Progress In Electromagnetics Research, Vol. 149, 173–182, 2014

A Third Way to Cloak an Object: Cover-up with a Background Object

Fei Sun[1, 2] and Sailing He[1, 2, *]

(*Invited Paper*)

Abstract—Based on a space folding transformation, we propose a new way to hide an object in full space, namely, to cover-up the scattering of the hidden object with the scattering of a background object so that only the scattering of the background object can be detected by an outside observer and the hidden object disappears electromagnetically (a very weak "ghost image" or perturbation may appear inside the strong background object image in an experiment). The present method is essentially different from previous methods of cloaking an object, namely, the optically isolated cloak and the scattering cancellation cloak, and thus provide a third way to cloak an object. Unlike all the previous methods of cloaking, the present full-space omni-directional invisibility simultaneously has the following features: (i) the hidden object can "see" the outside world without being detected; (ii) the cloak can still work when some characteristics (e.g., shape and medium) of the hidden object change; and (iii) there is no need to know the information of the incident wave in advance. The present work furthers efforts to achieve invisibility and conceal an object in a real environment in full space.

1. INTRODUCTION

Invisibility, which has been a long-time goal, has received many studies with transformation optics (TO), meta-materials and other methods in recent years [1–16]. Readers can get some background on this topic with some recent reviews (see e.g., [17, 18]). There are many different ways to classify the types of invisibility cloaks (e.g., inner or outer cloak, active or passive cloak). We can classify the previous invisibility cloaks into two main types based on the principle of how to achieve invisibility. One type is the optically isolated cloak (OIC), which is often designed with TO [1–5] or similar methods [6]. When an incoming electromagnetic wave enters the OIC, it will be smoothly guided around the concealed region and propagates back to its original direction (the wave front will not be disturbed). From the view of optics, the hidden region is isolated from the outside world "optically". The other type is the scattering cancellation cloak (SCC) [7–12]. An SCC works by eliminating or greatly reducing the scattering cross section of the hidden object. In this case, the hidden object, which is not optically isolated from the outside world, can receive electromagnetic waves of same frequency band from the outside world.

Based on TO, an OIC can be easily designed by extending a point in an enclosed region in the reference space into a volume (the hidden region) in the real space while keeping the space outside this enclosed region untransformed [1]. The hidden region in the real space that corresponds to the point in the reference space is optically isolated from the other regions in the real space, i.e., the incident beam from the outside world will never touch the hidden region. An identical transformation outside the enclosed region (the outer boundary of the cloak) ensures that the incident electromagnetic wave

Received 3 October 2014, Accepted 14 October 2014, Scheduled 15 October 2014

* Corresponding author: Sailing He (sailing@jorcep.org).

[1] Centre for Optical and Electromagnetic Research, Zhejiang Provincial Key Laboratory for Sensing Technologies, East Building #5, Zijingang Campus, Zhejiang University, Hangzhou 310058, China. [2] Department of Electromagnetic Engineering, JORCEP, School of Electrical Engineering, Royal Institute of Technology (KTH), S-100 44 Stockholm, Sweden.

will be undisturbed after it passes through the OIC. Note that the OIC can also be designed with other methods, besides point expansion [6]. Since a 3D free-space OIC requires fabrication of 3D metamaterial that is not easy to realize under current technologies, most studies are focused on the 2D OIC [13]. Many researchers try to make such OICs easier to construct (e.g., triangle transformations to eliminate the inhomogeneity of the OIC [14], Eikonal approximation to reduce the inhomogeneity [3]). Many methods have been proposed to remove the singularity at the inner surface of the point-expanded OIC, including sacrificing the performance of the cloak (e.g., extend an extremely small volume but not a point to a finite volume [15]), abolishing the requirement for phase preservation [16], or introducing Non-Euclidean space in the reference space (see e.g., [4] and references therein). However there is always an essential problem in an OIC: the detecting wave doesn't touch the hidden object (it is simply guided around it), which leads to the fact that the cloaked object cannot communicate with the outside world in the same frequency band. This means that we can hide an object (e.g., a person) in a perfect OIC at the cost of being blind to the outside world. This OIC is not an ideal invisibility cloak that appears in science fiction: we need to see the outside world without being seen.

The SCC does not have this problem, as the incident wave interacts with both the hidden object and the SCC. Therefore the hidden object can touch the wave from the outside world in an SCC, which means it is not blind and can see the outside world. An SCC can mainly be classified into two types. One is an active SCC that is composed of some active sources [11] or active Huygens-surfaces [12]. The other type is a passive SCC including a complementary medium method [10], plasmonic shells or mantle cloaks [7], and cloaks based on some optimization algorithms (e.g., genetic algorithm [8] and topology optimization algorithm [9]). For an active SCC (which often can work in a broadband frequency range), the user has to know the incident wave information (including both amplitude and phase information) in advance. This is the main drawback of the active SCC. For a passive SCC, it has the main drawback that the cloak will not work when the hidden object changes (e.g., its shape, size, or medium changes). This is a common drawback of a passive SCC regardless of the principle (e.g., complementary medium, plasmonic cloak or the cloak based on optimization algorithms) it is based on. We should also note that a plasmonic cloak and a cloak based on some optimization algorithm can often work in a relatively broadband range. They do not provide ideal, perfect invisibility but greatly reduce the scattering of the cloaked object (e.g., canceling the dominant scattering terms in the multipole expansion of the scattered field). A complementary medium-based SCC can achieve perfect scattering cancellation theoretically. However, a complementary medium-based SCC cannot conceal any object composed of a perfect electric conductor (PEC), as PECs do not have any corresponding complementary medium.

