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Wiener-Hopf Analysis of the *H*-Polarized Plane Wave Diffraction by a Finite Sinusoidal Grating

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(*Invited Paper*)

Abstract—The diffraction by a finite sinusoidal grating is analyzed for the H-polarized plane wave incidence using the Wiener-Hopf technique combined with the perturbation method. Assuming the depth of the grating to be small compared with the wavelength and approximating the boundary condition on the grating surface, the problem is reduced to the diffraction problem involving a flat strip with a certain mixed boundary condition. Introducing the Fourier transform for the unknown scattered field and applying an approximate boundary condition together with a perturbation series expansion for the scattered field, the problem is formulated in terms of the zero-order and first-order Wiener-Hopf equations. The Wiener-Hopf equations are solved via the factorization and decomposition procedure leading to the exact and asymptotic solutions. Taking the inverse Fourier transform and applying the saddle point method, the scattered field expression is explicitly derived. Scattering characteristics of the grating are discussed in detail via numerical examples of the far field intensity.

1. INTRODUCTION

Analysis of the scattering and diffraction by gratings and periodic structures is an important subject in electromagnetic theory and optics. Various analytical and numerical methods have been developed thus far and diffraction phenomena have been investigated for a number of periodic structures. It is well known that the Riemann-Hilbert problem technique [1–3], analytical regularization methods [3–5], the Yasuura method [6–8], the integral and differential method [9], the point matching method [10], and the Fourier series expansion method [11, 12] are efficient for the analysis of diffraction problems involving periodic structures. The Wiener-Hopf technique [13–16] is known as a powerful approach for analyzing electromagnetic wave problems associated with canonical geometries rigorously, and can be applied efficiently to problems of the diffraction by specific periodic structures such as gratings. There are significant contributions to the analysis of the diffraction by gratings and other related structures based on the Wiener-Hopf technique [17–23]. In the previous papers, we have analyzed the diffraction problems involving transmission-type gratings with the aid of the Wiener-Hopf technique [24–27], where rigorous solutions valid over a broad frequency range have been obtained.

It is to be noted that the analysis in most of the above-mentioned papers are restricted to periodic structures of infinite extent and plane boundaries. Therefore, it is important to investigate scattering problems involving periodic structures without these restrictions. As an example of infinite periodic structures with non-plane boundaries, Das Gupta [28] analyzed the plane wave diffraction by a half-plane with sinusoidal corrugation by means of the Wiener-Hopf technique together with the perturbation method. The method developed in [28] has been generalized thereafter by Chakrabarti and Dowerah [29]

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for the Wiener-Hopf analysis of the H-polarized plane wave diffraction by two parallel sinusoidal half-planes. In [30, 31], we have analyzed the problem considered by Chakrabarti and Dowerah via a different Wiener-Hopf approach for both E and H polarizations, and derived various new expressions of the scattered field inside and outside the waveguide. We have also considered a finite sinusoidal grating as another important generalization to Das Gupta [28], and analyzed the E-polarized plane wave diffraction via a hybrid Wiener-Hopf and perturbation approach [32, 33].

In this paper, we shall consider the same grating geometry as in our previous papers [32, 33], and analyze the diffraction problem for the H-polarized plane wave incidence. The method of solution presented in this paper for H polarization is similar to, but more complicated than, the analysis carried out for the E-polarized case [32, 33]. Assuming that the corrugation amplitude of the grating is small compared with the wavelength, the original problem is replaced by the problem of the H-polarized plane wave diffraction by a flat strip with a certain mixed boundary condition. The approximate boundary condition thus derived contains both the first- and second-order derivatives of the unknown scattered field, and this makes the problem formulation more complicated than in the E-polarized case [32, 33]. It is to be noted that, in the E polarization, the approximate boundary condition is expressed in terms of the first-order derivative of the scattered field. Using the approximate boundary condition for the H-polarized case, we expand the unknown scattered field into a perturbation series and separate the problem into the zero-order and the first-order boundary value problems.

Introducing the Fourier transform for the unknown scattered field and applying boundary conditions in the transform domain, the problem is formulated in terms of the zero- and first-order Wiener-Hopf equations, which are solved exactly via the factorization and decomposition procedure. However, the solution is formal in the sense that branch-cut integrals with unknown integrands are involved. By using a rigorous asymptotic method established recently by Kobayashi [34], we have derived high-frequency asymptotic expansions of these branch-cut integrals, and subsequently obtained asymptotic solutions of the Wiener-Hopf equations for the width of the grating large compared with the wavelength. Taking the Fourier inverse of the solution in the transform domain and applying the saddle point method, the scattered far field in the real space is derived. Representative numerical examples of the far field intensity are shown for various physical parameters, and scattering characteristics of the grating are discussed in detail. The results are also compared with our previous analysis for the E-polarized case [32, 33].

The time factor is assumed to be $e^{-i\omega t}$ and suppressed throughout this paper.

2. STATEMENT OF THE PROBLEM

We consider the diffraction of an H-polarized plane wave by a finite sinusoidal grating as shown in Figure 1, where the grating surface is assumed to be infinitely thin, perfectly electric conducting, and uniform in the y-direction, being defined by

$$x = h \sin mz, \quad |z| \le a, \tag{1}$$

where $2h$ is the depth of the grating, and $m(> 0)$ is the periodicity (surface roughness) parameter. Taking into account the grating geometry together with the fact that the magnetic field is parallel to the y-axis, this scattering problem is reduced to a two-dimensional problem.

Let us define the total magnetic field $\phi^t(x, z)[\equiv H_y^t(x, z)]$ by

$$\phi^t(x, z) = \phi^i(x, z) + \phi(x, z), \tag{2}$$

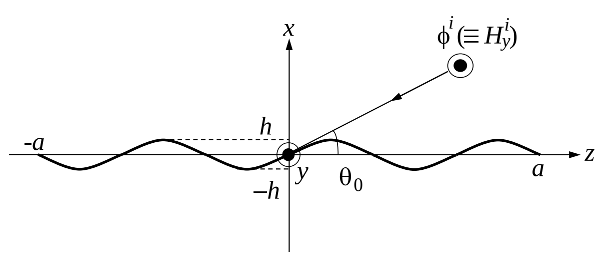


Figure 1. Geometry of the problem.

where $\phi^i(x, z)$ is the incident field of H polarization given by

$$\phi^i(x, z) = e^{-ik(x \sin\theta_0 + z \cos\theta_0)}, \quad 0 < \theta_0 < \pi/2 \tag{3}$$

with $k[\equiv \omega(\varepsilon_0\mu_0)^{1/2}]$ being the free-space wavenumber. The scattered field $\phi(x, z)$ satisfies the two-dimensional Helmholtz equation

$$\left(\partial^2/\partial x^2 + \partial^2/\partial z^2 + k^2\right)\phi(x, z) = 0. \tag{4}$$

