Ultra-Compact Electromagnetic Metamaterial Transmission Line and Its Application in Miniaturized Butler Matrix

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Abstract—A novel super compact electromagnetic metamaterial transmission line (EM-MTM TL) is proposed in this paper by using the structure of symmetric double spiral lines (SDSLs). The investigation results indicate that the proposed EM-MTM TL not only has controllable resonant frequency, but also has very compact size, and the circuit area is only $8.8 \,\mathrm{mm} \times 7.2 \,\mathrm{mm}$ (equal to $\lambda_0/32.16 \times \lambda_0/39.31$, where λ_0 is the free space wavelength at the resonant frequency) without the feed lines. Using the proposed structure, a 3-dB branch-line coupler and a 0-dB crossover operated at 0.86 GHz have been designed, fabricated and measured; the measured and simulated results are in good agreement. The two microwave devices realize 84.8% and 85.7% size reduction, respectively. Then, a compact Butler matrix is obtained by optimizing the combination of the branch-line couplers, 0 dB crossovers and 45-degree phase shifters. The measured and simulated results of the proposed Butler matrix agree well, showing that the proposed device operates at 0.86 GHz with very good electromagnetic performances. Moreover, the circuit area of the proposed Butler matrix is $109.0 \text{ mm} \times 89.3 \text{ mm}$, which realizes at least 80.9% size reduction in comparison with the conventional one (whose circuit area is at least $226.2 \text{ mm} \times 226.2 \text{ mm}$), and the miniaturization is considerable. Besides, these designed microwave devices, without any lumped elements, bonding wires, defected ground structure (DGS), and via-holes, are more suitable for modern wireless communication systems.

1. INTRODUCTION

Recently, electromagnetic (EM) metamaterials (MTMs) have attracted increasing attention in microwave engineering for the miniaturization, multi-band and other improved performances of Generally speaking, EM MTMs are a general designation of double microwave devices [1]. negative metamaterials (DNG-MTMs) [2,3], single negative metamaterials (SNG-MTMs) [4-6], composite right/left-handed transmission lines (CRLH TLs) [7–9], negative/zero-refractiveindex metamaterials [10, 11], gradient-index metamaterials [12], electromagnetic band-gap (EBG) structures [13], etc. EM MTMs are bringing a profound transformation in physical engineering from the microwave frequency band to optical spectrum due to their abundant electromagnetic characteristics that do not appear in nature [14, 15]. The ultra miniaturization of single negative metamaterials (SNG MTMs) unit cell is not only a very hot topic, but also a challenging and contradictive issue in microwave technology and communication industry [16]. Many researchers have done some resultful works in this field. Xu et al. proposed an ultra-small SNG electric MTM [17] in 2012, with a size of only $\lambda_0/26.8 \times \lambda_0/26.8$ (λ_0 is the free space wavelength at the resonant frequency), to reduce the electromagnetic coupling of microstrip antenna array. At least 8.4 dB coupling reduction is obtained. In paper [18], the authors realize a miniaturized artificial magnetic metamaterial by using the spiral resonators. Besides, lumped elements [19] and fractal theory [20] are also applied to realize the miniaturization of SNG MTMs unit cells. Despite these fruitful developments, there was still insufficient progress toward practical applications [17].

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Butler matrix has been widely applied in the beamforming network of multibeam smart antenna arrays and multichannel amplifiers due to its multiple inputs and outputs with equal power distribution and constant phase relation. It consists of fixed 45-degree phase shifters, 3-dB hybrids and 0-dB crossovers, which accounts for the large occupied area. According to the survey of recent literatures, miniaturized Butler matrixes have been designed by employing fractal geometry [21], slow-wave transmission lines [22], the distributed CRLH TLs [23] and the microstrip [24] technique, etc. However, these proposed matrixes exhibited considerable size reduction, and there were some obvious shortages. The fractal parts and slowwave transmission lines deteriorated the inband performances to a great extent. The matrix based on distributed CRLH TLs is very complicated, and the microstrip technique is always associated with a complicated design and fabrication process.