We can also mention carpet cloak [19]. So far, many experimental demonstrations are of such a kind of cloak. However carpet cloak does not provide a real sense of invisibility compared with the full space cloak mentioned above. The function of a carpet cloak is to hide an object under a curved ground (PEC ground or dielectric ground [20]) and make this curved ground looks like a flat one. We do not consider a carpet cloak in this paper.

To summarize the results of recent studies on invisibility cloaks: so far there is still no method to achieve full space omni-directional invisibility that simultaneously has the following features: (i) the hidden object can "see" the outside world without being detected; (ii) the cloak can still work when the hidden object changes; and (iii) there is no need to know the information of the incident wave in advance. In this work, we will propose a novel method to achieve a full space omni-directional invisibility cloak that has the above three features at the same time. The key point of our method is that the hidden object is not alone in a free space but there is some other background object (e.g., a tree, a car, a trash can, etc.) nearby. Note that this condition can be easily fulfilled in a real environment: we live in a space with many objects other than the object to be hidden.

The basic principle of our cloak is shown in Figs. 1(a) and 1(b). We use the idea of scattering coverage to conceal the object under some other object in the background. Such a scattering coverage can be achieved by using the space folding transformation. The key idea of the present novel method is that the hidden object (e.g., a mouse) is visually folded into the position of the background object by our cloak. If the size of the hidden object is smaller than the background object and the scattering of the background object (e.g., PEC) is very strong at the boundary (we choose PEC as the background object in the numerical simulation), the scattering of the hidden object can be covered by the scattering of the background object, and hence the observer can only see the background object.

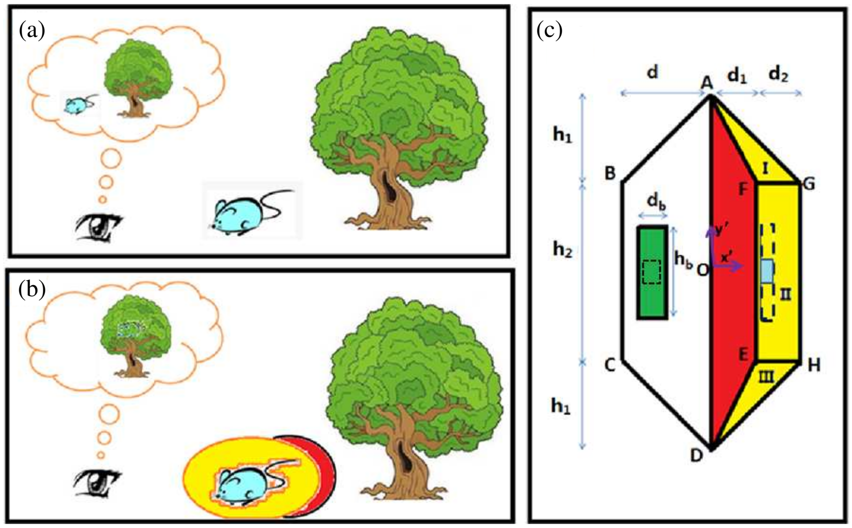

Figure 1. The basic principle of our invisibility cloak: the hidden object here is a mouse, and the background object is a tree. (a) If there is no cloak applied around the mouse, the observer can see both the mouse and the tree. (b) If we put the designed cloak (the yellow compression medium and the red space folding medium) around the mouse, the observer can only see the background tree [the mouse disappears; practically one may still see a very weak perturbation or "ghost image" (related to the scattering from the mouse) in a small region of the strong image of the tree]. (c) This specific structure is chosen as our cloak. The white region is air. We put the object to be hidden in the blue region, which will be folded into the part (indicated by the dashed line) inside the green region (the background object). The dashed line in the yellow region indicates the boundary which is exactly folded into the whole green region, which limits the maxima size of the object to be hidden.

We choose a specific structure shown in Fig. 1(c) to demonstrate our idea. A similar structure has been utilized in our recent work on transformation magneto-statics and illusions for magnets [21]. The background object is a green rectangle with width d_b and height h_b within a region of air. The red region is a space folding medium which folds boundary AFED to boundary ABCD. The yellow region is the compression medium: when boundary AFED is folded into boundary ABCD by the space folding medium, the yellow region AFEDHG filled with the space compression medium is mapped to the whole structure ABCDHG filled with air. The dashed rectangle in the yellow region is the maximum size of the hidden object (in the blue region), which is exactly folded into the green region. The object to be hidden in the blue region is folded into the dashed green part inside the green region (the background object). If the hidden object is no larger than the dashed rectangular region, it can be hidden by placing it inside the blue region (other redundant space is filled by the space compression yellow medium II in this paper). The red space folding medium and yellow compression medium together will fold the hidden object into the dashed region inside the background green object while keeping the other region to still be air. If the background green object is a PEC or some other kind of medium that electromagnetic waves will be strongly scattered at the boundary, the scattering feature of the hidden object will be covered by the strong scattering of the background object, as the hidden object is optically folded into the background object that the incident wave will be strongly scattered at the boundary.