Nonzero components of the scattered electromagnetic fields are derived from the following relation:

$$(H_y, E_x, E_z) = \left(\phi, \frac{1}{i\omega\varepsilon_0}\frac{\partial\phi}{\partial z}, \frac{i}{\omega\varepsilon_0}\frac{\partial\phi}{\partial x}\right). \tag{5}$$

According to the boundary condition on the grating surface, tangential components of the total electric field $E^t_{\tan}$ satisfies

$$E^t_{\tan} = \frac{\partial\phi^t(h\sin mz, z)}{\partial n} = 0, \quad |z| < a, \tag{6}$$

where $\partial/\partial n$ denotes the normal derivative on the grating surface. We assume that the grating depth $2h$ is small compared with the wavelength and expand (6) in terms of the Taylor series. Then by ignoring the $O(h^2)$ terms from the Taylor expansion, we obtain that

$$\frac{\partial\phi^t(0, z)}{\partial x} + h\left[\sin mz\frac{\partial^2\phi^t(0, z)}{\partial x^2} - m\cos mz\frac{\partial\phi^t(0, z)}{\partial z}\right] + O(h^2) = 0, \quad |z| < a. \tag{7}$$

Equation (7) is the approximate boundary condition used throughout the remaining part of this paper.

We express the unknown scattered field $\phi(x, z)$ in (2) using a perturbation series expansion in h as

$$\phi(x, z) = \phi^{(0)}(x, z) + h\phi^{(1)}(x, z) + O(h^2), \tag{8}$$

where $\phi^{(0)}(x, z)$ and $\phi^{(1)}(x, z)$ are the zero-order and the first-order terms contained in the scattered field, respectively. Substituting (8) into (4) and taking into account (2), (3), and (7), the original problem is separated into the two boundary value problems.

The zero-order and first-order scattered fields $\phi^{(n)}(x, z)$ for $n = 0$, 1 satisfy

$$\left(\partial^2/\partial x^2 + \partial^2/\partial z^2 + k^2\right)\phi^{(n)}(x, z) = 0, \tag{9}$$

where the boundary conditions are given by

$$\phi^{(0)}(+0, z) = \phi^{(0)}(-0, z)\left[\equiv \phi^{(0)}(0, z)\right], \tag{10}$$

$$\frac{\partial\phi^{(0)}(+0, z)}{\partial x} = \frac{\partial\phi^{(0)}(-0, z)}{\partial x}\left[\equiv \frac{\partial\phi^{(0)}(0, z)}{\partial x}\right], \tag{11}$$

$$\phi^{(1)}(+0, z) = \phi^{(1)}(-0, z)\left[\equiv \phi^{(1)}(0, z)\right], \tag{12}$$

$$\frac{\partial\phi^{(1)}(+0, z)}{\partial x} = \frac{\partial\phi^{(1)}(-0, z)}{\partial x}\left[\equiv \frac{\partial\phi^{(1)}(0, z)}{\partial x}\right] \tag{13}$$

for $|z| > a$, and

$$\phi^{(0)}(+0, z) - \phi^{(0)}(-0, z) = j^{(0)}(0, z), \tag{14}$$

$$\frac{\partial\phi^{(0)}(0, z)}{\partial x} = ik\sin\theta_0 e^{-ikz\cos\theta_0}, \tag{15}$$

$$\phi^{(1)}(+0, z) - \phi^{(1)}(-0, z) = j^{(1)}(0, z), \tag{16}$$

$$\frac{\partial\phi^{(1)}(0, z)}{\partial x} + \sin mz\frac{\partial^2\phi^{(0)}(0, z)}{\partial x^2} - m\cos mz\frac{\partial\phi^{(0)}(0, z)}{\partial z}$$
$$= \frac{ik}{2}\left[k\sin^2\theta_0\sum_{n=1}^{2}(-1)^n e^{-ikz\cos\theta_n} - m\cos\theta_0\sum_{n=1}^{2}e^{-ikz\cos\theta_n}\right] \tag{17}$$

for $|z| < a$ with

$$\cos\theta_1 = \cos\theta_0 - m/k, \quad \cos\theta_2 = \cos\theta_0 + m/k. \tag{18}$$

In (14) and (16), $j^{(0)}(0, z)$ and $j^{(1)}(0, z)$ are the zero-order and first-order terms of the unknown surface currents induced on the grating surface, respectively. As seen from the above discussion, the zero-order problem corresponds to the diffraction by a perfectly conducting flat strip [16, 35]. On the other hand, the first-order problem is important since it contains the effect due to the sinusoidal corrugation.

3. WIENER-HOPF EQUATIONS

For convenience of analysis, we assume that the medium is slightly lossy as in $k = k_1 + ik_2$ with $0 < k_2 \ll k_1$. The solution for real k is obtained by letting $k_2 \to +0$ at the end of analysis. In view of the radiation condition, it follows from (8) that, the zero- and first-order scattered fields $\phi^{(n)}(x, z)$ for $n = 0$, 1 behave like the diffracted field for fixed x as $|z| \to \infty$. Hence we can show that

$$\phi^{(n)}(x, z) \sim CH_0^{(1)}(k\rho) \sim C'\rho^{-1/2} e^{ik_1\rho} e^{-k_2\rho} = O\left(e^{-k_2|z|}\right) = O\left(e^{-k_2|z|\cos\theta_0}\right) \tag{19}$$

as $|z| \to \infty$, where $\rho = (x^2 + z^2)^{1/2}$, and C and C' are constants. In (19), $H_0^{(1)}(\cdot)$ is the Hankel function of the first kind. It is noted that the first- and second-order derivatives of the scattered field $\phi^{(n)}(x, z)$ with respect to x should also satisfy (19).

