Machine Copy for Proofreading, Vol. x, y–z, 2014

Two Uniform Linear Arrays for Non-coherent and Coherent Sources for Two Dimensional Source Localization

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Abstract—This paper presents a novel method for the two-dimensional direction of arrival (DOA) estimation based on QR decomposition. Two uniform linear antenna array (ULA) configuration is employed for the joint estimation of elevation (θ) and azimuth (ϕ) angles. **Q** data matrix will estimate the azimuth angle while **R** data matrix will estimate the elevation angle. The proposed method utilizes only a single snapshot of the received data and constructs a Toeplitz data matrix. This reduces the computational complexity of the proposed method to $O((N+1)^2)$ from $O(N^3)$ for SVD based methods. The structure of the data matrix also favors the 2D DOA estimation for both coherent and non-coherent source signals. Simulation results are presented and performance of the proposed method is compared with the Matrix Pencil method for 2D DOA estimation of multiple incident source signals.

1. INTRODUCTION

Direction of arrival (DOA) estimation in two-dimensions (i.e., azimuth, and elevation (ϕ, θ)) has assumed great importance in array signal processing in both commercial and military applications. Several 2D DOA estimation techniques exist such as the classic subspace techniques MUSIC and ESPRIT [1, 2] which are considered to be the most popular algorithms for the DOA estimation. However, these and other similar techniques have several drawbacks in terms of estimation accuracy and computational complexity. While there is a severe degradation in the accuracy of DOA estimation in the presence of partially correlated sources, these methods fail when highly correlated and coherent signals are present. A spatial smoothing technique was introduced in [3, 4] to improve the system performance and estimation accuracy. However, one side effect of this spatial smoothing technique is the increase in the computational complexity of the algorithm. In addition, these algorithms use the time averaging of the covariance matrix based on a large number of snapshots (e.g., hundreds of snapshots per trial) in order to obtain an acceptable performance resulting in further increase in the system complexity. Furthermore, the DOA of the signals must not change until all the input data is received to estimate the covariance matrix.

Recently, several interesting techniques have been developed to resolve the 2D DOA estimation of multiple incoming incident sources [5–14]. However, most of these techniques are only applicable when the signals are uncorrelated. In addition, multiple snapshots are needed to estimate the covariance matrix. In order to overcome the limitation of these algorithms especially when highly correlated and coherent signals are involved, a number of new techniques have been developed [14–16]; yet, these techniques still require high computational complexity due to employing spatial smoothing techniques and a large number of snapshots.

In this paper, we propose a method based on QR decomposition that utilizes only a single snapshot of the received data and constructs a Toeplitz data matrix [17]. The elevation (θ) is estimated from the Q matrix and the azimuth (ϕ) from the R matrix. The proposed method has several advantages

Received 19 May 2014
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over the methods in [5–16]. First, the proposed method does not require spatial smoothing [3, 4] which results in a reduction in the computational complexity and cost. Second, the proposed method can estimate the DOA of coherent sources at very low signal-to-noise ratios (SNR). Third, a new antenna array configuration consisting of two uniform linear arrays is proposed to estimate the 2D DOA estimation for both coherent and highly correlated sources, whereas most of the existing antenna array configurations that have been proposed estimate the 2D DOA only for non-coherent sources [10–14]. Fourth, the proposed method utilizes only a single snapshot of the received data to estimate the DOA for incident sources. This significantly reduces the computational load and makes the proposed method a superior candidate for real-time implementation, whereas the methods in [5–16] require a large number of snapshots (e.g., 100 to 200) in order to produce a quality estimate. Fifth, computing the QR factorization for the data matrix using Toeplitz structure [17] in the proposed method requires only $O((N+1)^2)$ operations while calculating regular data matrix with dimension $(N \times N)$ using the singular value decomposition (SVD) of the data matrix or the Eigen value decomposition (EVD) of the covariance matrix which have been heavily used by many existing schemes requires $O(N^3)$ operations [18].

The rest of this paper is organized as follows: in Section 2 we describe the system model. In Section 3 we present the performance of the proposed method and simulations results for multiple sources lying in the far-field region of our proposed array configuration of two ULAs. Finally, conclusions are drawn in Section 4.

2. SYSTEM MODEL

The proposed array configuration is shown in Figure 1 which consists of two uniform linear arrays (ULAs) with interspacing d equal to a half wavelength of incident coherent signals. Each uniform linear array in Figure 1 consists of $2N+1$ elements. One array is placed in the y-axis and another one in the y-z plane. We consider the element which is located at point (0, 0, 0) as a reference element. In the proposed method, we construct $(N+1) \times 1$ sub-arrays, each consisting of $N \times 1$ antennas. The $(N+1) \times 1$ from single snapshot data vector is obtained from the y-axis antenna elements $2N+1$. We consider K narrowband sources for coherent and non-coherent cases impinging on the antenna array with $(\theta_k,\ \phi_k)$, where θ_k and ϕ_k are elevation and azimuth angle for of the kth source, respectively.