The materials in each region can be calculated using TO [1, 18]:

$$\begin{cases} \varepsilon^{i'j'} = \dfrac{1}{\det(\Lambda)} \Lambda_i^{i'} \Lambda_j^{j'} \varepsilon^{ij} \\ \mu^{i'j'} = \dfrac{1}{\det(\Lambda)} \Lambda_i^{i'} \Lambda_j^{j'} \mu^{ij} \end{cases}, \tag{1}$$

where $\Lambda_i^{i'} = \partial x^{i'}/\partial x^i$. $\varepsilon^{i'j'}$ and $\mu^{i'j'}$ are the relative permittivity and permeability in the real space. ε^{ij}

and μ^{ij} are the relative permittivity and permeability in the reference space. The reference here is free space (e.g., $\varepsilon^{ij} = \mu^{ij} = 1$). Note that the quantities with or without prime correspond to the ones in the real and reference space, respectively. We apply a space folding transformation to the red region:

$$x' = -\frac{d_1}{d}x; \quad y' = y; \quad z = z. \tag{2}$$

The space folding medium in the red region ADEF can be calculated with the help of Eq. (1):

$$\varepsilon'_{red} = \mu'_{red} = \mathrm{diag}\left(-\frac{d_1}{d}, -\frac{d}{d_1}, -\frac{d}{d_1}\right). \tag{3}$$

We apply the space compression transformation to the yellow region. We divide the yellow region into three sub-regions: triangular region I, rectangular region II, and triangular region III. The coordinate transformation in each region can be given as:

$$x' = \begin{cases} \dfrac{d_2}{d+d_1+d_2}x - \dfrac{(d+d_1)(d_1+d_2)}{h_1(d+d_1+d_2)}y + \dfrac{(d+d_1)(d_1+d_2)}{h_1(d+d_1+d_2)}\left(h_1+\dfrac{h_2}{2}\right), & \text{yellow region I} \\ \dfrac{d_2}{d+d_1+d_2}x + \dfrac{(d+d_1)(d_1+d_2)}{d+d_1+d_2}, & \text{yellow region II} \\ \dfrac{d_2}{d+d_1+d_2}x + \dfrac{(d+d_1)(d_1+d_2)}{h_1(d+d_1+d_2)}y + \dfrac{(d+d_1)(d_1+d_2)}{h_1(d+d_1+d_2)}\left(h_1+\dfrac{h_2}{2}\right), & \text{yellow region III} \end{cases} ;\ y'=y;\ z'=z. \tag{4}$$

The space compression medium in the yellow region can also be calculated with the help of Eq. (1):

$$\begin{cases} \varepsilon'_{\text{yellow,I}} = \mu'_{\text{yellow,I}} = \begin{bmatrix} \dfrac{P^2+Q^2}{P} & -\dfrac{Q}{P} & 0 \\ -\dfrac{Q}{P} & \dfrac{1}{P} & 0 \\ 0 & 0 & \dfrac{1}{P} \end{bmatrix} \\ \varepsilon'_{\text{yellow,II}} = \mu'_{\text{yellow,II}} = \mathrm{diag}(P, \dfrac{1}{P}, \dfrac{1}{P}) \\ \varepsilon'_{\text{yellow,III}} = \mu'_{\text{yellow,III}} = \begin{bmatrix} \dfrac{P^2+Q^2}{P} & \dfrac{Q}{P} & 0 \\ \dfrac{Q}{P} & \dfrac{1}{P} & 0 \\ 0 & 0 & \dfrac{1}{P} \end{bmatrix} \end{cases}, \tag{5}$$

where

$$P = \frac{d_2}{d+d_1+d_2}; \quad Q = \frac{(d+d_1)(d_1+d_2)}{h_1(d+d_1+d_2)}. \tag{6}$$

Note that Eqs. (3) and (5) are expressed in the Cartesian coordinate system. The blue region is filled by the object to be hidden and the green region is the background object. In the following numerical example, we choose a PEC object (so that the scattered field is strong) as the background object while the object to be hidden can be either a PEC object (different from illusion optics, where PEC does not have any corresponding complementary medium) or a dielectric object.