Let us introduce the Fourier transform of the scattered field $\phi^{(n)}(x, z)$ as in

$$\Phi^{(n)}(x, \alpha) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} \phi^{(n)}(x, z) e^{i\alpha z} dz, \tag{20}$$

where $\alpha = \mathrm{Re}\alpha + i\mathrm{Im}\alpha (\equiv \sigma + i\tau)$. It follows from (19) and (20) that $\Phi^{(n)}(x, \alpha)$ for $n = 0$, 1 are regular in the strip $|\tau| < k_2 \cos\theta_0$ of the complex α-plane. We also introduce the Fourier integrals as

$$\Phi_{\pm}^{(n)}(x, \alpha) = \pm(2\pi)^{-1/2} \int_{\pm a}^{\pm\infty} \phi^{(n)}(x, z) e^{i\alpha(z \mp a)} dz, \tag{21}$$

$$\Phi_1^{(n)}(x, \alpha) = (2\pi)^{-1/2} \int_{-a}^{a} \phi^{(n)}(x, z) e^{i\alpha z} dz, \tag{22}$$

$$J_1^{(n)}(0, \alpha) = (2\pi)^{-1/2} \int_{-a}^{a} j^{(n)}(0, z) e^{i\alpha z} dz. \tag{23}$$

Then it is seen that $\Phi_{\pm}^{(n)}(x, \alpha)$ are regular in $\tau \gtrless \mp k_2 \cos\theta_0$ whereas $\Phi_1^{(n)}(x, \alpha)$ and $J_1^{(n)}(0, \alpha)$ are entire functions. It follows from (20)–(22) that

$$\Phi^{(n)}(x, \alpha) = e^{-i\alpha a}\Phi_{-}^{(n)}(x, \alpha) + \Phi_1^{(n)}(x, \alpha) + e^{i\alpha a}\Phi_{+}^{(n)}(x, \alpha). \tag{24}$$

Taking the Fourier transform of (9) and making use of (19), we derive that

$$\left[d^2/dx^2 - \gamma^2(\alpha)\right] \Phi^{(n)}(x, \alpha) = 0, \tag{25}$$

where $\gamma(\alpha) = (\alpha^2 - k^2)^{1/2}$. Since $\gamma(\alpha)$ is a double-valued function of α, we choose its proper branch so that $\gamma(\alpha)$ reduces to $-ik$ when $\alpha = 0$. The solution of (25) is then expressed as

$$\begin{aligned} \Phi^{(n)}(x, \alpha) &= A^{(n)}(\alpha) e^{-\gamma(\alpha)x}, \quad x > 0, \\ &= B^{(n)}(\alpha) e^{\gamma(\alpha)x}, \quad x < 0, \end{aligned} \tag{26}$$

where $A^{(n)}(\alpha)$ and $B^{(n)}(\alpha)$ for $n = 0$, 1 are unknown functions. Using the boundary conditions as given by (10)–(17), we deduce that

$$\left.\begin{matrix} A^{(0)}(\alpha) \\ B^{(0)}(\alpha) \end{matrix}\right\} = \pm \frac{J_1^{(0)}(\alpha)}{2}, \tag{27}$$

$$\begin{aligned} \left.\begin{matrix} A^{(1)}(\alpha) \\ B^{(1)}(\alpha) \end{matrix}\right\} = \pm \frac{J_1^{(1)}(\alpha)}{2} + \frac{1}{4i\gamma(\alpha)} \Big\{ &\left[\gamma^2(\alpha + m) - m(\alpha + m)\right] J_1^{(0)}(\alpha + m) \\ &- \left[\gamma^2(\alpha - m) + m(\alpha - m)\right] J_1^{(0)}(\alpha - m) \Big\}, \end{aligned} \tag{28}$$

where

$$J_1^{(n)}(\alpha) = \Phi_1^{(n)}(+0,\alpha) - \Phi_1^{(n)}(-0,\alpha) \tag{29}$$

for $n = 0$, 1. Substituting (27) and (28) into (26), we obtain that

$$\Phi^{(0)}(x,\alpha) = \pm\frac{J_1^{(0)}(\alpha)}{2}e^{\mp\gamma(\alpha)x}, \tag{30}$$

$$\Phi^{(1)}(x,\alpha) = \pm\frac{J_1^{(1)}(\alpha)}{2}e^{\mp\gamma(\alpha)x} + \frac{1}{4i\gamma(\alpha)}\Big\{\big[\gamma^2(\alpha+m) - m(\alpha+m)\big]J_1^{(0)}(\alpha+m) - \big[\gamma^2(\alpha-m) + m(\alpha-m)\big]J_1^{(0)}(\alpha-m)\Big\}e^{\mp\gamma(\alpha)x} \tag{31}$$

for $x \gtrless 0$. Equations (30) and (31) are the zero- and first-order scattered fields in the Fourier transform domain, respectively.

Setting $x = \pm 0$ in (30) and (31) and carrying out some manipulations with the aid of the boundary conditions, we are led to

$$e^{-i\alpha a}U_-(\alpha) + K(\alpha)J_1^{(0)}(\alpha) + e^{i\alpha a}U_{(+)}(\alpha) = 0, \tag{32}$$

$$e^{-i\alpha a}V_-(\alpha) + K(\alpha)J_1^{(1)}(\alpha) + e^{i\alpha a}V_{(+)}(\alpha) = 0 \tag{33}$$

for $|\tau| < k_2\cos\theta_0$, where

$$U_-(\alpha) = \Phi_-^{(0)\prime}(0,\alpha) + \frac{A_0}{\alpha - k\cos\theta_0}, \tag{34}$$

$$U_{(+)}(\alpha) = \Phi_+^{(0)\prime}(0,\alpha) - \frac{B_0}{\alpha - k\cos\theta_0}, \tag{35}$$

$$V_-(\alpha) = \Psi_-(\alpha) - \sum_{n=1}^{2}(-1)^n\frac{A_nC_n}{\alpha - k\cos\theta_n}, \tag{36}$$

$$V_{(+)}(\alpha) = \Psi_+(\alpha) + \sum_{n=1}^{2}(-1)^n\frac{B_nC_n}{\alpha - k\cos\theta_n}, \tag{37}$$

$$K(\alpha) = \gamma(\alpha)/2 \tag{38}$$

with

$$\Psi_\pm(\alpha) = \Phi_\pm^{(1)\prime}(0,\alpha) + (1/2i)\Big\{[\gamma^2(\alpha+m) - m(\alpha+m)]e^{\pm ima}\Phi_\pm^{(0)}(0,\alpha+m) - [\gamma^2(\alpha-m) - m(\alpha-m)]e^{\mp ima}\Phi_\pm^{(0)}(0,\alpha+m) \pm (2\pi)^{-1/2}m\cos ma\phi^{(0)}(0,a)\Big\}, \tag{39}$$

$$\left.\begin{matrix}A_0\\B_0\end{matrix}\right\} = -\frac{k\sin\theta_0 e^{\pm ika\cos\theta_0}}{(2\pi)^{1/2}}, \tag{40}$$

$$\left.\begin{matrix}A_n\\B_n\end{matrix}\right\} = \frac{e^{\pm ika\cos\theta_n}}{(2\pi)^{1/2}}, \quad n = 1, 2, \tag{41}$$

$$C_n = (k/2)\left[k\sin^2\theta_0 - (-1)^n m\cos\theta_0\right], \quad n = 1, 2. \tag{42}$$

In the above equations, the prime denotes differentiation with respect to x. Equations (32) and (33) are the zero- and first-order Wiener-Hopf equations, respectively.