The received data from the elements on the y-axis for the first sub-array $\mathbf{Y}_1$ at time t can be written as

$$\mathbf{Y}_1(t) = [y_0(t) \quad y_1(t) \quad \dots \quad y_N(t)]^T = \mathbf{A}(\theta, \phi)\,\mathbf{S} + \mathbf{n}_1(t), \tag{1}$$

where $\mathbf{S}(t) = [s_1(t) \quad s_2(t) \quad \dots \quad s_K(t)]^T$ is a signal vector for the K sources, $\mathbf{n}_1$ is the $(N+1) \times 1$ additive white Gaussian noise (AWGN) vectors whose elements has zero mean and variance σ^2, T represents

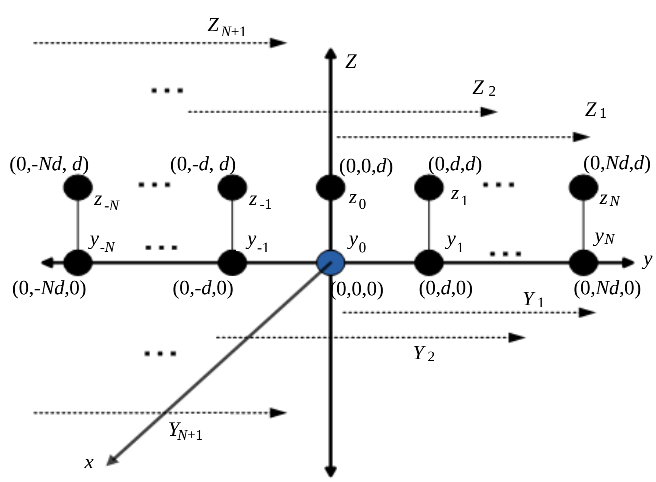


Figure 1. Proposed array configuration used for the joint elevation and azimuth $(\theta,\ \phi)$ DOA estimation.

transpose function, $\mathbf{A}(\theta,\phi) = [\mathbf{a}(\theta_1,\phi_1) \quad \mathbf{a}(\theta_2,\phi_2) \quad \ldots \quad \mathbf{a}(\theta_K,\phi_K)]$ is the collection of array response matrix, $\mathbf{a}(\theta_k,\phi_k) = \begin{bmatrix}1 & u_k & \ldots & u_k^{N-1}\end{bmatrix}^T$ where $u_k = \exp(-j\frac{2\pi d \sin\theta_k \sin\phi_k}{\lambda})$, λ is the wavelength of the signal, and d is the array element spacing. The snapshot time-index t will be dropped since the proposed algorithm uses a single snapshot. The elements of the snapshot vector from the second sub-array $\mathbf{Y}_2$ can be written as

$$\mathbf{Y}_2 = [y_{-1} \quad y_0 \quad \ldots \quad y_{N-1}]^T = \mathbf{A}(\theta,\phi)\,\mathbf{\Phi}_1^*\mathbf{S} + \mathbf{n}_2, \tag{2}$$

where,

$$\mathbf{\Phi}_1^*(\theta,\phi) = \operatorname{diag}(u_1^* \quad u_2^* \quad \ldots \quad u_K^*), \tag{3}$$

where $\mathbf{\Phi}_1^*(\theta,\phi)$ is a $K\times K$ diagonal matrix whose diagonal elements are u_k, $k = 1, \ldots, K$. The output observation from the $N+1$ sub-array $\mathbf{Y}_{N+1}$ can be given as

$$\mathbf{Y}_{N+1} = \begin{bmatrix}y_{-N} & y_{-(N-1)} & \cdots & y_0\end{bmatrix}^T = \mathbf{A}(\theta,\phi)(\mathbf{\Phi}_\mathbf{1}^*)^N\mathbf{S} + \mathbf{n}_{N+1} \tag{4}$$

With the single snapshot the collected data is formulated from all sub-arrays as

$$\mathbf{YY} = [\mathbf{Y}_1 \quad \mathbf{Y}_2 \quad \ldots \quad \mathbf{Y}_{N+1}] = \begin{bmatrix} y_0 & y_{-1} & y_{-2} & \cdots & y_{-N} \\ y_1 & y_0 & y_{-1} & \cdots & y_{-(N-1)} \\ y_2 & y_1 & y_0 & \cdots & y_{-(N-2)} \\ \ldots & \ldots & \ldots & \ddots & \vdots \\ y_N & y_{N-1} & y_{N-2} & \cdots & y_0 \end{bmatrix} \tag{5}$$

The data matrix $\mathbf{YY}$ with a dimension of $(N+1)(N+1)$ has a Hermitian Toeplitz structure. The advantage of the Toeplitz structure is the ability to estimate DOAs from up to N coherent or non-coherent sources because the incident sources become de-correlated and the rank of $\mathbf{Y}$ can be higher than that of the covariance matrix of the signal vector.