When we put an object to be hidden inside our cloak (in the blue region in Fig. 1(c)), the transformation media (red and yellow regions) will fold the object to be hidden into the green PEC rectangle (the background object) electromagnetically. As the incident wave can hardly penetrate into the PEC background object (the green region) where the object to be hidden in the blue region is virtually folded into, the object to be hidden in the blue region will produce no additional scattering to the incident wave because the overall green region will still behave like a PEC (the ghost image of the object to be hidden is folded into the green PEC region). For simplicity in the simulation, we choose a specific geometrical structure with $d_1 = d_2 = d/2$ and $h_1 = h_2/2 = d$ in our following simulations. In this case, Eqs. (3) and (5) can be reduced to

$$\begin{cases} \varepsilon'_{red} = \mu'_{red} = \mathrm{diag}\left(-\frac{1}{2}, -2, -2\right) \\ \varepsilon'_{\text{yellow,I}} = \mu'_{\text{yellow,I}} = \begin{bmatrix} 2.5 & -3 & 0 \\ -3 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \\ \varepsilon'_{\text{yellow,II}} = \mu'_{\text{yellow,II}} = \mathrm{diag}(0.25, 4, 4) \\ \varepsilon'_{\text{yellow,III}} = \mu'_{\text{yellow,III}} = \begin{bmatrix} 2.5 & 3 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \end{cases} \tag{7}$$

In the following studies, we keep the background object (a rectangular PEC with width d_b and height h_b) and the cloak described by Eq. (7) invariant, and only change the object to be hidden in the blue region in Fig. 1(c). We use numerical simulation based on finite element method to verify the performance of our novel cloak. The FEM simulation is performed by using commercial software COMSOL Multiphysics. We first simulate cases that only the background object exists (see Figs. 2(a) and 2(b)). Then we simulate the case when both the hidden object (a rectangular PEC in our numerical example) with the cloak and the background object exist (see Figs. 2(c) and 2(d)). The total field distributions are the same in these two cases, which verify the fact that the scattering of the hidden object is covered up perfectly by the background object. Note that in this case the PEC object to be hidden is filled up the whole dashed rectangular yellow region in Fig. 1(c). We also simulate the case when both the object to be hidden (but without cloak) and the background object exist (see Figs. 2(e) and 2(f)). Compared to Figs. 2(b), 2(d) and 2(f), we can obviously see that when we put the designed cloak around the object to be hidden beside the background object, the scattering feature of the whole system is the same as the scattering of only a single background object, and clearly different from the case without the cloak. To describe the cloaking effect quantitatively, we introduce the averaged relative change γ of the scattered field, which is defined as:

$$\gamma := \frac{\int |E_2^s - E_1^s|\, dl}{\int |E_1^s| dl} \tag{8}$$

where E_1^s is the scattered field when only the background object exists and E_2^s is the scattering field when other objects (e.g., cloak and the object to be hidden) are introduced aside the original background object. The integration is applied on the whole left or top edge which stands for the observer's field of view (we use the subscript 'left' or 'top' to indicate the location of the observer). For Figs. 2(c) and 2(d) (with our cloak), we have very small perturbation of the scattered field, namely, $\gamma_{\text{left}} = 1.66\%$, and $\gamma_{\text{top}} = 1.67\%$, respectively. For Figs. 2(e) and 2(f) (without our cloak), we have much larger averaged relative changes, namely, $\gamma_{\text{left}} = 55.63\%$ and $\gamma_{\text{top}} = 109.64\%$, respectively.

Next we will study the case when the object to be hidden is not a PEC but some dielectric medium (same size as the object to be hidden in Fig. 2 and the cloak is identical to the one used for Fig. 2). As shown in Fig. 3, the cloaking effect is still kept in this case. For the case with our cloak in Figs. 3(a) and 3(b), we have $\gamma_{\text{left}} = 4.38\%$, and $\gamma_{\text{top}} = 17.29\%$, respectively. For the case without cloak in Figs. 3(c) and 3(d), we have much larger averaged relative changes, namely, $\gamma_{\text{left}} = 9.59\%$ and $\gamma_{\text{top}} = 86.45\%$, respectively. Note that the slight imperfection of our cloak (i.e., small perturbation in the scattered field) is mainly due to the overlapped boundary of the PEC boundary and the boundary of the transformed dielectric object. This overlapped boundary effect can be reduced significantly if the size of the dielectric object to be hidden is reduced slightly (e.g., will not fill up the whole dashed yellow region in Fig. 1(c)), which corresponds to the case in Figs. 3(e) and 3(f): we have $\gamma_{\text{left}} = 1.29\%$, and $\gamma_{\text{top}} = 7.66\%$, respectively. If we further reduce the size of the dielectric object to be hidden (see Figs. 3(g) and 3(f)), we have $\gamma_{\text{left}} = 0.059\%$, and $\gamma_{\text{top}} = 4.48\%$, respectively. For a dielectric object case, the smaller size of the object to be hidden, the better cover-up effect of our cloak.