4. EXACT AND ASYMPTOTIC SOLUTIONS

In this section, we shall solve the zero-order and first-order Wiener-Hopf equations to derive exact and asymptotic solutions. First we note that the kernel function $K(\alpha)$ defined by (38) is factorized as

$$K(\alpha) = K_+(\alpha)K_-(\alpha), \tag{43}$$

where

$$K_{\pm}(\alpha) = 2^{-1/2} e^{-i\pi/4} (k \pm \alpha)^{1/2}. \tag{44}$$

Multiplying both sides of (32) by $e^{\pm i\alpha a}/K_{\pm}(\alpha)$ and applying the decomposition procedure, we arrive at the exact solution with the result that

$$U_{(+)}(\alpha) = K_{+}(\alpha) \left\{ -\frac{B_0}{K_{+}(k\cos\theta_0)(\alpha - k\cos\theta_0)} + \frac{1}{2}[u_s(\alpha) - u_d(\alpha)] \right\}, \tag{45}$$

$$U_{-}(\alpha) = K_{-}(\alpha) \left\{ \frac{A_0}{K_{-}(k\cos\theta_0)(\alpha - k\cos\theta_0)} + \frac{1}{2}[u_s(-\alpha) + u_d(-\alpha)] \right\}, \tag{46}$$

where

$$u_{s.d}(\alpha) = \frac{1}{\pi i} \int_k^{k+i\infty} \frac{e^{2i\beta a} U_{(+)}^{s,d}(\beta)}{K_{-}(\beta)(\beta + \alpha)} d\beta, \tag{47}$$

$$U_{(+)}^{s,d}(\alpha) = U_{(+)}(\alpha) \pm U_{-}(-\alpha). \tag{48}$$

Equations (45) and (46) are formal since branch-cut integrals with the unknown integrands $U_{(+)}^{s,d}(\beta)$ are involved. Applying a rigorous asymptotic method developed by Kobayashi [34], we obtain a high-frequency solution explicitly as in

$$U_{-}(\alpha) \sim -\frac{k\sin\theta_0 e^{ika\cos\theta_0}(\alpha - k)^{1/2}}{(2\pi)^{1/2}(k\cos\theta_0 - k)^{1/2}(\alpha - k\cos\theta_0)} + K_{-}(\alpha)\left[C_1^u \xi(-\alpha) + B_0 \eta_0^b(-\alpha)\right], \tag{49}$$

$$U_{(+)}(\alpha) \sim \frac{k\sin\theta_0 e^{-ika\cos\theta_0}(\alpha + k)^{1/2}}{(2\pi)^{1/2}(k\cos\theta_0 + k)^{1/2}(\alpha - k\cos\theta_0)} + K_{+}(\alpha)\left[C_2^u \xi(\alpha) + A_0 \eta_0^a(\alpha)\right] \tag{50}$$

for $ka \to \infty$, where

$$C_{1,2}^u = \frac{K_{+}(k)\left[\chi_0^{a,b}(k) + K_{+}(k)\xi(k)\chi_0^{b,a}(k)\right]}{1 - K_{+}^2(k)\xi^2(k)}, \tag{51}$$

$$\xi(\alpha) = -\frac{2a^{1/2}e^{2ika}}{\pi}\Gamma_1(1/2, -2i(\alpha + k)a), \tag{52}$$

$$\eta_0^{a,b}(\alpha) = \frac{\xi(\alpha) - \xi(\pm k\cos\theta_0)}{\alpha \mp k\cos\theta_0} \tag{53}$$

with

$$\chi_0^a(\alpha) = A_0 \eta_0^a(\alpha) + B_0 P_0^b(\alpha), \tag{54}$$

$$\chi_0^b(\alpha) = B_0 \eta_0^b(\alpha) + A_0 P_0^a(\alpha), \tag{55}$$

$$P_0^{a,b}(\alpha) = \frac{1}{\alpha \pm k\cos\theta_0}\left[\frac{1}{K_{+}(\alpha)} - \frac{1}{K_{\mp}(k\cos\theta_0)}\right]. \tag{56}$$

In (52), $\Gamma_1(\cdot,\cdot)$ is the generalized gamma function introduced by Kobayashi [36] and is defined by

$$\Gamma_m(u, v) = \int_0^{\infty} \frac{t^{u-1}e^{-t}}{(t + v)^m} dt \tag{57}$$

for $\mathrm{Re}\, u > 0$, $|v| > 0$, $|\arg v| < \pi$, and positive integer m. This completes the solution of the zero-order Wiener-Hopf equation (32).

A similar procedure may also be applied to the first-order Wiener-Hopf Equation (33). The factorization and decomposition procedure leads to the exact solution with the result that

$$V_{-}(\alpha) = -K_{-}(\alpha)\left\{\sum_{n=1}^{2}(-1)^n \frac{A_n C_n}{K_{-}(k\cos\theta_n)(\alpha - k\cos\theta_n)} + \frac{1}{2}[v_s(-\alpha) + v_d(-\alpha)]\right\}, \tag{58}$$

$$V_{(+)}(\alpha) = K_{+}(\alpha)\left\{\sum_{n=1}^{2}(-1)^n \frac{B_n C_n}{K_{+}(k\cos\theta_n)(\alpha - k\cos\theta_n)} + \frac{1}{2}[v_s(\alpha) - v_d(\alpha)]\right\}, \tag{59}$$

where

$$v_{s,d}(\alpha)=\frac{1}{\pi i}\int_k^{k+i\infty}\frac{e^{2i\beta a}V_{(+)}^{s,d}(\beta)}{K_-(\beta)(\beta+\alpha)}d\beta \tag{60}$$

with

$$V_{(+)}^{s,d}(\alpha)=V_{(+)}(\alpha)\pm V_-(-\alpha). \tag{61}$$

Carrying out asymptotic evaluation of the branch-cut integrals defined by (60) for large $|k|a$ and making some arrangements, we obtain that

$$V_-(\alpha)\sim-\sum_{n=1}^{2}(-1)^n\frac{C_n e^{ika\cos\theta_n}(\alpha-k)^{1/2}}{(2\pi)^{1/2}(k\cos\theta_n-k)^{1/2}(\alpha-k\cos\theta_n)}+K_-(\alpha)\Big[D_1^v\xi(-\alpha)+B_1\eta_1^b(-\alpha)+B_2\eta_2^b(-\alpha)\Big], \tag{62}$$

$$V_{(+)}(\alpha)\sim\sum_{n=1}^{2}(-1)^n\frac{C_n e^{-ika\cos\theta_n}(\alpha+k)^{1/2}}{(2\pi)^{1/2}(k\cos\theta_n+k)^{1/2}(\alpha-k\cos\theta_n)}+K_+(\alpha)\Big[D_2^v\xi(\alpha)+A_1\eta_1^a(\alpha)+A_2\eta_2^a(\alpha)\Big] \tag{63}$$