The data matrix $\mathbf{YY}$ can be rewritten as follows

$$\mathbf{YY}=\Big[\mathbf{A}(\theta,\phi)\,\mathbf{S} \quad \mathbf{A}(\theta,\phi)\,\mathbf{\Phi}_1^*\mathbf{S} \quad \mathbf{A}(\theta,\phi)(\mathbf{\Phi}_1^*)^2\mathbf{S} \quad \ldots \quad \mathbf{A}(\theta,\phi)\left(\mathbf{\Phi}_1^*\right)^N\mathbf{S}\Big]+[\mathbf{n}_1 \quad \mathbf{n}_2 \quad \mathbf{n}_3 \quad \ldots \quad \mathbf{n}_{N+1}] \tag{6}$$

The collected data from the elements along the y-axis will not be sufficient to estimate 2D DOA (both azimuth and elevation) angles for each source, so extra information needs to be collected to complete the estimation. The proposed method achieves that by employing the second ULA elements located in the y-z plane.

The observed data from the antenna elements in the y-z plane will be used to construct $(N+1)\times 1$ sub-arrays in the same manner as in $\mathbf{YY}$ data. The received signal vector from the subarray $\mathbf{Z}_1$ can be expressed as

$$\begin{aligned}
\mathbf{Z}_1(t) &= [z_0(t) \quad z_1(t) \quad \ldots \quad z_N(t)]^T = \mathbf{A}(\theta,\phi)\,\mathbf{\Phi}_2^*\mathbf{S}(t) + \mathbf{n}_{11}(t), && (7)\\
\mathbf{\Phi}_2^*(\theta) &= \operatorname{diag}[v_1 \quad v_2 \quad \ldots \quad v_k], && (8)\\
v_k &= \exp\left(-j\frac{2\pi d\cos\theta_k}{\lambda}\right), && (9)
\end{aligned}$$

The snapshot time-index t will be dropped again since proposed algorithm is using a single snapshot. Next, the elements of the snapshot vector $\mathbf{Z}_2$ from the second sub-array can be denoted as,

$$\mathbf{Z_2} = [z_{-1} \quad z_0 \quad \ldots \quad z_{N-1}]^T = \mathbf{A}(\theta,\phi)\mathbf{\Phi}_2^*\mathbf{\Phi}_1^*\mathbf{S}(t) + \mathbf{n}_{22}(t) \tag{10}$$

Finally, the output observation from the $N+1$ sub-array $\mathbf{Z}_{N+1}$ can be written as,

$$\mathbf{Z}_{N+1} = \begin{bmatrix}z_{-N} & z_{-(N-1)} & \cdots & z_0\end{bmatrix}^T = \mathbf{A}(\theta,\phi)\,\mathbf{\Phi}_2^*(\mathbf{\Phi}_1^*)^N\mathbf{S} + \mathbf{n}_{(N+1)(N+1)} \tag{11}$$

For a single snapshot, the received data $\mathbf{ZZ}$ from the antenna elements placed on the y-z plane for all subarrays can be formulated as

$$\mathbf{ZZ} = [\mathbf{Z}_1 \quad \mathbf{Z}_2 \quad \ldots \quad \mathbf{Z}_{N+1}] = \begin{bmatrix} z_0 & z_{-1} & z_{-2} & \cdots & z_{-N} \\ z_1 & z_0 & z_{-1} & \cdots & z_{-(N-1)} \\ z_2 & z_1 & z_0 & \cdots & z_{-(N-2)} \\ \ldots & \ldots & \ldots & \ddots & \vdots \\ z_N & z_{N-1} & z_{N-2} & \cdots & z_0 \end{bmatrix} \tag{12}$$

The observation data in Equation (12) can be written as follows:

$$\begin{aligned}\mathbf{ZZ} \; = & \; \left[\mathbf{A}\left(\theta,\phi\right)\boldsymbol{\Phi}_2^*\mathbf{S} \quad \mathbf{A}\left(\theta,\phi\right)\boldsymbol{\Phi}_2^*\boldsymbol{\Phi}_1^*\mathbf{S} \quad \mathbf{A}\left(\theta,\phi\right)\boldsymbol{\Phi}_2^*\left(\boldsymbol{\Phi}_1^*\right)^2\mathbf{S} \quad \ldots \quad \mathbf{A}\left(\theta,\phi\right)\boldsymbol{\Phi}_2^*\left(\boldsymbol{\Phi}_1^*\right)^N\mathbf{S}\right] \\ & + \left[\mathbf{n}_{11} \quad \mathbf{n}_{22} \quad \mathbf{n}_{33} \quad \ldots \quad \mathbf{n}_{(N+1)(N+1)}\right] \end{aligned} \tag{13}$$