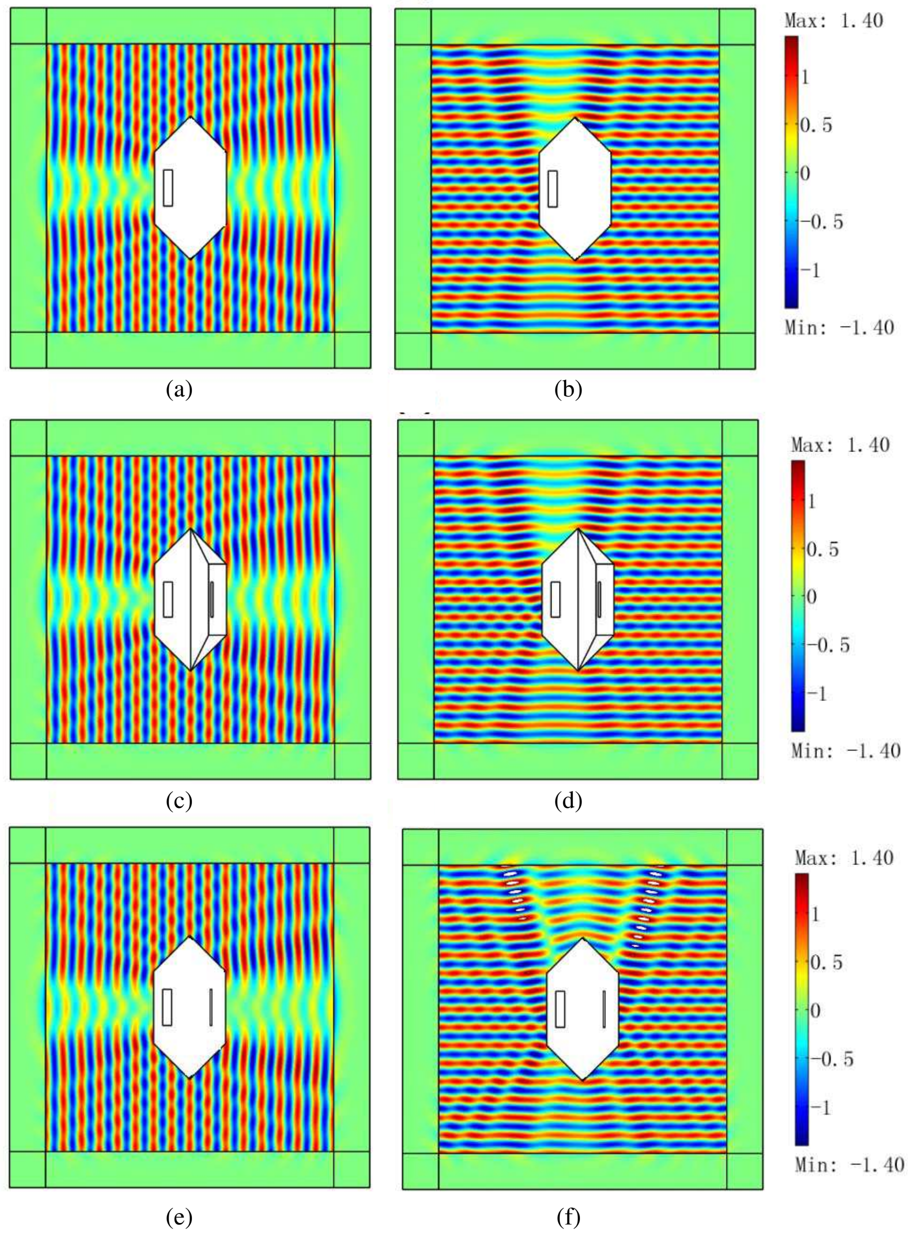

Figure 2. FEM simulation results of the total electric field distribution when a harmonic plane wave with normalized amplitude is incident along the x direction [from the left to the right; (a), (c) and (e)] or along the y direction [from the bottom to the top; (b), (d) and (f)]. We choose $d_1 = d_2 = d/2$ and $h_1 = h_2/2 = d = 2\lambda$. In this case the cloak can be described by Eq. (7). The background object is a PEC rectangle with width $d_b = 0.5\lambda$ and height $h_b = 2\lambda$. The hidden object is a rectangular PEC filled up the whole dashed rectangle in Fig. 1(c) (width 0.125λ and height 2λ). (a) and (b): only the background object exists. (c) and (d): the hidden object with the cloak is put aside the background object. (e) and (f): the object (to be hidden) without the cloak is put aside the background object.

Next we change both the shape and medium of the object to be hidden, which is composed of three dielectric cylinders (our numerical examples are for 2D for simplicity) with radius 0.04λ and relative permittivity $\varepsilon = 20$, 30, and 50 filled in the blue region (other redundant space in the blue region is the space compression yellow medium II). The simulation results are given in Fig. 4, which shows that

