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Accurate Coupling Matrix Synthesis for Microwave Filters with Random Initial Value

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Abstract—A hybrid optimization method that synthesizes coupling matrices for cross-coupled microwave filters is presented. This method consists of a general solvopt algorithm and fmincon algorithm, respectively. To avoid divergence from the coupling matrix, two cost functions are built, where the first one is constructed from the eigenvalues of the coupling matrix and its principal sub-matrices, while another one is dependent on the determinant of the coupling matrix and one of its cofactors. The values of non-zero elements of the coupling matrix serve as the independent variables to minimize the cost functions by using solvopt and fmincon. Although the stochastic initial values are not sufficiently close to the global optimum, the hybrid optimization procedure is still robust to find multiple coupling matrices to overcome the initial problem. It is significant that the suitable coupling matrix can be chosen from the multiple solutions to meet the given requirements in practice. For demonstrating the proposed hybrid optimization algorithm, some extraordinary prototype topologies are provided which validate the efficiency of the proposed synthesis procedure.

1. INTRODUCTION

The rapid growing wireless communication systems have led to more stringent requirements for RF/microwave filters. Cross-coupling presented by Atia and William [1, 2], has been used to meet with these stringent requirements because transmission zeros can be arbitrarily placed in the stopband where it is needed. In the design, the coupling matrix is one of the most important parameters. It cannot be obtained easily and is indeed one of the most difficult problems.

Cameron [3] extended this work and proposed more advanced synthesis technique from scattering polynomials. In his work the so-called transversal coupling matrix often includes unwanted or unrealizable coupling elements. In order to eliminate the unwanted or unrealizable coupling elements for practical realization, the synthesis procedures generally have two categories: similarity transformation synthesis procedure and numerical synthesis procedure. The main drawback of similarity transformation is that there is no general rule for determining the sequence of matrix rotations. As the complexity of the response to be synthesized increases, analytical methods become extremely intricate, time-consuming and difficult to realize, and in some cases impossible to perform. Although Macchiarella has derived the sequence of transformations allowing annihilation of the unwanted elements in [4], the method is based on multiple matrix rotations (similarity transforms) and numerical optimization. Additionally, there are cases where the iterative process of similarity transformation formulated as an optimization problem is not able to find a minimum of a global problem, converging to the local minimum.

Relative to similarity transformation, numerical synthesis approach for coupling matrix synthesis is based on direct optimization, where the cost function is well constructed [5, 6]. It offers the advantage of automating the search for the coupling matrix, thus simplifying the synthesis process. It also avoids

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the need for matrix transformations by enforcing the filter topology throughout the process. Therefore it has the potential to overcome the problems experienced with analytical methods. The first example of such a synthesis method was presented in [5]. The cost function was based on the values of the characteristic function at its zeros and poles respectively, and the ripple is also invoked in the error function by calculating the value of S_{11} at the edge of the passband. The optimization was performed directly on the entries of the coupling matrix using a standard gradient unconstrained minimization algorithm. Lamecki et al. [6] proposed a cost function from zeroes and poles of short circuit input and output admittances. The initial coupling matrix in a tridiagonal form was employed by solving the Jacobi inverse eigenvalue problem, leading to fewer calculations and yielding rapid convergence rates. In 2012, Szydlowski et al. [7] boosted it to synthesize a new class of filters with frequency-dependent coupling matrix. In [8], cross-coupled filters were synthesized using a gradient-based optimization technique employing determinants.

Although optimization is a very powerful tool, it must be applied judiciously. Additionally, there are cases where the cost function usually converges to local minimum rather than to global minimum. In general, the definition of cost function is crucial for the optimization process. The cost function with only one minimum and a rapidly convergent is perfect, which, in most cases is impossible. Hence the efficient cost function should be chosen reasonably to produce less local minima as soon as possible. In fact, given an initial value not sufficiently close to the global optimum, the most elegant optimization procedure may not be able to find an acceptable solution. Although global optimizers are robust to find the global minimum to overcome the initial problem, they tend to suffer from slow convergence to the best solution and may lack accuracy in a final solution.

