results show again the advantages of proposed algorithm under low SNR. Meanwhile we can also find that the performance of GRPM is very close to root-MUSIC especially at higher than 8 dB.

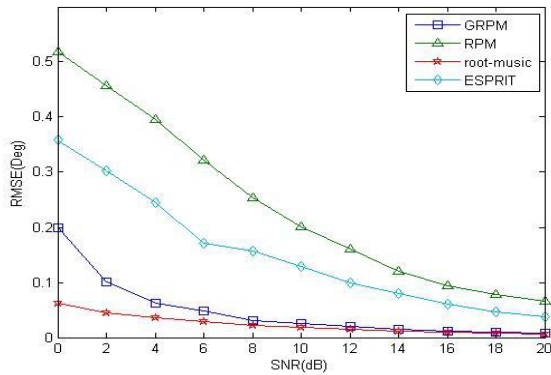


Figure 4. The estimation RMSE versus SNR (Snapshots = 200).

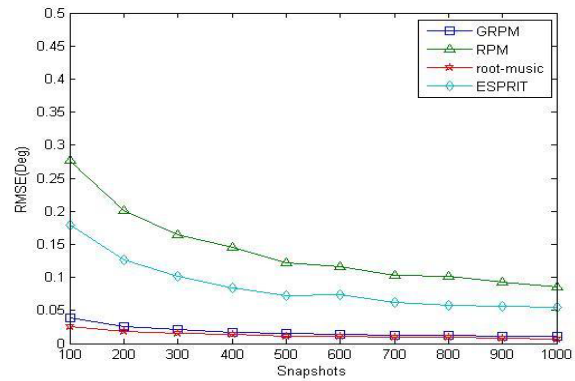


Figure 5. The estimation RMSE versus snapshots (SNR = 10 dB).

5.4. Experiment 4

The SNR is fixed at 10 dB with the snapshots changing from 100 to 1000. From Fig. 5 we can find that under different snapshots the RMSE of GRPM and root-MUSIC is smaller than 0.05, but more than 0.1 for RPM and 0.08 for ESPRIT. Obviously, the estimation effect of GRPM is better than RPM and ESPRIT. And the estimation accuracy of GRPM is nearly the same as root-MUSIC in this case.

6. SUMMARY AND DISCUSSION

In this paper, a GPM is proposed to estimate the DOA of narrow-band incident signals in uniform linear array. The method is an expansion of traditional PM. It uses the different block structures of array manifold to get a new spectral function without EVD, by which the array received data can be utilized more effectually. Simulation results show that GPM is much better than PM at low SNR and snapshots. Corresponding GRPM is also obtained from GPM like root-MUSIC. Its performance is very close to root-MUSIC, but much better than RPM and ESPRIT. So this study can provide a good reference for associated problem estimation of DOA.

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