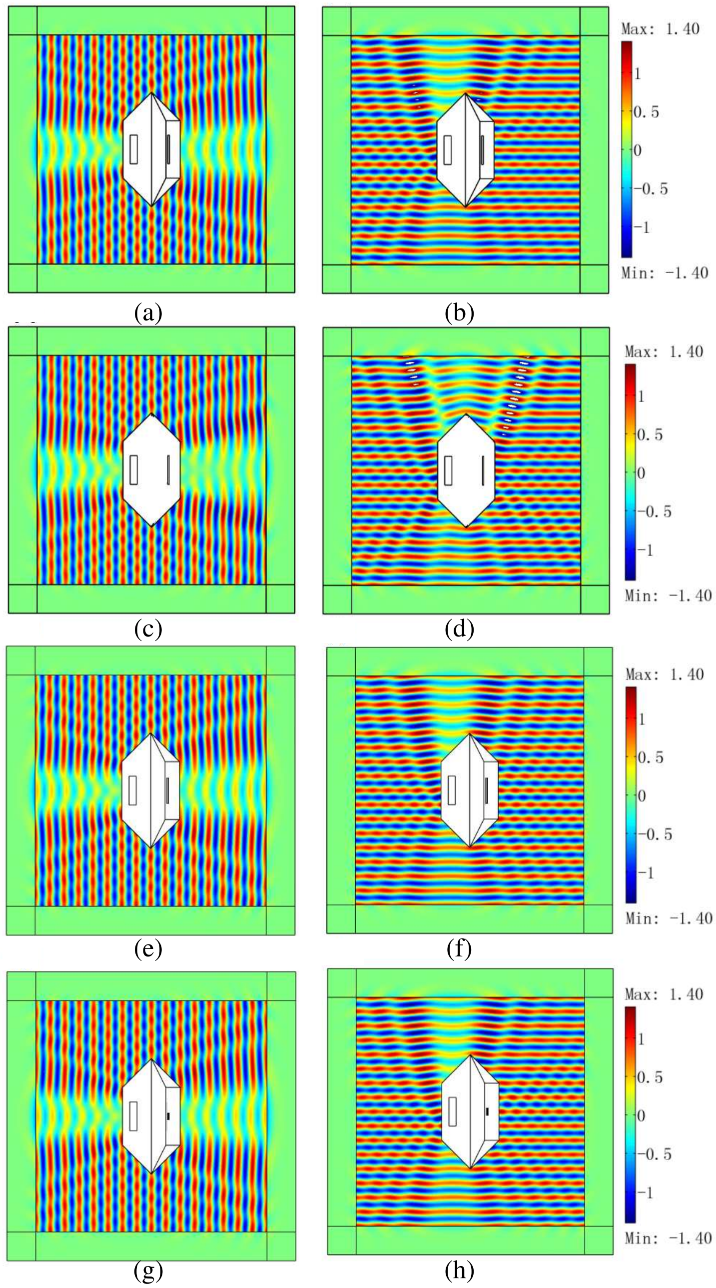

Figure 3. FEM simulation results of the total electric field distribution when a harmonic plane wave with normalized amplitude is incident along the x direction [from the left to the right; (a), (c), (e) and (g)] or along the y direction [from the bottom to the top; (b), (d), (f), and (h)]. The geometrical parameters of the cloak and the background object are the same as the ones in Fig. 2. The object to be hidden is a dielectric rectangle with relative permittivity $\varepsilon = 60$ and relative permeability $\mu = 1$. (a) and (b): the object to be hidden with the cloak is put aside the background object. The object to be hidden has a width of 0.125λ, and a height of 2λ (i.e., filling up the whole dashed yellow rectangle in Fig. 1(c)). (c) and (d): the object (to be hidden) without the cloak is put aside the background object. (e) and (f) the object to be hidden with the cloak is put aside the background object. The object to be hidden here has a width of 0.1λ, and a height of 1.8λ (it does not filling up the whole dashed yellow rectangle in Fig. 1(c)). (g) and (h) the object with the cloak is put aside the background object. The object to be hidden has a width of 0.04λ, and a height of 0.5λ (filling up only a small part of the dashed yellow rectangle in Fig. 1(c)).