Therefore, hybrid method is widely adopted for optimization problems with many local minima [9, 10]. In this work two cost functions were formulated to avoid a trap of local optimum, where the entries of coupling matrix were used as independent variables in the optimization process. The gradient-based hybrid method combines fmincon with solvopt algorithm, which performs a local search within only a limited number of iterations with good fitness values. This hybrid method can provide good accuracy to find the final solution, while maintaining the speed of search. Compared with conventional techniques, the initial coupling matrix with random values rather than with guess has the potential to be useful in the synthesis of coupling matrices when local optimization methods, which rely upon on the provision of a good initial guess at the solution, fail. The proposed method was verified and illustrated by numerical tests which show the effectiveness of the procedure developed in the paper.

2. POLYNOMIAL DEFINITIONS AND CIRCUIT MODEL OF FILTERS

Generally, the transmission coefficient S_{21} and reflection coefficient S_{11} of a lossless filter network can be described as a ratio of two polynomials [3]:

$$S_{21}(s) = \frac{P(s)}{\varepsilon E(s)} \tag{1}$$

$$S_{11}(s) = \frac{F(s)}{\varepsilon_R E(s)} \tag{2}$$

where $s = j\omega$ is a complex frequency variable, and ε and ε_R are ripple constants related to the maximum return loss. Polynomial $P(s)$ is determined from finite transmission zeros. Once N, the passband return loss and the transmission zeros s_t are given, polynomial $F(s)$ can be calculated using a recursion formula [3]. From the conservation of energy, the relation between numerator polynomials and denominator polynomials can be expressed as

$$E(s)\,E(s)^* = \frac{F(s)\,F(s)^*}{\varepsilon_R^2} + \frac{P(s)\,P(s)^*}{\varepsilon^2} \tag{3}$$

where superscript * denotes the complex conjugate, $\varepsilon_R = 1$ when the degree of $F(s)$ is greater than that of $P(s)$. If $F(s)$ and $P(s)$ have the same degree,

$$\varepsilon_R = \frac{\varepsilon}{\sqrt{\varepsilon^2 - 1}} \tag{4}$$

S-parameters can also be directly related to the coupling coefficients as follows:

$$\begin{aligned} S_{21} &= 2\sqrt{R_1 R_2} I_N = -2j\sqrt{R_1 R_2}\left[A^{-1}\right]_{N1} \\ S_{11} &= 1 - 2R_1 I_1 = 1 + 2jR_1\left[A^{-1}\right]_{11} \end{aligned} \tag{5}$$

where the matrix A is given by

$$A = M - jR + \omega U \tag{6}$$

Here, U is the identity matrix, $R = \text{diag}\{R_1, \ldots, R_2\}$ the diagonal matrix with all entries zero except for $R_{11} = R_1$ and $R_{NN} = R_2$, and M the symmetric square coupling matrix.

3. COST FUNCTIONS AND OPTIMIZATION ALGORITHM

An appropriate cost function is one of the important things for successful optimization. Many cost functions [5–10] minimize the cost by evaluating the amplitude of S_{21} and S_{11} at the critical frequencies such as, transmission zeros, reflection zeros and passband edges, and compare them to the ideal polynomial response. In [6, 7], the cost function was constructed from the related eigenvalues of matrix with initial coupling matrix by solving the Jacobi inverse eigenvalue problem. Zeros and poles were used to construct the cost function in [8–10], requiring less iterations along with better chances of convergence compared to other techniques. But in some case, the frequency response from the obtained solution is very different with the original one when the optimization process falls into local optimum from an initial value far from global minimum. In this paper, two cost functions which combined the cost function proposed in [6] and parts of the cost function in [8] are considered. It is worth mentioning that optimization results from only one cost function are often likely to be ambiguous.