In this paper, we propose an ultra-small electromagnetic metamaterial transmission line (EM-MTM TL) by using symmetric double spiral lines (SDSLs). The equivalent circuit model and simulated results are provided to validate the negative permittivity of the proposed EM-MTM TL at the resonant frequency. The sizes of the proposed structure are only $\lambda_0/32.16 \times \lambda_0/39.31$ (λ_0 is the free space wavelength at the resonant frequency) without feed lines. Using the 90-degree phase shift characteristic in the passband of the proposed EM-MTM TL, a 3-dB branch-line coupler with 84.8% size reduction and 0-dB crossover with 85.7% size reduction are designed, fabricated and measured. Based on the above, a miniaturized Butler matrix is accomplished by optimizing the combination of 45-degree phase shifters, 3-dB branch-line couplers and 0-dB crossovers. Compared with the conventional Butler matrix, the fabricated one in this paper achieves at least 80.9% size reduction. Moreover, the proposed Butler matrix has supreme miniaturization in comparison with the references.

2. CHARACTERIZATION AND MODELING OF THE PROPOSED EM-MTM TL

The geometry structure, equivalent circuit model and simulation settings model of the proposed EM-MTM TL unit cell are shown in Figure 1. The unit cell is built on a commonly utilized Rogers RT/duroid 5880 substrate with a dielectric constant $\varepsilon_r = 2.2$, thickness h = 0.508 mm and loss tangent tan $\delta = 0.001$. As shown in Figure 1, the schematic and geometrical parameters of the proposed EM-MTM TL are given. The geometrical parameters are: w = 1.6 mm, $w_1 = 0.3$ mm, $w_2 = 0.2$ mm, $l_1 = 7.2$ mm and $l_2 = 8.2$ mm. The boundary condition of the microstrip structure and the ground is set as a perfect E. The excitation at the two input ports is set as a lumped port. The simulated S-parameters and phase coming from Ansoft HFSS 13.0 are shown in Figure 2(a).

In the circuit model, the capacitor (C_2) models the coupling capacitor between the loops and the



Figure 1. Geometry structure, equivalent circuit model and simulation settings of the proposed EM-MTM TL.



Figure 2. The electromagnetic parameters of the proposed EM-MTM TL. (a) *S*-parameters and phase. (b) The distribution of electric field at 1.25 GHz. (c) Effective constitutive EM parameters. (d) Propagation constant.

ground. The inductor (L_1) in series is produced by the main microstrip lines. The capacitor (C_1) and inductor (L_2) in parallel are produced by the double spiral-line loops. According to the simulated results shown in Figure 2(a), by using Ansoft Serenade, we can get the values of elements in equivalent circuit model: $L_1 = 5.65 \text{ nH}$, $C_1 = 6.28 \text{ pF}$, $L_2 = 3.49 \text{ nH}$, $C_2 = 2.74 \text{ pF}$.

Figure 2 plots the simulated S-parameters and phase, the distribution of electric field and the retrieved constitutive EM parameters against frequency of the proposed EM-MTM TL. Referring to Figure 2(a), there is a very obvious stopband. As shown in Figure 2(d), the phase constant is zero in this stopband, indicating that no power can transmit within the stopband. A frequency point with -90-degree phase shift exists in the passband with the return loss better than 10 dB, which means that the proposed EM-MTM TL can be used to replace the quarter-wavelength microstrip transmission line in the design of some microwave devices. Figure 2(b) shows the distribution of electric field at 1.25 GHz of the proposed EM-MTM TL. It is obvious that the electric field has been astricted in the structure of double spiral-line loop at 1.25 GHz, namely, this stopband is brought by the electric resonance of the proposed EM-MTM TL. As a result, a single negative permittivity around 1.25 GHz is expected in Figure 2(c). Besides, as shown in Figure 2(c), both the permittivity and permeability are negative from 1.10 GHz to 1.20 GHz, and the results in Figure 2(d) are good evidence for the back-wave transmission characteristic from 1.10 GHz to 1.20 GHz of the proposed EM-MTM TL.