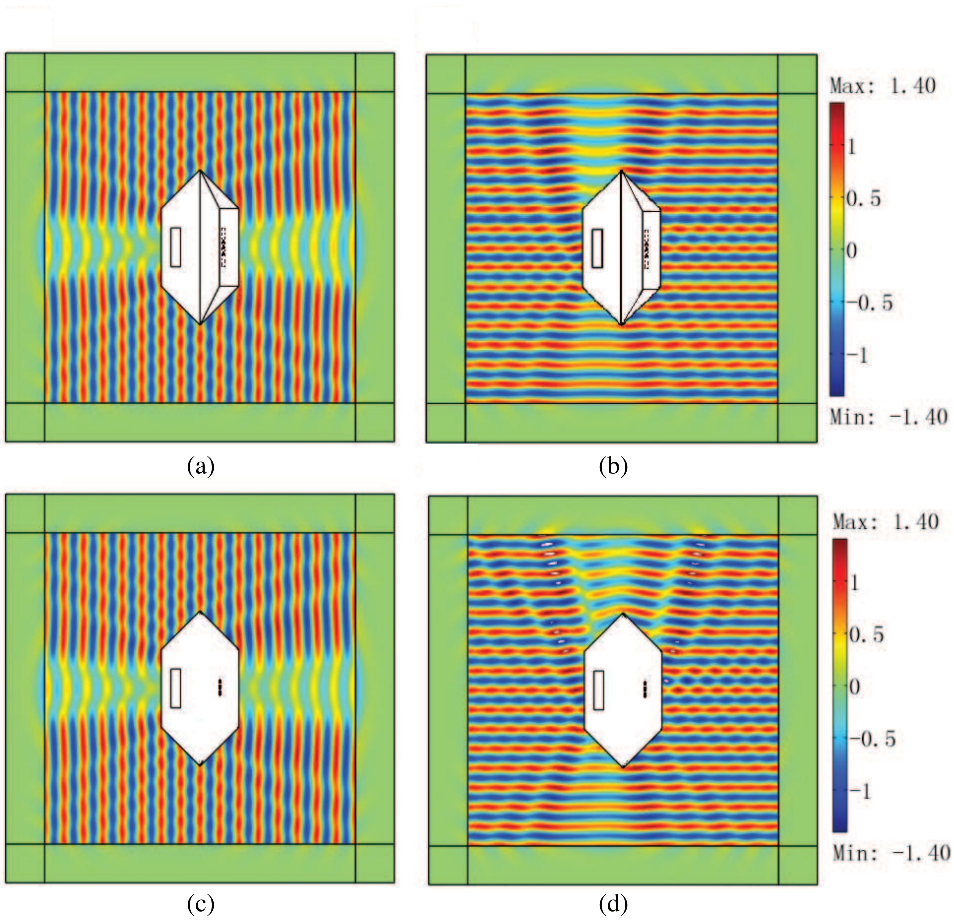

Figure 4. FEM simulation results of the total electric field distribution when a harmonic plane wave with normalized amplitude is incident along the x direction [from the left to the right; (a) and (c)] or along the y direction [from the bottom to the top; (b) and (d)]. The geometrical parameters of the cloak and the background object are the same as the ones in Fig. 2. The objects to be cloaked here are three dielectric cylinders with radius 0.04λ and relative permittivity $\varepsilon = 20$, 30, and 50, filled in the blue rectangular region with width 0.125λ, height 2λ, with the space compression yellow medium II filled in the redundant space in the blue region in Fig. 1(c). (a) and (b): the object (to be hidden) with the cloak is put aside the background object. (c) and (d): the object (to be hidden) without the cloak is put aside the background object.

our cloak (identical to the cloak used for Fig. 2) can still provide perfect scattering cover-up effect. We should note that the electromagnetic wave can touch the hidden object, which means that the cloaked region is not isolated from the outside world in our cloak. In Figs. 4(a) and 4(b) (with our cloak), we have $\gamma_{\text{left}} = 0.49\%$ and $\gamma_{\text{top}} = 4.33\%$, respectively. In Figs. 4(c) and 4(d) (without our cloak), we have much larger averaged relative changes, namely, $\gamma_{\text{left}} = 4.12\%$ and $\gamma_{\text{top}} = 78.34\%$, respectively.

All above analyses indicate that our cloaking effect is nearly ideal theoretically if the background object is PEC. In practice, there are many kinds of objects (not a PEC) for which an incident wave will be largely scattered at the outer boundary (e.g., a wall, door, car, etc.), and the present method can still work.

The main limitation of the proposed cloak here is that it requires negative refraction medium (the red space folding medium) to achieve a space folding effect, which will limit its bandwidth. One possible solution is to find some other kinds of transformations that do not involve the space folding but can still cover-up the scattering of the object to be hidden. The proposed cloak can also be realized using layered homogeneous isotropic media (similar to how an OIC was realized [28–30]).

2. CONCLUSION AND DISCUSSION

We should note that although we use the space folding transformation mathematically similar to the illusion optics (or special SSC) based on some complementary medium design. The physical essence of our cloak is totally different from the illusion optics (special SSC) based on the complementary medium. We list the following differences between these two methods: (i) The design based on illusion optics includes two kinds of media: one is the complementary medium to cancel the scattering of the object to be hidden; the other is the restoring medium to create the desired optical illusion. Our cloak in this paper doesn't need the complementary medium (that needs to be designed for the specific object to be hidden in illusion optics). Neither do we need to cancel the scattering of the object to be hidden (we just need to optically fold the image of the object to be hidden to the position where the background object is). The scattering of the object to be hidden is covered up by the strong scattering of the background object, but not be canceled. This is essentially different from the previous design based on the complementary medium. (ii) The previous design based on the complementary medium cannot hide a PEC object, as there is no complementary medium for the PEC. However, our cloak in this paper can hide a PEC object (as shown in the example of Fig. 2). (iii) The design based on the complementary medium is very sensitive to the object to be hidden: if the shape or material of the object to be hidden changes, the complementary medium cannot cancel its scattering any more. However, our cloak is not so sensitive to the characteristics (e.g., shape and medium) of the object to be hidden (as we have shown that the identical cloak works for all the examples in Figs. 2–4). (iv) The design based on the complementary medium can be achieved without any background medium. The cloak effect in the present paper cannot be achieved if there is no background object (with strong scattering) near the object to be hidden.

A new finding in the field of invisibility has been made in this paper by covering-up the scattering of the object to be hidden with the scattering of some other background object. In an experiment one would see only the background object (possibly with a very weak "ghost image" or perturbation inside the strong background object image). A device designed by the space folding transformation has been utilized to demonstrate this idea. The numerical simulations have shown that even if some characteristics (e.g., shape and medium) of the object to be cloaked change, the same cloak device would still work practically. Compared with the OIC, the hidden object inside the cloak proposed in the present paper can communicate with the outside world (as the electromagnetic wave touches the cloaked object). Compared with the active SCC, the cloak proposed here does not need to know the information of incident waves in advance. Compared with the passive SCC, the cloaking effect in this paper can still be maintained even if we change some characteristics of the cloaked object.