3.1. Cost Function Based on Zeros, Poles and Its Gradient

For an Nth-degree network capable of generating N_z transmission zeros ($N_z \leq N - 2$), the first cost function is

$$f_1 = \sum_{i=1}^{N_z} |P(s_{ti})|^2 + \sum_{i=1}^{N} |F(s_{ri})|^2 + \sum_{i=1}^{N} |E(s_{pi})|^2 + \left(\text{Num}\left(|S_{11}|_{s=\pm j} - 10^{-RL/20}\right)\right)^2 \tag{7}$$

where s_t is transmission zeros, s_r represents reflections zeros, and s_p denotes poles, $\text{Num}(f(x))$ indicates numerator of $f(x)$, RL is related to return loss in decibels. The above cost function is a slight modification of that proposed in [8]: the third term is an addition to the cost function given in [8], and the final term is attributed to return loss at $s = \pm j$. It can be seen that the proposed cost function is a sum of four polynomials rather than rational functions. Hence, with two accessorial terms it produces less local minima, requiring less iteration and better chances of convergence compared to other techniques. Each evaluation of the cost function is fast due to its relying only upon the determinant and a cofactor of the matrix at the required frequencies. According to (5), (7) is rewritten as

$$\begin{aligned} f_1 = &\sum_{i=1}^{Nz} 4R_1R_2 \left|cof\left(A_{N1}(s_{ti})\right)\right|^2 + \sum_{i=1}^{N} \left|\det\left(A(s_{ri})\right) + 2jR_1 cof\left(A_{11}(s_{ri})\right)\right|^2 + \sum_{i=1}^{N} \left|\det\left(A(s_{pi})\right)\right|^2 \\ &+ \left(\left|\det\left(A(s)\right) + 2jR_1 cof\left(A_{11}(s)\right)\right| - 10^{-RL/20}\left|\det\left(A(s)\right)\right|\right)^2_{s=\pm j} \end{aligned} \tag{8}$$

where $cof(A_{mn}(s))$ is the cofactor of matrix A evaluated by removing row m and column n of matrix A, and $\det(A)$ is the determinant of matrix A. Different from [8], the constant $4R_1R_2$ multiplied by the first term of the cost function is taken into account for optimization.

The derivative of a matrix determinant with respect to the matrix elements themselves is [8]:

$$\frac{d\,\det(M)}{dm_{jk}} = adj_{jk}(M) \tag{9}$$

where m_{jk} are the elements of matrix M, $adj(M)$ represents the adjoint of the matrix. The gradient of a cofactor can be expressed as [8]:

$$\frac{d\,cof_{mn}(M)}{dm_{jk}} = \begin{cases} adj_{j-1,k-1}\left[\text{minor}_{mn}(M)\right] & \text{for } j,k \neq 1 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where $[\text{minor}_{mn}(M)]$ is the minor of matrix M by deleting the row and column corresponding to element $(m,\, n)$. The gradient of the error function (8) can be thus analytically calculated using (9) and (10).

3.2. Cost Function Based on Short Circuit Parameters and Its Gradient

The short-circuit transfer admittance $y_{22}(s)$ from (6) can be deduced by putting R_1 and $R_2 = 0$ [3]:

$$y_{22}(s)|_{R_1,R_2=0} = -j\left[M+\omega U\right]^{-1}_{NN} = -j\frac{\det\left[M'-jsU'\right]}{\det\left[M-jsU\right]} \tag{11}$$

where $U' \in R^{(N-1)\times(N-1)}$ is the identity matrix, M' is the upper principal sub-matrix obtained by deleting the last row and column from the matrix M. From (11) it can be seen that the poles of y_{22} are eigenvalues of matrix M multiplied by $-j$, while zeros of y_{22} are $-j$ times of eigenvalues of matrix M'. The similar result holds for short circuit input admittance y_{11} except that zeros are the eigenvalues of M''-obtained by deleting the first row and column of matrix M.

It is well known that rational functions y_{11} and y_{22} can be easily obtained from [3]. Assume that λ_{zi} and λ_{pi} are the roots of polynomials in the numerator and denominator of y_{22} which is constructed by polynomial synthesis technique in [3]. The second cost function [5] is used in this work:

$$f_2 = \sum_{i=1}^{N}\left(\lambda_{pi}-\lambda'_{pi}\right)^2 + \sum_{i=1}^{N-1}\left(\lambda_{zi}-\lambda'_{zi}\right)^2 + \sum_{i=1}^{N-1}\left(\lambda_{zi}-\lambda''_{zi}\right)^2 \tag{12}$$

where λ'_{pi}, λ'_{zi} and λ''_{pi} are the eigenvalues of the coupling matrix M, M' and M'', respectively, all multiplied by $-j$. The last term of the above equation is considered since the network is symmetrical $(y_{11} = y_{22})$.