In order to explore the relation between the geometry parameters and the electromagnetic characteristics of the proposed EM-MTM TL, the optimization in Ansoft HFSS 13.0 has been applied, and the results are shown in Figure 3. It can be seen that the resonant frequency (f_0) of the stopband



Figure 3. The dependence of S-parameters on the geometry parameters.

can be modulated by the geometrical parameters. f_0 increases when l_1 shrinks, because when l_1 shrinks, the length of the spiral line will decrease, which means that the electric current path at f_0 decreases. As shown in Figure 3(c), f_0 will decreases when w_2 rises; however, the length of the spiral line increases when w_2 rises. In a word, the resonant frequency (f_0) only rests with the length of the spiral line. The results in Figure 3(b) and Figure 3(c) indicate that the bandwidth of the stopband identified by the isolation better than 10 dB increases with the augment of the width of the spiral line and the gap. Figure 3(d) shows that the width of the feed line plays a negligible role in affecting f_0 .

According to the simulated results, the electromagnetic characteristics of the proposed EM-MTM TL can be summarized as follows: The proposed EM-MTM TL has a simple structure and can be adjusted easily. The resonant frequency (f_0) only rests with the length of the spiral line, and the performances will be better by adjusting the other geometry parameters. Moreover, the circuit area of the proposed EM-MTM TL is only $8.8 \text{ mm} \times 7.2 \text{ mm}$ (equal to $\lambda_0/32.16 \times \lambda_0/39.31$, where λ_0 is the free space wavelength at the resonant frequency) without the feed lines and can be applied in the miniaturization of microwave devices.

3. DESIGN AND MEASUREMENT OF THE MINIATURIZED BUTLER MATRIX

3.1. Design of 50.0, 35.3 and 25.0 ohm EM-MTM Transmission Lines

According to the design principles introduced in [25], as shown in Figure 4, the branch-line coupler consists of two branch lines and two parallel lines, whose electrical length is 90 degrees, and impedances are $Z_1 = 50.0$ ohm and $Z_2 = 35.3$ ohm, while the electrical length and impedances of the structures in crossover are 90 degrees, $Z_1 = 50.0$ ohm, $Z_2 = 35.3$ ohm, and $Z_2 = 25.0$ ohm. In Figure 4, the circuit area of the conventional branch-line coupler and crossover are at least $\lambda_g/4 \times \lambda_g/4$ and $\lambda_g/2 \times \lambda_g/4$



Figure 4. The basic structure of branch-line coupler and crossover. (a) 3-dB branch-line coupler. (b) 0-dB crossover.



Figure 5. The simulated results of the three sections. (a) S-parameters. (b) Phase.

Table 1. The geometry parameters of the three EM-MTM TL sections (mm).

	l_1	l_2	w	w_1	w_2
$50.0\mathrm{ohm}~\mathrm{TL}$	7.2	8.2	1.6	0.3	0.2
$35.3\mathrm{ohm}\mathrm{TL}$	7.2	8.2	2.5	0.3	0.2
$25.0\mathrm{ohm}~\mathrm{TL}$	10	6.2	4.0	0.3	0.2

 $(\lambda_g \text{ is the guided wavelength at the operating frequency})$, so the large occupied circuit area is with the same problem of these two devices, which needs to be solved.

In order to realize the miniaturization of branch-line coupler and crossover by using the proposed EM-MTM TL structure, the key procedure is to design 50.0, 35.3 and 25.5 ohm EM-MTM TL sections with specified phase shift, typically as 90-degree respectively, and then assemble them in integrated circuits to form the miniaturized branch-line coupler and crossover. According to [25], we know that the impedance of the proposed structure depends on the width of feed lines, and the widths of the designed 50.0, 35.3 and 25.5 ohm EM-MTM TL sections are 1.6 mm, 2.5 mm and 4.0 mm. As shown in Figure 5, the simulated S-parameters and phase of these three sections are shown, indicating that these sections obviously operate at 0.86 GHz with exact 90-degree phase shift. The transmission loss is low and the impedance match with return loss better than 25 dB is good. Table 1 gives the geometry parameters of the three EM-MTM TL sections.