The two observation matrices, $\mathbf{YY}$ and $\mathbf{ZZ}$, which are collected from the two ULAs along the y- axis and y-z plane can provide two estimations, $\hat{\theta}_k$ and $\hat{\phi}_k$. We define a new data matrix $\mathbf{W}$ with dimension $(2N+2)\times(N+1)$ as follow

$$\begin{aligned}\mathbf{W} \; = & \; \begin{bmatrix}\mathbf{YY}\\ \mathbf{ZZ}\end{bmatrix} = \begin{bmatrix}\mathbf{A}\left(\theta,\phi\right)\mathbf{S} & \mathbf{A}\left(\theta,\phi\right)\boldsymbol{\Phi}_1^*\mathbf{S} & \mathbf{A}\left(\theta,\phi\right)\left(\boldsymbol{\Phi}_1^*\right)^2\mathbf{S} & \ldots & \mathbf{A}\left(\theta,\phi\right)\left(\boldsymbol{\Phi}_1^*\right)^N\mathbf{S} \\ \mathbf{A}\left(\theta,\phi\right)\boldsymbol{\Phi}_2^*\mathbf{S} & \mathbf{A}\left(\theta,\phi\right)\boldsymbol{\Phi}_2^*\boldsymbol{\Phi}_1^*\mathbf{S} & \mathbf{A}\left(\theta,\phi\right)\boldsymbol{\Phi}_2^*\left(\boldsymbol{\Phi}_1^*\right)^2\mathbf{S} & \ldots & \mathbf{A}\left(\theta,\phi\right)\boldsymbol{\Phi}_2^*\left(\boldsymbol{\Phi}_1^*\right)^N\mathbf{S}\end{bmatrix} \\ & + \begin{bmatrix}\mathbf{n}_1 & \mathbf{n}_2 & \mathbf{n}_3 & \ldots & \mathbf{n}_{N+1} \\ \mathbf{n}_{11} & \mathbf{n}_{22} & \mathbf{n}_{33} & \ldots & \mathbf{n}_{(N+1)(N+1)}\end{bmatrix}\end{aligned} \tag{14}$$

The matrices $\boldsymbol{\Phi}_1^*(\theta,\phi)$ and $\boldsymbol{\Phi}_2^*(\theta)$ in (14) are $K\times K$ diagonal matrices containing information that is used to extract the elevation angle θ_k and the azimuth angle ϕ_k.

Computing the QR factorization for the data matrix $\mathbf{YY}$ in (5) or $\mathbf{ZZ}$ in (12) using Toeplitz structure [17] in the proposed method requires only $O((N+1)^2)$ operations while calculating regular data matrix with dimension $(N\times N)$ using the singular value decomposition (SVD) of the date matrix or the Eigen value decomposition (EVD) of the covariance matrix which have been heavily used by many existing schemes requires $O(N^3)$ operations. This makes the proposed method a potential candidate for real-time implementation since computational complexity and storage memory are significantly reduced compared with the EVD and SVD methods. QR factorization is also very efficient in parallel applications and adaptive filtering techniques.

2.1. Estimation of Elevation Angle θ_k — Step 1

The proposed method employs QR factorization to estimate both azimuth and elevation angles of incident sources for both coherent and non-coherent signals in two steps using signal spaces of the factorized data matrices $\mathbf{Q}$ and $\mathbf{R}$, respectively. In step 1, $\mathbf{Q}$ matrix provides the estimation for the elevation angle θ_k. In step 2, $\mathbf{R}$ matrix provides the estimation for the azimuth angle ϕ_k using the estimated θ_k obtained from step 1.

We perform QR decomposition on the data matrix $\mathbf{W}$ as given in (14). The factorization of the data matrix $\mathbf{W}$ can be written as:

$$\mathbf{W} = \mathbf{QR} = [\mathbf{Q}]\,[\mathbf{R}]\,, \tag{15}$$

where $\mathbf{Q}$ is the data matrix of size $(2N+2)\times(2N+2)$, and $\mathbf{R}$ is the data matrix of size $(2N+2)\times(N+1)$. The QR factorization in (15) is called rank-revealing QR factorization. The $\mathbf{Q}$ data matrix can be decomposed into two spaces as follows:

$$\mathbf{Q}s = [q_1 \quad q_2 \quad \ldots \quad q_k]\,, \tag{16}$$

$$\mathbf{Q}n = [q_{k+1} \quad q_{k+2} \quad \ldots \quad q_{2N+2}]\,, \tag{17}$$

where $\mathbf{Q}s$ is $(2N+2)\times k$ column vectors representing the signal space and $\mathbf{Q}_n$ is $(2N+2)\times(2N+2-k)$ column vectors representing the noise space. The column vectors of the signal space $\mathbf{Q}s$ in (16) will be used to estimate the elevation angle θ_k.

We partition the data matrix $\mathbf{Q}s$ into two $((N+1)\times k)$ sub-matrices such that:

$$\mathbf{Q}s = \begin{bmatrix}\mathbf{Q}_{s1}\\ \hline \mathbf{Q}_{s2}\end{bmatrix}, \tag{18}$$

Columns vector of $\mathbf{Q}_{s1}$ and $\mathbf{Q}_{s2}$ based on shift invariant properties are related by the matrix $\boldsymbol{\Phi}_2^*(\theta)$.