Expression for sensitivity of the ith eigenvalue λ_i to the change of j, kth, element of matrix M is [6, 7]:

$$\frac{\delta\lambda_i}{\delta M_{jk}} = x_i^T P^{(kj)} x_i \tag{13}$$

where x_i is the ith eigenvector of M, $P^{(kj)}$ a $N \times N$ symmetric matrix whose all entries are zero except for $P_{kj} = P_{jk} = 1$. A similar result can be implemented to find sensitivities for elements of matrix M' and M''.

3.3. Hybrid Optimization Algorithm

MATLAB is chosen as the operational platform. Both solvopt and fmincon are used to minimize the cost function (8) and (12), respectively. Solvopt algorithm is an effective method to provide a general optimization tool applicable for a wide class of nonlinear optimization problems. It seems useless to apply it for solving linear and quadratic programming problems. However, fmincon based on the sequential quadratic programming algorithm finds a minimum of a constrained nonlinear multivariable function. Fig. 1 shows the whole optimization procedure. When one of cost functions reaches the goals, another cost function should be calculated too. Hence the optimization algorithm proposed here will iteratively change the values of coupling coefficients until two cost functions reach a value below 10^{-12} synchronously, denoting the convergence to the function global minimum and the exact synthesis of the network. Otherwise, the synthesis is only approximate. To avoid being trapped in a local optimum solution, a new coupling matrix is randomly reproduced (Fig. 1) when either the two cost functions don't reach a value below 10^{-12} or when 10 iterations have been performed.

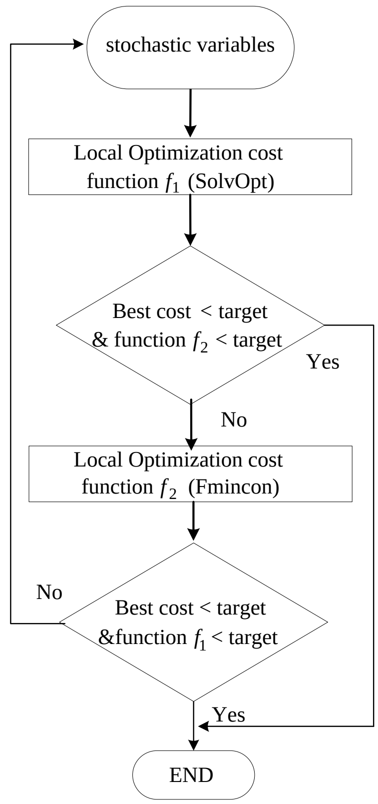

Figure 1. Flowchart of hybrid method.

4. NUMERICAL RESULTS

For verification of the hybrid method described above, three examples are tested in this section. Each of them is characterized by a different topology scheme (Fig. 2) and number of transmission zeros. In all presented examples, the algorithm begins by generating random numbers for non-zero elements of the coupling matrix, whose values lie within specified limits $[-1.5, 1.5]$. The lower- and upper-bounds for the control variables are -1.5, and 1.5, respectively. The process will terminate until both cost function f_1 and cost function f_2 drop below 10^{-12}.

4.1. Example A

The first synthesized network is a symmetric 8-pole filter with 3 pairs of finite transmission zeros, as shown in Fig. 2(a). Its electrical specifications are:

return loss: 20 dB;

transmission zeros: $s_t = [\pm j1.17, \pm j2.8, \pm j8]$;

Applying the hybrid procedure, the coupling matrix was obtained in few seconds. Several different coupling matrices can be obtained from randomly generated initial values. Three of matrices are outlined in Table 1. The only difference between these matrices is symbols of corresponding elements (electric coupling or magnetic coupling), which can be changed readily through coupling theory. The randomly generated initial matrix can find different coupling matrices from multiple cost functions using the hybrid method proposed in this paper, which demonstrates that the solution generated by the procedure may be non-unique. Since coupling elements for microwave filters can generally be realized for a limited range of coupling values, it is very practical to offer several solutions of coupling matrices to meet the requirements.