3.2. Design and Measurement of the 3-dB Branch-Line Coupler and 0-dB Crossover

According to the above analysis, we assemble the 50.0, 35.3 and 25.5 ohm EM-MTM TL sections in integrated circuits to form the miniaturized branch-line coupler and crossover. Figure 6 gives photographs of the fabricated 3-dB branch-line coupler and 0-dB crossover. The information in the photographs shows that the effective circuit area of the miniaturized branch-line coupler and crossover are $36.0 \text{ mm} \times 20.3 \text{ mm}$ and $62.0 \text{ mm} \times 20.3 \text{ mm}$, compared with the conventional structures (the circuit area of conventional branch-line coupler is $74.5 \text{ mm} \times 64.5 \text{ mm}$, and the conventional crossover is $134.0 \text{ mm} \times 65.5 \text{ mm}$). 84.8% and 85.7% size reduction can be obtained, respectively.

Figure 7 and Figure 8 show the comparison between the measured and simulated results of the fabricated 3-dB branch-line coupler and 0-dB crossover. It can be seen that the measured and simulated results are in good agreement with each other. Referring to the measured results of the branch-line coupler, the central frequency is located at 0.86 GHz, and the measured $|S_{21}|$ and $|S_{31}|$ at 0.86 GHz are 3.15 dB and 3.37 dB, respectively. The measured return loss is better than 30 dB, and the isolation is more than 35 dB. The main difference between the measurements and simulations is a slight shift of the operating frequency point; however, this main difference is less than 0.3%, while as shown in Figure 8, the measured results indicate that the fabricated 0-dB crossover owns comparable performance in terms of 0 dB transmission loss bandwidth and 20 dB isolation bandwidth. Although the isolation of the proposed crossover is not so good as that of conventional one, the low insertion loss of $|S_{31}|$ around 0.27 dB should be highlighted.

Table 2 is the comparison between the fabricated miniaturized branch-line coupler and the same works in the references. We can see that the work in this paper has the minimal circuit area and maximal size reduction.



Figure 6. Photographs of the fabricated branch-line coupler and crossover. (a) Photograph of branch-line coupler. (b) Photograph of 0-dB crossover.



Figure 7. The measured and simulated results of the fabricated branch-line coupler. (a) S-parameters. (b) Phase difference.

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References	[21]	[23]	[28]	[29]	[30]	This work
center frequency	$1.16\mathrm{GHz}$	$0.98\mathrm{GHz}$	$1.00\mathrm{GHz}$	$2.00\mathrm{GHz}$	$2.00\mathrm{GHz}$	$0.86\mathrm{GHz}$
effective area	$0.0377\lambda_g^2$	$0.0309 \ \lambda_g^2$	$0.0196 \ \lambda_g^2$	$0.0215\lambda_g^2$	$0.0141\lambda_g^2$	$0.0112\lambda_g^2$
size reduction	55.7%	62.8%	66.0%	72.0%	77.0%	84.8%

Table 2. The comparison between the branch-line coupler and the references.



Figure 8. The measured and simulated results of the fabricated 0-dB crossover. (a) Simulated S-parameters. (b) Measured S-parameters.



Figure 9. The basic principle structure of Butler matrix.



Figure 10. Photograph of the fabricated Butler matrix.

3.3. Design and Measurement of the Miniaturized Butler Matrix

As shown in Figure 9, Butler matrix consists of two 0-dB crossovers, two 45-degree phase shifters and four 3-dB branch-line couplers, and it has occupied large circuit area, which is at least $\lambda_g \times \lambda_g$ (λ_g is the guided wavelength at the operating frequency). With the purpose of miniaturization, we apply the proposed 0-dB crossover and 3-dB branch-line coupler to realize the miniaturization of Butler matrix. The designed Butler matrix can realize great size reduction, due to the proposed branch-line coupler and crossover which achieve 84.8% and 85.7% size reduction, respectively. The designed Butler matrix can realize great size reduction, respectively. The designed Butler matrix can realize great size reduction area is only 109.0 mm × 89.3 mm, while the area of the conventional one is more than 226.2 mm × 226.2 mm, achieving at least 80.9% size reduction.