as $ka\to\infty$, where

$$D_{1,2}^v = \frac{K_+(k)}{1-K_+^2(k)\xi^2(k)}\sum_{n=1}^{2}\Big[\chi_n^{a,b}(k)+K_+(k)\xi(k)\chi_n^{b,a}(k)\Big], \tag{64}$$

$$\eta_n^{a,b}(\alpha) = -(-1)^n C_n\frac{\xi(\alpha)-\xi(\pm k\cos\theta_n)}{\alpha\mp k\cos\theta_n} \tag{65}$$

with

$$\chi_n^a(\alpha) = A_n\eta_n^a(\alpha)-(-1)^n B_n C_n P_n^b(\alpha), \tag{66}$$

$$\chi_n^b(\alpha) = B_n\eta_n^b(\alpha)-(-1)^n A_n C_n P_n^a(\alpha), \tag{67}$$

$$P_n^{a,b}(\alpha) = \frac{1}{\alpha\pm k\cos\theta_n}\left[\frac{1}{K_+(\alpha)}-\frac{1}{K_\mp(k\cos\theta_n)}\right] \tag{68}$$

for $n=1$, 2. Equations (49), (50) and (62), (63) yield high-frequency asymptotic solutions of the zero- and first-order Wiener-Hopf equations (32) and (33), respectively, and hold uniformly in incidence angle θ_0.

It is important to note here that the analysis presented in this paper reduces to the solution of the diffraction problem involving a sinusoidal half-plane by taking the limit $a\to\infty$, and we can show that the results are identical to Das Gupta [28].

5. SCATTERED FAR FIELD

In this section, we shall derive analytical expressions of the scattered field by using the results obtained in Section 4. The scattered field $\phi^{(n)}(x,z)$ with $n=0$, 1 in the real space can be derived by taking the inverse Fourier transform of (30) and (31) according to the formula

$$\phi^{(n)}(x,z)=(2\pi)^{-1/2}\int_{-\infty+ic}^{\infty+ic}\Phi^{(n)}(x,\alpha)e^{-i\alpha z}d\alpha, \tag{69}$$

where c is a constant satisfying $|c|<k_2\cos\theta_0$. In the following, we shall derive asymptotic expressions of the zero- and first-order scattered far fields explicitly.

Substituting the zero-order field representation in (30) into (69) with $n=0$ and taking into account (32), an integral representation of the zero-order scattered field $\phi^{(0)}(x,z)$ is expressed as

$$\phi^{(0)}(x,z)=\mp(2\pi)^{-1/2}\int_{-\infty+ic}^{\infty+ic}\frac{e^{-i\alpha a}U_-(\alpha)+e^{i\alpha a}U_{(+)}(\alpha)}{2K(\alpha)}e^{\mp\gamma(\alpha)x-i\alpha z}d\alpha \tag{70}$$

for $x\gtrless 0$, where c is a constant such that $|c|<k_2\cos\theta_0$. Since the integrand of (70) has branch points at $\alpha=\pm k$, evaluation in closed form is in general difficult. However, we may apply the saddle

point method to derive an asymptotic expression at large distances from the origin. Introducing the cylindrical coordinate centered at the origin as

$$x = \rho \sin\theta, \quad z = \rho\cos\theta, \quad -\pi < \theta < \pi \tag{71}$$

and applying the saddle point method, we derive a far field asymptotic expression with the result that

$$\phi^{(0)}(\rho,\theta) \sim \mp \frac{e^{ika\cos\theta}U_-(-k\cos\theta) + e^{-ika\cos\theta}U_{(+)}(-k\cos\theta)}{2K(-k\cos\theta)} k\sin|\theta| \, \frac{e^{i(k\rho-\pi/4)}}{(k\rho)^{1/2}} \tag{72}$$

for $x \gtrless 0$ as $k\rho \to \infty$. Using (49) and (50) in (72), we may derive an explicit expression of the high-frequency scattered far field for large $|k|a$.

We now derive a far field expression of the first-order scattered field $\phi^{(1)}(x,z)$. Substituting (31) into (69) with $n = 1$ and using (32) and (33), we obtain an integral representation of the first-order scattered field as in

$$\phi^{(1)}(x,z) = \phi_v^{(1)}(x,z) + \phi_u^{(1)}(x,z)\,, \tag{73}$$

where

$$\phi_v^{(1)}(x,z) = \mp(2\pi)^{-1/2}\int_{-\infty+ic}^{\infty+ic} \frac{e^{-i\alpha a}V_-(\alpha) + e^{i\alpha a}V_{(+)}(\alpha)}{2K(\alpha)}\, e^{\mp\gamma(\alpha)x - i\alpha z}d\alpha, \tag{74}$$

$$\begin{aligned}\phi_u^{(1)}(x,z) = -(2\pi)^{-1/2}\int_{-\infty+ic}^{\infty+ic} \frac{1}{8iK(\alpha)}\Bigg\{ &\left[\gamma^2(\alpha+m) - m(\alpha+m)\right] \\ &\cdot\frac{e^{-i(\alpha+m)a}U_-(\alpha+m) + e^{i(\alpha+m)a}U_{(+)}(\alpha+m)}{K(\alpha+m)} - \left[\gamma^2(\alpha-m) - m(\alpha-m)\right] \\ &\cdot\frac{e^{-i(\alpha-m)a}U_-(\alpha-m) + e^{i(\alpha-m)a}U_{(+)}(\alpha-m)}{K(\alpha-m)}\Bigg\} e^{\mp\gamma(\alpha)x - i\alpha z}d\alpha.\end{aligned} \tag{75}$$

Applying the saddle point method with the aid of the cylindrical coordinate defined by (71), it is found that $\phi_v^{(1)}(x,z)$ has an asymptotic expression.

$$\phi_v^{(1)}(\rho,\theta) \sim \mp \frac{e^{ika\cos\theta}V_-(-k\cos\theta) + e^{-ika\cos\theta}V_{(+)}(-k\cos\theta)}{2K(k\cos\theta)} k\sin|\theta| \, \frac{e^{i(k\rho-\pi/4)}}{(k\rho)^{1/2}} \tag{76}$$

for $x \gtrless 0$ as $k\rho \to \infty$.