The frequency response of the synthesized filter and group delay are shown in Fig. 3, which are indistinguishable from the prototype response, because the final cost function is less than 10^{-12}.

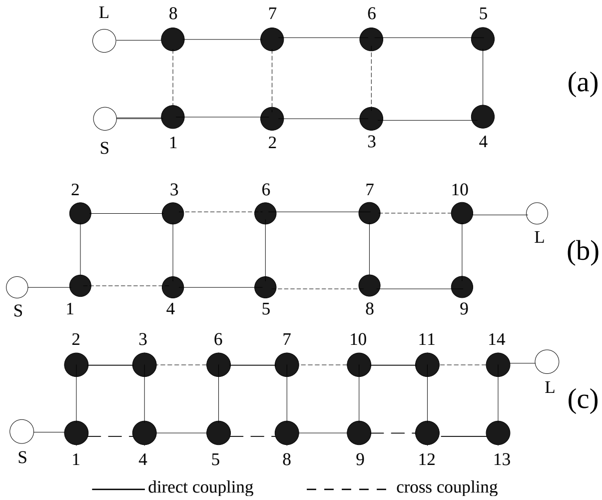

Figure 2. Different topologies used for testing.

$M_{i,j}$	M	M	M
$M_{1,2}$	-0.8119	0.8119	-0.8119
$M_{2,3}$	0.5828	0.5828	0.5828
$M_{3,4}$	-0.4867	-0.4867	0.4867
$M_{4,5}$	-0.7631	0.7631	0.7631
$M_{5,6}$	0.4867	-0.4867	-0.4867
$M_{6,7}$	-0.5828	0.5828	0.5828
$M_{7,8}$	0.8119	0.8119	0.8119
$M_{1,8}$	-0.0001	-0.0001	-0.0001
$M_{2,7}$	-0.0118	0.0118	-0.0118
$M_{3,6}$	-0.2528	-0.2528	0.2528

Table 1. Coupling matrix M or example A.

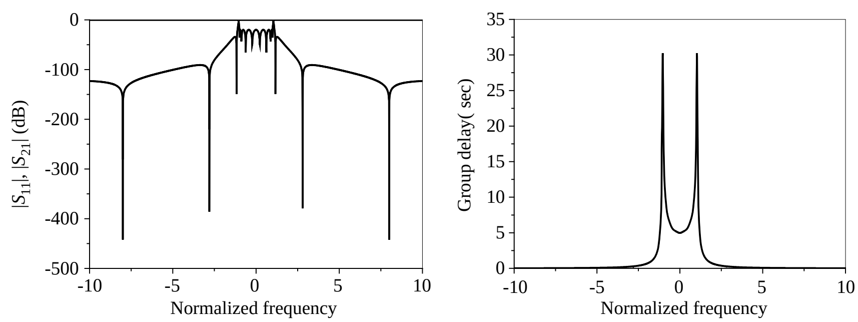

Figure 3. Frequency response and group delay for example A.

Table 2. Incorrect coupling matrix M for example A.

$M_{i,j}$	M	$M_{i,j}$	M
$M_{1,2}$	0.6158	$M_{6,7}$	0.2264
$M_{2,3}$	0.2264	$M_{7,8}$	0.6158
$M_{3,4}$	0.0243	$M_{1,8}$	0.5292
$M_{4,5}$	1.0962	$M_{2,7}$	0.2053
$M_{5,6}$	0.0243	$M_{3,6}$	0.8555

The coupling matrix shown in Table 2 was achieved by minimizing the cost function (12) ($f_2 \approx 2.2474 \times 10^{-15}$). However, it was found that another cost function (8) was far greater than 10^{-12} ($f_1 \approx 1.3622 \times 10^{11}$) at this time despite the fact that f_2 reached the goal. It is obvious that there are tremendous divergence between the frequency response from the characteristic of the resonator filter and the frequency response obtained from the coupling matrix. The main reason of this phenomenon is that any cost function for the coupling matrix is necessary but not sufficient condition. To avoid this situation, two cost functions should be built in this paper.