The measured results of the fabricated Butler matrix are shown in Figure 11. It can be seen that the fabricated Butler matrix has very good electromagnetic performances at the operating frequency. The return losses at the eight ports are better than 25 dB. The insertion losses at port 1 and port 2 range from 5.8 dB to 7.6 dB, and the average is 7.15 dB. Meanwhile, at port 3 and port 4, they range from 6.1 dB to 7.7 dB, and the average is 7.3 dB. By all appearances, the measured results indicate that this proposed miniaturized Butler matrix has very good transmission characteristic at 0.86 GHz. Figure 11(d) and Figure 11(e) are the measured isolations of the Butler matrix at input ports and



Figure 11. The measured results of the fabricated Butler matrix. (a) The measured return losses at eight ports. (b) The measured insertion losses at port 1 and 2. (c) The measured insertion losses at port 3 and 4. (d) The measured isolation between the input ports. (e) The measured isolation between the output ports.

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Phase difference	theory	measurement	Phase difference	theory	measurement
$\angle S_{81} - \angle S_{71}$	-45°	-43.2°	$\angle S_{83} - \angle S_{73}$	-135°	-132.6°
$\angle S_{71} - \angle S_{61}$	-45°	-48.7°	$\angle S_{73} - \angle S_{63}$	-135°	-130.7°
$\angle S_{61} - \angle S_{51}$	-45°	-47.3°	$\angle S_{63} - \angle S_{53}$	-135°	-139.5°
$\angle S_{82} - \angle S_{72}$	135°	131.2°	$\angle S_{84} - \angle S_{74}$	45°	42.3°
$\angle S_{72} - \angle S_{62}$	135°	133.4°	$\angle S_{74} - \angle S_{64}$	45°	47.1°
$\angle S_{62} - \angle S_{52}$	135°	139.1°	$\angle S_{64} - \angle S_{54}$	45°	41.2°

Table 3. The theoretical and measured phase difference of the output ports.

Table 4. The comparison of Butler matrix between this paper and the references.

References	[26]	[27]	[31]	[32]	This work
center frequency	$1.80\mathrm{GHz}$	$1.80\mathrm{GHz}$	$1.50\mathrm{GHz}$	-	$0.86\mathrm{GHz}$
effective area (mm^2)	67.7×63.7	109×119	-	-	109 imes 89.3
size reduction	66%	55%	75%	71%	80.9%

output ports. It can be seen that when Butler matrix operating at 0.86 GHz, the minimum, maximum and average isolations at the input ports are 23.8 dB, 34.1 dB and 28.7 dB and at the output ports are 22.5 dB, 37.7 dB and 29.4 dB.

Table 3 gives the phase of the output ports when the Butler matrix operated at 0.86 GHz. The minimum, maximum and average phase deviation of the output ports relative to the theory are 1.6° , 4.5° and 3.1° . It means that the proposed miniaturized Butler matrix has good performances.

Table 4 is the comparison between this work and the references. It can be seen that the proposed Butler matrix has the minimum circuit area and the maximum size reduction. Besides, there is no any lumped elements, bonding wires, defected ground structure (DGS), or via-holes in the proposed microwave devices in this paper, and they are more suitable for modern wireless communication systems.

4. CONCLUSION

In this paper, an ultra small electromagnetic metamaterial transmission line (EM-MTM TL) is proposed and applied to realize the miniaturization of microwave devices. According to the results, the size of the proposed EM-MTM TL is only $\lambda_0/32.16 \times \lambda_0/39.31$. The designed miniaturized branch-line coupler, 0-dB crossover and Butler matrix not only realize 84.8%, 85.7% and 80.9% size reduction respectively, but also have very good performances. Moreover, compared with the same work in the references, the miniaturization of microwave devices in this paper is the best, and they are more suitable for modern wireless communication systems.

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