Since range of $\Re[\mathbf{Q}_{s1}] = \Re[\mathbf{Q}_{s2}]$ and $\mathbf{Q}_{s1}$ and $\mathbf{Q}_{s2}$ span the same signal space it follows that they are related by a nonsingular transform ψ as follows:

$$\mathbf{Q}_{s2} = \mathbf{Q}_{s1}\boldsymbol{\psi}, \tag{19}$$

The Eigen values of the matrix $\boldsymbol{\psi}$ are the diagonal elements of $\mathbf{\Phi}_2^*(\theta)$. Finding the Eigen values of $\boldsymbol{\psi}$ will lead to obtaining the elevation angle DOAs for incident sources.

Equation (19) can be solved using least square approach (LS-ESPRIT) which minimizes the difference between $\mathbf{Q}_{s2}$ and $\mathbf{Q}_{s1}\boldsymbol{\psi}$

$$\boldsymbol{\psi} = \arg\min_{(\psi)} \|\mathbf{Q}_{s2} - \mathbf{Q}_{s1}\boldsymbol{\psi}\|_F^2 = \arg\min_{(\psi)} tr\left\{[\mathbf{Q}_{s2} - \mathbf{Q}_{s1}\boldsymbol{\psi}]^H [\mathbf{Q}_{s2} - \mathbf{Q}_{s1}\boldsymbol{\psi}]\right\} \tag{20}$$

The least square solution of (20) can be found as:

$$\boldsymbol{\psi} = \left[\mathbf{Q}_{s1}^H \mathbf{Q}_{s1}\right]^{-1} \mathbf{Q}_{s1}\mathbf{Q}_{s2}, \tag{21}$$

The Eigen values can be calculated using the Eigen decomposition of $\boldsymbol{\psi}$,

$$\hat{\mathbf{\Phi}}_2^*(\theta) = \lambda_k(\boldsymbol{\psi}), \quad k = 1, 2, \ldots, \hat{K}. \tag{22}$$

Using the estimated Eigen values for the k sources in (22), elevation angle can be found as follows

$$\hat{\theta}_k = \cos^{-1}\left[\frac{\text{angle}(\hat{\mathbf{\Phi}}_2^*(\theta))}{2\pi d/\lambda}\right], \tag{23}$$

where angle $(\cdot)$ represents the phase angles.

2.2. Estimation of Azimuth Angle (ϕ_k) — Step 2

The estimate $\hat{\theta}$ obtained in Step 1 will be used to estimate the azimuth DOA ϕ_k in Step 2. The $\mathbf{R}$ data matrix in (15) will be used to estimate the azimuth angle ϕ_k.

The data matrix $\mathbf{R}$ can be divided into two spaces. They are $\mathbf{R}_s$ the signal space and $\mathbf{R}_n$ the noise space which can be expressed as follows:

$$\mathbf{R}_s = [r_1 \quad r_2 \quad \ldots \quad r_k]^T, \tag{24}$$

$$\mathbf{R}_n = [r_{k+1} \quad r_{k+2} \quad \ldots \quad r_{2N+2}]^T, \tag{25}$$

where $\mathbf{R}_s$ is $k \times (N+1)$ row vectors representing the signal space and $\mathbf{R}_n$ is $(2N+2-k) \times (N+1)$ row vectors representing the noise space.

The row vectors of the signal space $\mathbf{R}_s$ will be used to estimate the azimuth angle ϕ_k. We transform the row vectors of the data matrix $\mathbf{R}_s$ to column vectors as follows:

$$\mathbf{R}_{ss} = \mathbf{R}_s^T = \begin{bmatrix} r_{11} & r_{21} & \cdots & r_{k1} \\ r_{12} & r_{22} & \cdots & r_{k2} \\ \vdots & \vdots & \ddots & \vdots \\ r_{1(N+1)} & r_{2(N+1)} & \cdots & r_{k(N+1)} \end{bmatrix}. \tag{26}$$

We create two submatrices $\mathbf{R}_{ss1}$ and $\mathbf{R}_{ss2}$ as,

$$\mathbf{R}_{ss1} = \mathbf{J}_1\mathbf{R}_{\mathrm{SS}} \quad \text{and} \quad \mathbf{R}_{ss2} = \mathbf{J}_2\mathbf{R}_{\mathrm{SS}}, \tag{27}$$

where $\mathbf{J}_1 = [\mathbf{I}_N \quad \mathbf{0}]$ and $\mathbf{J}_2 = [\mathbf{0} \quad \mathbf{I}_N]$ are the selection matrices, $\mathbf{I}_N$ is the N dimension identity matrix and $\mathbf{0}$ the zero vector. Equation (27) can be expressed as follows:

$$\mathbf{R}_{ss2} = \mathbf{R}_{ss1}\mathbf{\Phi}_1^*(\theta, \phi). \tag{28}$$

By applying the least squares approach to Equation (28), $\mathbf{\Phi}_1^*(\theta, \phi)$ can be obtained as

$$\mathbf{\Phi}_1(\theta, \phi) = \mathbf{R}_{ss1}^{\#}\mathbf{R}_{ss2}, \tag{29}$$

where $\mathbf{R}_{ss1}^{\#} = (\mathbf{R}_{ss1}^H\mathbf{R}_{ss1})^{-1}\mathbf{R}_{ss1}^H$ is the Moore-Penrose pseudoinverse of $\mathbf{R}_{ss1}$. We apply an Eigen decomposition on $\mathbf{\Phi}_1^*(\theta, \phi)$ to get k Eigen values. From the estimated Eigen values, we can estimate the azimuth angle ϕ_K as follows:

$$\phi_k = \cos^{-1}\left[\frac{\lambda\,\text{angle}\left(\hat{\mathbf{\Phi}}_1^*(\theta, \phi)\right)}{2\pi d \sin\left(\hat{\theta}\right)}\right]. \tag{30}$$