4.2. Example B

The next example is the 10th order filter (Fig. 2(b)) with four asymmetrically located transmission zeros and 20 dB return loss. The positions of transmission zeros are. $s_t = [j1.10929,\ j1.19518,\ \pm 0.75877 -$

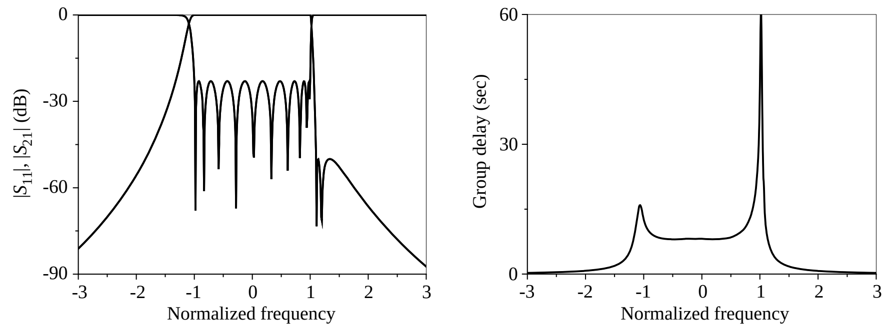

Figure 4. Frequency response and group delay for example B.

Table 3. Coupling matrix M for example B.

$M_{i,j}$	M	M	M
$M_{1,1}$	0.0145	0.0145	0.0145
$M_{2,2}$	0.0546	−0.8698	0.2500
$M_{3,3}$	−0.0885	−0.0195	0.1742
$M_{4,4}$	−0.8704	0.2916	0.0125
$M_{5,5}$	−0.6087	0.0453	−0.0566
$M_{6,6}$	0.1007	0.0394	0.0760
$M_{7,7}$	0.4270	−0.8951	0.0634
$M_{8,8}$	−0.1042	0.0520	−0.6036
$M_{9,9}$	−0.0011	0.2655	−1.0065
$M_{10,10}$	0.0145	0.0145	0.0145
$M_{1,2}$	−0.8450	−0.4207	0.1028
$M_{2,3}$	0.6220	−0.2448	0.5125
$M_{3,4}$	0.1859	0.5052	−0.4901
$M_{4,5}$	0.1087	−0.1295	0.4258
$M_{5,6}$	0.2521	−0.5224	−0.3134
$M_{6,7}$	0.4112	0.0035	0.5713
$M_{7,8}$	0.4554	0.2043	0.0176
$M_{8,9}$	−0.5085	−0.5188	0.0360
$M_{9,10}$	−0.8460	0.7648	0.1820
$M_{1,4}$	0.1768	−0.7539	0.8571
$M_{3,6}$	−0.5703	0.4237	−0.4144
$M_{5,8}$	−0.3439	0.5397	−0.4720
$M_{7,10}$	0.1719	0.4004	−0.8439

$j0.13761]$. The coupling matrix for this topology is not easily synthesized by conventional methods. Neither fmincon nor solvopt can find an acceptable solution even if the initial values of matrix entries are obtained by solving the Jacobi inverse eigenvalue problem [6]. Randomly generated initial values of matrix entries enable the coupling matrices to be different sets of coupling values after execution of the program. More than 20 coupling matrices were obtained and three of them are shown in Table 3. It is once again noted that the solutions generated by the procedure may be non-unique.

The insertion and return loss of the synthesized filter are depicted in Fig. 4. The difference of frequency responses between the required specification and the obtained coupling matrices is not visible. When compared to the technique put forward by [6, 8–10], the optimization algorithm proposed here proves more robust and more successful in converging to global minima.

4.3. Example C

The third example (Fig. 2(c)) is the 14th order filter with two symmetric transmission zeros located at $s_t = [\pm j1.05, \pm j1.2, \pm j1.4]$ and a passband return loss of 20 dB. In this case the above method provides many coupling matrices with stochastic initial values of matrix entries. Five matrices are given in Table 4. Fig. 5 shows return loss, attenuation, and group delay. Note that the prototype synthesized produces a frequency response indistinguishable from the theoretical one.