ACKNOWLEDGMENT

This work is partially supported by the National High Technology Research and Development Program (863 Program) of China (No. 2012AA030402), the National Natural Science Foundation of China (Nos. 61178062 and 60990322), the Program of Zhejiang Leading Team of Science and Technology Innovation, Swedish VR grant (# 621-2011-4620) and AOARD. Fei Sun thanks the China Scholarship Council (CSC) No. 201206320083.

REFERENCES

1. Pendry, J. B., D. Schurig, and D. R. Smith, "Controlling electromagnetic fields," *Science*, Vol. 312, 1780–1782, 2006.
2. Schurig, D., J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, "Metamaterial electromagnetic cloak at microwave frequencies," *Science*, Vol. 314, No. 5801, 977–980, 2006.
3. Ma, Y., Y. Liu, L. Lan, T. Wu, W. Jiang, C. K. Ong, and S. He, "First experimental demonstration of an isotropic electromagnetic cloak with strict conformal mapping," *Sci. Rep.*, Vol. 3, 2182, 2013.
4. Xu, T., Y. C. Liu, Y. Zhang, C. K. Ong, and Y. G. Ma, "Perfect invisibility cloaking by isotropic media," *Phys. Rev. A*, Vol. 86, No. 4, 043827, 2012.

5. Kanté, B., D. Germain, and A. de Lustrac, "Experimental demonstration of a nonmagnetic metamaterial cloak at microwave frequencies," *Phys. Rev. B*, Vol. 80, No. 20, 201104, 2009.
6. Howell, J. C., J. B. Howell, and J. S. Choi, "Amplitude-only, passive, broadband, optical spatial cloaking of very large objects," *Appl. Opt.*, Vol. 53, No. 9, 1958–1963, 2014.
7. Alù, A., "Mantle cloak: Invisibility induced by a surface," *Phys. Rev. B*, Vol. 80, No. 24, 245115, 2009.
8. Xu, S., X. Cheng, S. Xi, R. Zhang, H. O. Moser, Z. Shen, and H. Chen, "Experimental demonstration of a free-space cylindrical cloak without superluminal propagation," *Phys. Rev. Lett.*, Vol. 109, No. 22, 223903, 2012.
9. Lan, L., F. Sun, Y. Liu, C. K. Ong, and Y. Ma, "Experimentally demonstrated a unidirectional electromagnetic cloak designed by topology optimization," *Appl. Phys. Lett.*, Vol. 103, No. 12, 121113, 2013.
10. Lai, Y., H. Chen, Z. Q. Zhang, and C. T. Chan, "Complementary media invisibility cloak that cloaks objects at a distance outside the cloaking shell," *Phys. Rev. Lett.*, Vol. 102, No. 9, 093901, 2009.
11. Vasquez, F. G., G. W. Milton, and D. Onofrei, "Active exterior cloaking for the 2D Laplace and Helmholtz equations," *Phys. Rev. Lett.*, Vol. 103, No. 7, 073901, 2009.
12. Selvanayagam, M. and G. V. Eleftheriades, "Discontinuous electromagnetic fields using orthogonal electric and magnetic currents for wavefront manipulation," *Opt. Express*, Vol. 21, No. 12, 14409–14429, 2013.
13. Zhang, P., Y. Jin, and S. He, "Obtaining a nonsingular two-dimensional cloak of complex shape from a perfect three-dimensional cloak," *Appl. Phys. Lett.*, Vol. 93, No. 24, 243502, 2008.
14. Zhang, J., L. Liu, Y. Luo, S. Zhang, and N. A. Mortensen, "Homogeneous optical cloak constructed with uniform layered structures," *Opt. Express*, Vol. 19, No. 9, 8625–8631, 2011.
15. Chen, H. and B. Zheng, "Broadband polygonal invisibility cloak for visible light," *Sci. Rep.*, Vol. 2, 255, 2012.
16. Chen, H., B. Zheng, L. Shen, H. Wang, X. Zhang, N. I. Zheludev, and B. Zhang, "Ray-optics cloaking devices for large objects in incoherent natural light," *Nat. Commun.*, Vol. 4, 2652, 2013.
17. Fleury, R. and A. Alù, "Cloaking and invisibility: A review," *Forum for Electromagnetic Research Methods and Application Technologies (FERMAT)*, Vol. 1, No.7, 1–24, 2014.
18. Pendry, J. B., "Controlling Light on the Nanoscale," *Progress In Electromagnetics Research*, Vol. 147, 117–126, 2014.
19. Li, J. and J. B. Pendry, "Hiding under the carpet: A new strategy for cloaking," *Phys. Rev. Lett.*, Vol. 101, No. 20, 203901, 2008.
20. Zhang, P., M. Lobet, and S. He, "Carpet cloaking on a dielectric half-space," *Opt. Express*, Vol. 18, No. 17, 18158–18163, 2010.
21. Sun, F. and S. He, "Transformation magneto-statics and illusions for magnets," *Sci. Rep.*, Vol. 4, 6593, 2014.