3. SIMULATION RESULTS

The performance of the proposed methods is analyzed and compared with well-known matrix pencil method [19]. We consider multiple sources lying in the far-field region of our proposed array configuration of two ULAs where each ULA consists of $N = 12$ elements. The pencil parameter is chosen as $L = 11$. The interspacing d between the ULAs is set to half wavelength of the incoming signals. The value of the SNR is varied from 5–30 dB where for each SNR value 100 Monte Carlo trials are performed. The RMSE for the joint DOA estimation is computed using the following equation.

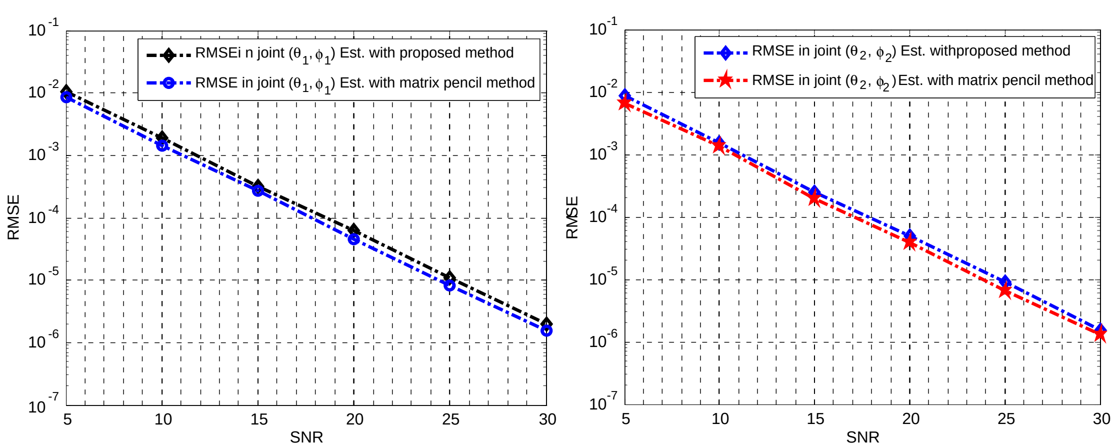
$$\text{RMSE} = \sqrt{E\left[\left(\hat{\theta}_i - \theta_i\right)^2 + \left(\hat{\phi}_i - \phi_i\right)^2\right]}, \tag{31}$$

where i represents the source index, $E[X]$ denotes the expectation of a random variable X, and $(\hat{\theta}_1, \hat{\phi}_1)$ are the pair of estimated elevation and azimuth angle estimates.

In the first scenario, we consider two non-coherent sources ($K = 2$) with DOAs of $(\theta_1, \phi_1) = (50°, 40°)$ and $(\theta_2, \phi_2) = (70°, 60°)$ respectively. Figure 2 shows the plot of RMSE in the joint azimuth and elevation angles for varying SNR with the proposed QR-2D DOA method and the Matrix Pencil estimation method. The figure shows estimation accuracy of the proposed methods for the 2D-DOA estimates of both sources as depicted by the low values of RMSE for varying SNR. From the figure, it can also be observed that the proposed QR-2D method compares favorably with the Matrix Pencil method.

For the second set of simulations, we consider three coherent sources ($K = 3$) with DOAs of $(\theta_1, \phi_1) = (50°, 40°)$, $(\theta_2, \phi_2) = (70°, 60°)$ and $(\theta_3, \phi_3) = (100°, 80°)$ respectively. The plot of RMSE in the joint azimuth and elevation angles for varying SNR with the proposed QR-2D DOA method and the Matrix Pencil estimation method is shown in Figure 3. The figure verifies the accuracy of the proposed 2D-DOA estimation of the proposed method for three coherent sources as indicated by the less estimation errors for varying SNR values. It can be observed that the proposed QR-2D method again compares favorably with the Matrix Pencil method in estimating the DOAs of the three sources.

Further, the performance analysis of the proposed QR-2D DOA method with the Matrix Pencil method is performed by computing RMSE in angles estimate for varying source location in a two dimensional (θ, ϕ) plane. The angles (elevation and azimuth) are both varied from 0°–90° and RMSE in angle estimates are computed. The RMSE in θ and ϕ estimates for the proposed methods are shown in Figures 4 and 5 respectively. From Figure 4, it can be observed that the estimation error in elevation angle increases as ϕ approaches 0°. For all other pairs of elevation and azimuth angles the proposed methods provides accurate angle estimates. As the azimuth angle estimates depend upon the estimated elevation angles, thus the RMSE in ϕ estimates is observed to be higher as compared to the RMSE in θ estimates as shown in Figure 5. The figure also shows that the estimation errors with the proposed methods are relatively higher in a region when both θ and ϕ are close to 0°.