In the end, a series of experiments were performed to verify whether the cost functions may be effectively minimized. The technique has been verified by synthesizing over 30 prototype filters with orders from 4 to 14, and transmission zeros ranging from 1 to 8. The experiment was repeated 100 times and the convergence in all tests was excellent, so the coupling matrix was identified correctly. Although hybrid method in [9, 10] is also robust to find the global minimum to overcome the initial problem, they require high computation time to the best solution. Numerical tests show that the multiple cost functions with hybrid method simplify the process of synthesizing coupling matrices within very little iteration regardless of complex problems. The proposed hybrid algorithm is widely adopted to synthesize resonant filters with an arbitrary topology.

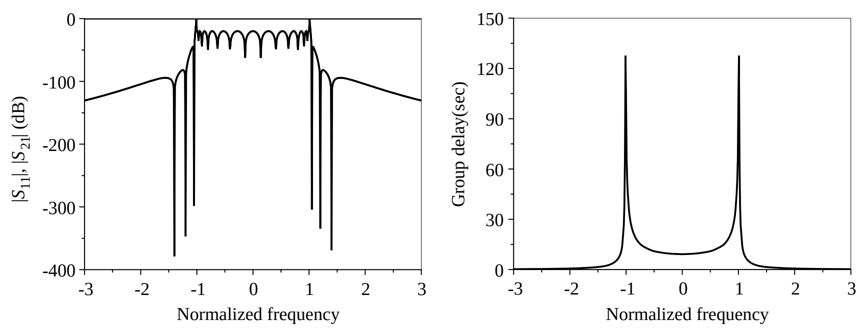

Figure 5. Frequency response and group delay for example C.

Table 4. Coupling matrix M for example C.

$M_{i,j}$	M	M	M	M	M
$M_{1,2}$	0.7935	0.5176	0.7789	0.6065	0.7970
$M_{2,3}$	0.5470	0.9393	0.6951	0.7466	0.6206
$M_{3,4}$	0.1319	0.2491	0.4939	0.0795	0.4630
$M_{4,5}$	0.9599	0.5175	0.5164	0.3940	0.4253
$M_{5,6}$	0.0004	0.4669	0.0655	0.0520	0.0882
$M_{6,7}$	0.4482	0.3649	0.9315	0.7019	0.9271
$M_{7,8}$	0.4863	0.2205	0.1415	0.1841	0.1142
$M_{8,9}$	0.5061	0.6955	0.4474	0.5055	0.5085
$M_{9,10}$	0.2963	0.3506	0.4611	0.4323	0.4385
$M_{10,11}$	0.7525	0.6871	0.5406	0.5449	0.7598
$M_{11,12}$	0.4788	0.2506	0.4942	0.0630	0.4743
$M_{12,13}$	0.6349	0.6715	0.7037	1.0006	0.6017
$M_{13,14}$	0.5198	0.7071	0.7755	0.1064	0.7993
$M_{1,4}$	0.1075	−0.6109	−0.1860	−0.5229	−0.0775
$M_{3,6}$	−0.4726	0.0067	−0.0110	−0.2417	−0.2061
$M_{5,8}$	0.1408	0.2030	0.5151	0.7109	0.4967
$M_{7,10}$	0.0004	−0.5600	−0.1860	−0.2687	−0.0040
$M_{9,12}$	−0.4129	−0.2692	−0.0360	−0.0539	−0.2574
$M_{11,14}$	−0.6091	−0.3757	−0.1996	0.7936	−0.0474

5. CONCLUSIONS

A hybrid optimization method combining fmincon and solvopt algorithm has been presented. Although the randomly generated initial values are not sufficiently close to the global optimum, the hybrid optimization procedure is still able to find acceptable solutions. Since the hybrid method may be run from random starting points, multiple coupling matrices can usually be captured, which meet the given requirements in practice. Several examples show that the hybrid method offers a significant advantage over the traditional algorithm without loss of fast convergence and good accuracy. It has the potential to be used for synthesis of resonator filters with arbitrary topology.

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