For $\phi_u^{(1)}(x,z)$ defined by (75), asymptotic evaluation is in general difficult since the integrand has branch points at $\alpha = \pm k + m$, $\pm k - m$ as well as $\alpha = \pm k$. For simplicity, we assume $|m/k| \ll 1$, which implies that the period of the grating is large compared with the wavelength. Then in the process of asymptotic evaluation, we can ignore contributions from branch-cut integrals occurring due to the branch points at $\alpha = \pm k + m$, $\pm k - m$. Therefore the simple saddle point method may be employed to obtain a far field expression with the result that

$$\begin{aligned}\phi_u^{(1)}(\rho,\theta) \sim \frac{1}{8iK(k\cos\theta)}\sum_{n=1}^{2}(-1)^n\Bigg\{ &\left[4K^2(k\cos\theta^{(n)}) - (-1)^n mk\cos\theta^{(n)}\right] \\ &\cdot\frac{e^{ika\cos\theta^{(n)}}U_-(-k\cos\theta^{(n)}) + e^{-ika\cos\theta^{(n)}}U_{(+)}(-k\cos\theta^{(n)})}{K(k\cos\theta^{(n)})}\Bigg\} k\sin|\theta| \, \frac{e^{i(k\rho-\pi/4)}}{(k\rho)^{1/2}}\end{aligned} \tag{77}$$

for $x \gtrless 0$ as $k\rho \to \infty$, where

$$\theta^{(1)} = \cos^{-1}(\cos\theta - m/k), \quad \theta^{(2)} = \cos^{-1}(\cos\theta + m/k)\,. \tag{78}$$

Substituting (76) and (77) into (73), an asymptotic expression of the first-order scattered field is derived as

$$\phi^{(1)}(\rho,\theta) \sim \mp \frac{e^{ika\cos\theta}V_-(-k\cos\theta) + e^{-ika\cos\theta}V_{(+)}(-k\cos\theta)}{2K(k\cos\theta)} k\sin|\theta| \, \frac{e^{i(k\rho-\pi/4)}}{(k\rho)^{1/2}}$$

$$+\frac{1}{8iK(k\cos\theta)}\sum_{n=1}^{2}(-1)^n\Bigg\{\Big[4K^2(k\cos\theta^{(n)})-(-1)^n mk\cos\theta^{(n)}\Big]$$
$$\cdot\frac{e^{ika\cos\theta^{(n)}}U_-(-k\cos\theta^{(n)})+e^{-ika\cos\theta^{(n)}}U_{(+)}(-k\cos\theta^{(n)})}{K(k\cos\theta^{(n)})}\Bigg\}k\sin|\theta|\,\frac{e^{i(k\rho-\pi/4)}}{(k\rho)^{1/2}} \tag{79}$$

for $x \gtrless 0$ as $k\rho \to \infty$. It is to be noted that (79) gives a uniform asymptotic expression of the first-order scattered far field, and holds for arbitrary incidence and observation angles.

6. NUMERICAL RESULTS AND DISCUSSION

In this section, we shall present numerical examples of the far field intensity and discuss scattering characteristics of the grating in detail. For convenience, let us introduce the normalized far field intensity as in

$$|\phi(\rho,\theta)|\,[\text{dB}] = 20\log_{10}\left[\frac{\lim\limits_{\rho\to\infty}\left|(k\rho)^{1/2}\phi(\rho,\theta)\right|}{\max\limits_{|\theta|<\pi}\lim\limits_{\rho\to\infty}\left|(k\rho)^{1/2}\phi(\rho,\theta)\right|}\right], \tag{80}$$

where

$$\phi(\rho,\theta) = \phi^{(0)}(\rho,\theta) + h\phi^{(1)}(\rho,\theta). \tag{81}$$

We have used the asymptotic expressions given by (72) and (79) in computing (81). As has been mentioned in Section 2, it is important to reduce the original grating problem to the diffraction by a flat strip with a Leontovich-type boundary condition as given by (7) under the small-depth approximation. By careful numerical experimentation, we have found that, if the grating depth $2h$ satisfies $2h \leq 0.1\lambda$ with λ being the free-space wavelength, then (7) can be employed to simulate a perfectly conducting sinusoidal surface with sufficient accuracy. On the other hand, in order to validate the far field asymptotic expression of $\phi_u^{(1)}(x,z)$ in (77), the ratio m/k has been taken as $m/k \leq 0.2$ in numerical computations. Under this condition, contributions from branch-cut integrals due to the branch points at $\alpha = \pm k + m$ and $\pm k - m$ arising in the process of asymptotic evaluation of (75) are small compared with the saddle point contribution and hence, (77) can be used with reasonable accuracy.

Figures 2, 3, 5, and 6 show the scattered far field intensity as a function of observation angle θ for the grating width $2a = 10\lambda$, 50λ, 25λ, and 45λ, respectively, where the incidence angle θ_0 is fixed as $60°$, and $N(\equiv (2a/\lambda)(m/k))$ implies the number of periods of the grating. In these figures, red and blue lines denote the results for H and E polarizations, respectively, where numerical results for E polarization have been generated based on the analysis presented in our previous papers [32, 33]. In each figure, the grating depth has been chosen as $2h = 0.02\lambda$, 0.1λ for investigating the effect of sinusoidal corrugation of the grating. It is important to investigate the differences of scattering characteristics between the perfectly-conducting flat strip and the sinusoidal grating. As mentioned earlier, the zero-order term $\phi^{(0)}(\rho,\theta)$ contained in (81) is the scattered far field for the flat strip. Therefore the far field intensity for the flat strip has been computed using the Wiener-Hopf solution obtained by Kobayashi [34, 35], and the results for the strip width $2a = 10\lambda$, 50λ, 25λ, and 45λ are shown in Figures 4(a), 4(b), 7(a), and 7(b), respectively. There is another important parameter in numerical computation, which is the periodicity (surface roughness) parameter m/k, and is chosen as 0.1 and 0.2, in Figures 2, 3 and 5, 6, respectively.

We see from all the figures that the far field intensity has maximum peaks at $\theta = -120°$ and $120°$, which correspond to the incident and reflected shadow boundaries, respectively. We now compare the results for a finite sinusoidal grating in Figures 2, 3, 5, and 6 with those for a flat strip in Figures 4(a), 4(b), 7(a), and 7(b), respectively. Then it is seen that the effect of sinusoidal corrugation for the grating is noticeable in the reflection region $90° < \theta < 180°$ and the far field intensity has sharp peaks at two particular observation angles around the specularly reflected direction at $\theta = \pi - \theta_0 (= 120°)$. Consideration on the structure of an infinite sinusoidal grating may offer physical understanding of the scattering mechanism at these particular observation angles. Referring to (18), it is seen that $\pi - \theta_1$ and $\pi - \theta_2$ are, respectively, propagation directions of the (-1) and $(+1)$ order diffracted waves involved in the Floquet mode arising in periodic structures of infinite extent [15]. The angles $\pi - \theta_1$, $\pi - \theta_2$ are,