(a) (b)

Figure 2. RMSE in joint (a) (θ_1, ϕ_1) estimates, and (b) (θ_2, ϕ_2) estimates using QR-2D & Matrix Pencil DOA methods for two sources at $(\theta_1, \phi_1) = (50°, 40°)$, and $(\theta_2, \phi_2) = (70°, 60°)$.

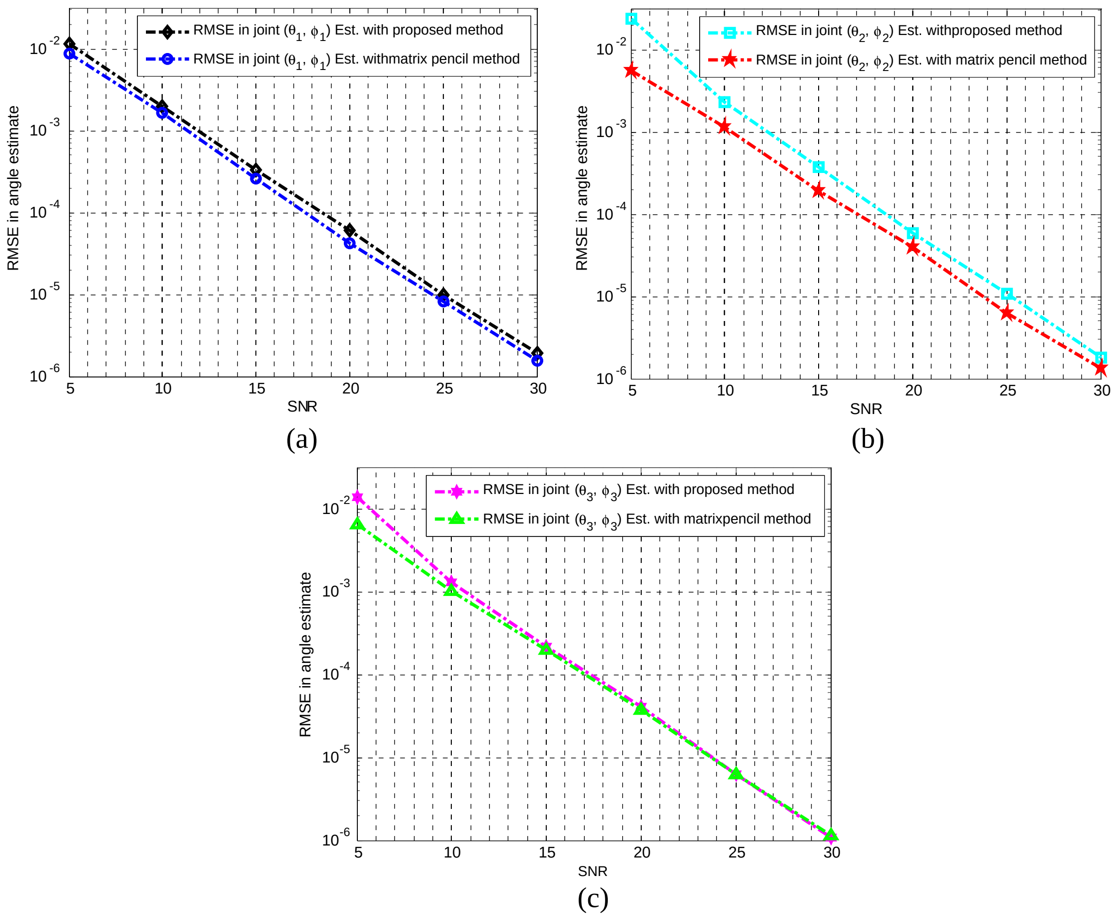


Figure 3. RMSE in joint (a) (θ_1, ϕ_1) est., (b) (θ_2, ϕ_2)) est., and (c) (θ_3, ϕ_3)) est., using QR-2D DOA and Matrix Pencil methods for three sources at $(\theta_1, \phi_1) = (50°, 40°)$, $(\theta_2, \phi_2) = (70°, 60°)$, and $(\theta_3, \phi_3) = (100°, 80°)$.

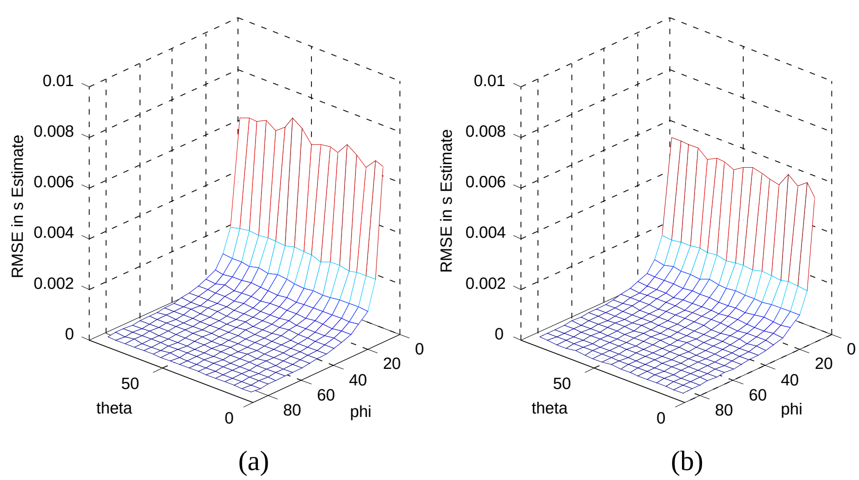
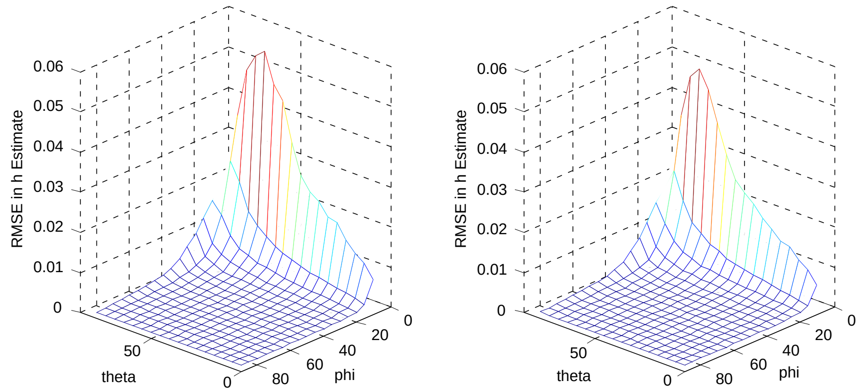


Figure 4. (a) RMSE in θ Estimates using proposed QR-2D & (b) Matrix Pencil methods with both θ and ϕ varying from 0° to 90°.

The computational times of the proposed QR-2D DOA and matrix pencil estimation methods for the cases of one, two and three sources are listed in Table 1. These computational times correspond to the execution times of the proposed methods on MATLAB software. From the table it can be observed that the proposed QR-2D DOA estimation method consumes less computational time as compared to the matrix pencil method for all three cases. The higher computational time of the matrix pencil



(a) (b)

Figure 5. (a) RMSE in ϕ Estimates using proposed QR-2D & (b) Matrix Pencil methods with both θ and ϕ varying from $0°$ to $90°$.

Table 1. Computation time comparison for 100 Monte Carlo iterations.

No. of Sources	Estimation Method	Computation Time (μs)
1	QR-2D DOA Est.	4525
	Matrix Pencil Est.	5175
2	QR-2D DOA Est.	4890
	Matrix Pencil Est.	5530
3	QR-2D DOA Est.	5210
	Matrix Pencil Est.	5665

Table 2. Comparison of computation complexity of QR proposed method and MP method.

Major Processing Steps	**Proposed QR based method**	**MP method**
Construction of the data matrix	$O(N+1)$	$O(2N-L+1)$
Decomposition (QR for proposed method, and SVD for matrix pencil)	$O(N+1)^2$	$2*O((2N-L)(L+1)^2)+11*O(L+1)^3$
Calculation of Moore-Penrose pseudo-inverse	$O(k^2N)$	$O\left(k^2N\right)$

method is due to the higher cost of computing SVD as compared to the QR.

Table 2 summarizes the comparison of computational complexity of major processing steps used in the proposed QR based method and the matrix pencil (MP) method. It is observed that the implementation of QR decomposition of the data matrix requires $O(N+1)^2$ FLOPS whereas the MP method requires $2*O(2N-L)(L+1)^2+11*O(L+1)^3$ FLOPS. The parameter L, called the pencil parameter, is normally chosen between $N/3$ and $N/2$ for efficient noise filtering. It is observed form Table 2 that MP Method has the highest complexity when $L = N/2$. The lower computational complexity of the proposed method mainly comes from the construction of the data matrix in Toeplitz form and utilization of QR decomposition.

4. CONCLUSIONS

A method based on QR decomposition for the two-dimensional direction of arrival (DOA) estimation called QR-2D DOA is proposed. The method employs a two uniform linear antenna array (ULA) configuration for the joint estimation of elevation (θ) and azimuth (ϕ) angles. The proposed method employs a single snapshot of the received data and constructs Toeplitz structured data matrix which favors the 2D DOA estimation for both coherent and in-coherent source signals. Simulation results of the proposed method are presented to demonstrate successful 2D DOA estimation for both two and three incident source signals. The RMSE in the DOA estimates and the computational times of the proposed method is compared with the matrix pencil method which is based on SVD. The proposed QR-2D DOA method is observed to provide very close 2D DOA estimates as compared to the matrix pencil method with the advantage of reduced computational time.

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