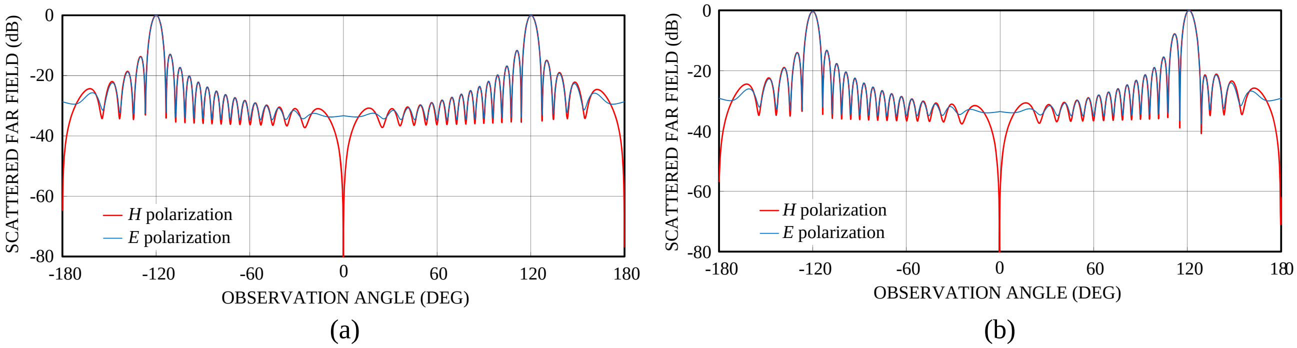


Figure 2. Scattered far field for $\theta_0 = 60°$, $N = 1$, $2a = 10\lambda$, $m/k = 0.1$. (a) $2h = 0.02\lambda$. (b) $2h = 0.1\lambda$.

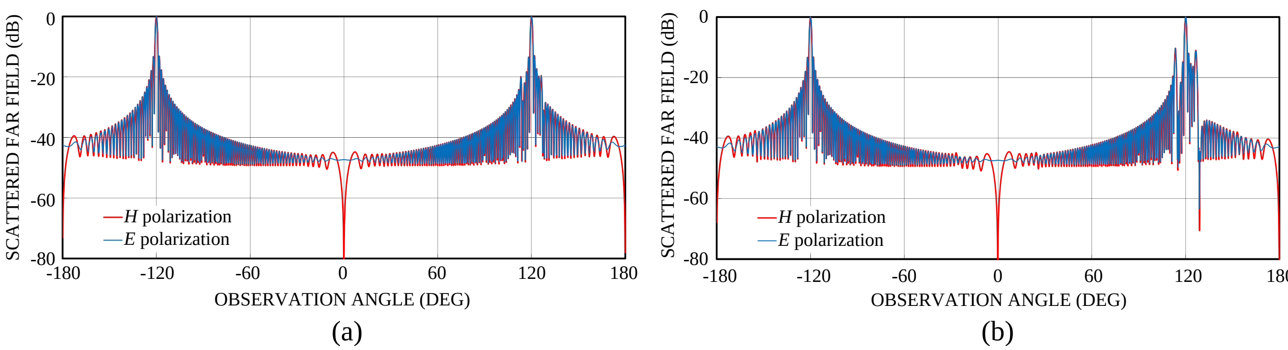


Figure 3. Scattered far field for $\theta_0 = 60°$, $N = 5$, $2a = 50\lambda$, $m/k = 0.1$. (a) $2h = 0.02\lambda$. (b) $2h = 0.1\lambda$.

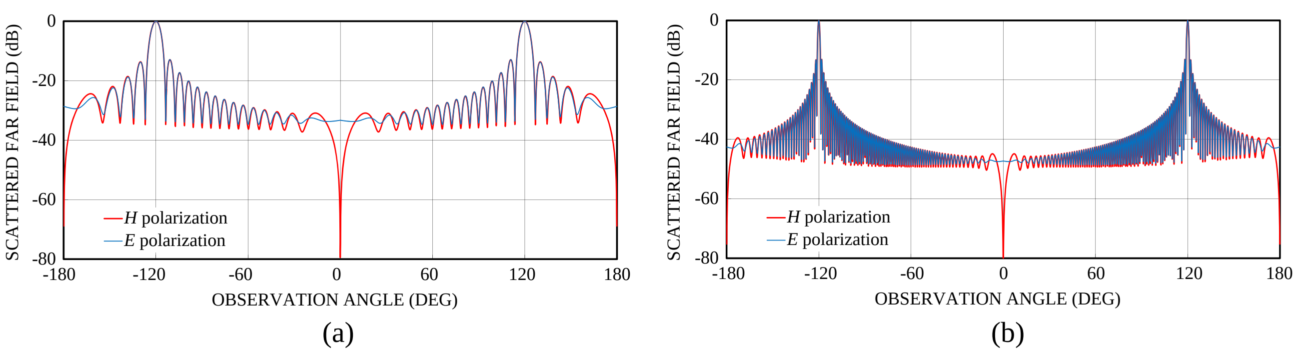
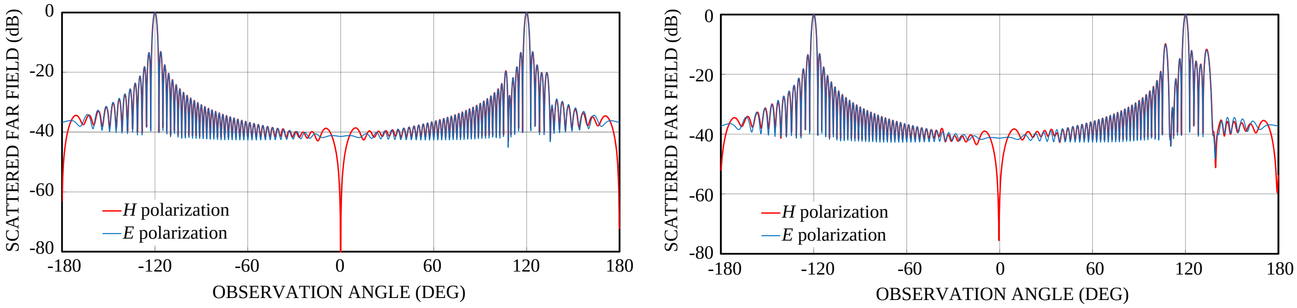


Figure 4. Scattered far field of a flat strip for $\theta_0 = 60°$. (a) $2a = 10\lambda$. (b) $2a = 50\lambda$.

respectively, 113.6°, 126.9° in Figures 2 and 3, and 107.5°, 134.4° in Figures 5 and 6, where somewhat large reflection can be expected. In fact, we see that the observation angles associated with the two peaks around $\pi - \theta_0$ in Figures 2, 3, 5, and 6 are precisely coincident with the directions at $\pi - \theta_1$ and $\pi - \theta_2$. On the other hand, the peaks along the specular reflection $\pi - \theta_0$ is also expected from the grating theory since they exactly correspond to the propagation direction of the zero-order Floquet mode. Therefore it is confirmed that the three peaks at $\pi - \theta_0$, $\pi - \theta_1$, and $\pi - \theta_2$ in the results for the sinusoidal grating are due to the periodicity of the grating surface.

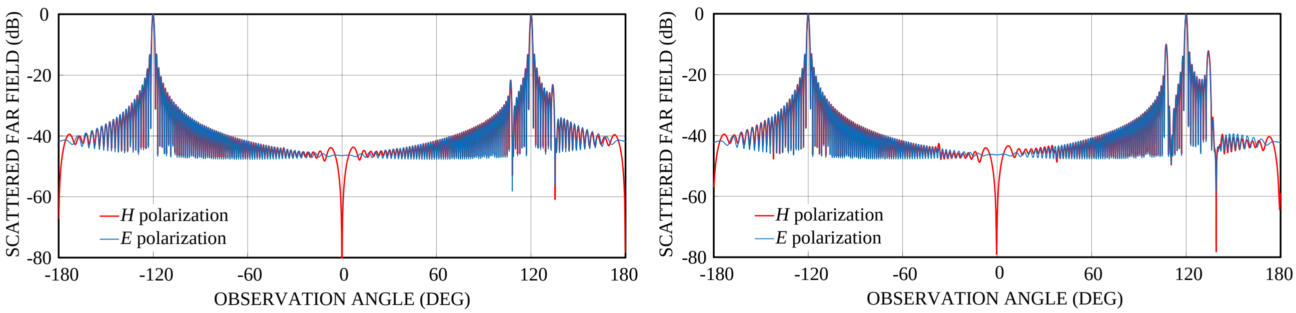
It can also be observed by comparing Figures 3(a), 5(a), and 6(a) with Figures 3(b), 5(b), and 6(b) that the peaks occurring at the $\pi - \theta_1$ and $\pi - \theta_2$ directions become sharper with an increase of $2h$. On comparing Figures 2(a), (b) for $N = 1$ and Figures 5(a), (b) for $N = 5$ with Figures 3(a), (b) for $N = 5$ and Figures 6(a), (b) for $N = 9$, respectively, we see the peaks along $\pi - \theta_1$ and $\pi - \theta_2$ more clearly for larger N. This is because, if N increases, then the structure approaches an infinite sinusoidal grating and hence, waves along the propagation directions of the particular Floquet modes are strongly excited.

We shall now make some comparisons between the two different polarizations. As mentioned earlier, we have already analyzed the diffraction problem involving the same grating geometry for the



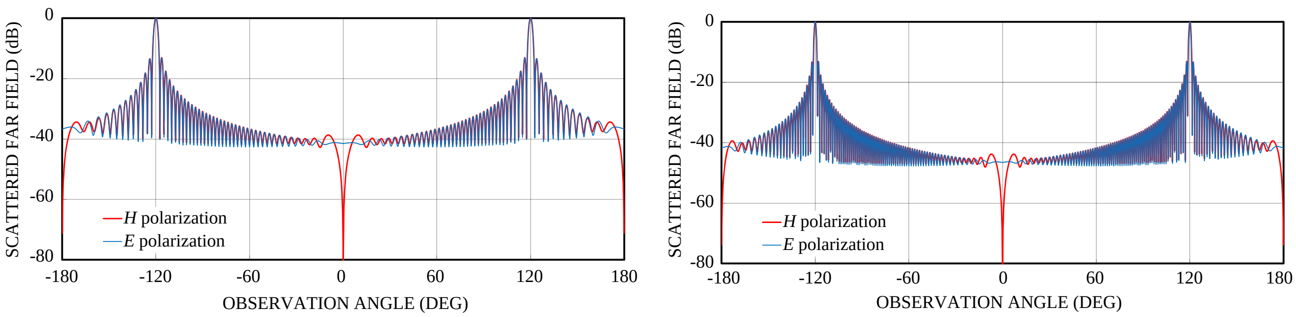
(a) (b)

Figure 5. Scattered far field for $\theta_0 = 60°$, $N = 5$, $2a = 25\lambda$, $m/k = 0.2$. (a) $2h = 0.02\lambda$. (b) $2h = 0.1\lambda$.



(a) (b)

Figure 6. Scattered far field for $\theta_0 = 60°$, $N = 9$, $2a = 45\lambda$, $m/k = 0.2$. (a) $2h = 0.02\lambda$. (b) $2h = 0.1\lambda$.



(a) (b)

Figure 7. Scattered far field of a flat strip for $\theta_0 = 60°$. (a) $2a = 25\lambda$. (b) $2a = 45\lambda$.

E-polarized plane wave incidence using the Wiener-Hopf technique together with the perturbation method [32, 33]. Comparing the results for H polarization (red lines) with those for E polarization (blue lines) in Figures 2–7, some differences can be seen in the scattering characteristics depending on the incident polarization. In particular, stronger oscillation is observed in the H-polarized case in comparison to the E polarization. In addition, we observe nulls around $\theta = 0°$, $\pm 180°$ for the H-polarized case, but such nulls are not seen in the E polarization. Except these differences, general characteristics for both polarizations show similar features.

7. CONCLUDING REMARKS

In this paper, we have analyzed the diffraction by a finite sinusoidal grating for the H-polarized plane wave incidence using the Wiener-Hopf technique combined with the perturbation method. Assuming that the corrugation amplitude is small compared with the wavelength and expanding the scattered field in the form of a perturbation series, the problem has been reduced to the diffraction by a flat strip with a certain mixed boundary condition. Using this approximate boundary condition, the problem has been formulated in terms of the zero- and first-order Wiener-Hopf equations. The Wiener-Hopf equations have been solved via the factorization and decomposition procedure leading to the exact and high-frequency solutions.

Taking the inverse Fourier transform and applying the saddle point method, we have derived a far field asymptotic expression of the scattered field, which is shown to be valid for arbitrary incidence and observation angles. Based on the results, we have carried out numerical computation of the far field intensity and investigated the effect of sinusoidal corrugation of the grating in detail. As a result, it has been confirmed that, in the reflection region, scattered waves are strongly excited along the specific directions corresponding to the three dominant Floquet modes.

Our final solution is valid for the case where the depth and period of the grating is small and large compared with the wavelength. Hence, the method of solution developed in this paper may become less accurate for deep or dense gratings. Our idea for approximating the boundary condition on the grating surface is to expand it in terms of a Taylor series around the average surface and use the zero- and first-order terms in the Wiener-Hopf analysis. By keeping up to the second and higher order terms in the Taylor series, it may be possible to extend the range of applicability of the method. This can be investigated as a future issue.

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