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A Survey of Motion-Based Multitarget Tracking Methods

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Abstract—Multitarget tracking (MTT) in surveillance system is extremely challenging, due to uncertain data association, maneuverable target motion, dense clutter disturbance, and real-time processing requirements. A good many methods have been proposed to cope with these challenges. However, no up-to-date survey is available in the literature that can help to select suitable tracking algorithm for practical problem. This paper provides a comprehensive review of the state-of-the-art motion-based MTT techniques, classifies existing methods into two groups, i.e., the detect-before-track (DBT) scheme and the track-before-detect (TBD) scheme. The DBT scheme is employed to achieve robust and tractable tracking performance when the signal-noise-ratio (SNR) is strong. The TBD scheme is used in the scenarios of low SNR, and it aims to cumulate target energy by multiple sensor frames. Furthermore, depending on the data association mechanism, the DBT methods can be classified into two categories, data association based approaches and finite set statistics (FISST) based approaches. And the TBD methods can be classified into non-Bayesian approaches and Bayesian approaches depending on the basis theory used for tracking. For each category, this paper provides the detailed descriptions of the representative algorithms, and examines their pros and cons. Finally, new trends for future research directions are offered.

1. INTRODUCTION

Multitarget tracking (MTT) aims to estimate the number, position, velocity and acceleration of interested targets simultaneously, and provides the target tracks continually. In contrast with classical single target tracking (STT), the interested target number of MTT is multiple and varying, they may appear, disappear, merge, or split in the field of surveillance at any time. MTT plays an important role in various sensing systems, such as infrared (IR), radar, sonar, including both military and civilian applications. For example, the forward-looking IR (FLIR) systems and ground moving target indicator (GMTI) radar systems have been developed to search and track ground vehicles [1, 2]. In a civilian air traffic control (ATC) area, the radar MTT systems [3, 4] are used to check the standard separation between pairs of targets for maintenance of safety conditions and regularity of traffic flow. The visible and IR camera MTT systems have been used in the applications of ballistic missile guidance [5] and space-based early warning [6], which are required to identify and track several hundred targets in real time [7]. For underwater target tracking, sonar MTT systems are deployed on an airplane or ship, and submerged for monitoring the underwater acoustics [8, 9].

In practical applications, motion measurements are the most important information that is used for MTT. Motion measurements typically consist of range, bearing, elevation and range rate (Doppler) measurements. Target state vector is often used to formulate the target position, velocity and acceleration, so the task of target tracking is to determine the target state. Motion-based tracking method deems that the target motion is a stochastic process, and determining the target state is a stochastic estimation problem. It also implies that the interested target is point target ignoring spatial

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information. In practice, other measurements may be also available for target tracking, such as texture feature in imaging sensors, and amplitude feature in radar or sonar systems. However, the motion-based tracking usually builds the main tracking framework, and other measurements can be incorporated into the motion-based framework to achieve improved performance.

In this paper, we concern the motion-based MTT problem with single sensor, which is the foundation for more complex tracking. Motion-based MTT is extremely difficult in practice and confronts several tremendous challenges:

-Uncertain data association. In the situation of multitarget, the associations between the measurements and tracks are uncertain. Data association is a burdensome problem especially when the target number is unknown, which is the main reason that leads to the complexity of MTT. MTT algorithms should address the data association problem explicitly, or avoid it by some sophisticated strategy.

-Dense clutter disturbance. The background of the sensor is influenced by the surveillance surrounding objects, weather conditions, sensor system noise, and so on. So dense clutter exists in the measurements, and it would cause false tracks and missing tracks.

-Maneuverable motion. In the surveillance, interested targets are generally with maneuvering motions, while the sensor mounted platforms are also with high ego-motion. So we need sophisticated models to model target motions and measurements, which turn out to be a nonlinear and non-Gaussian estimation problem.

-Real-time processing requirements. To process data in real-time is absolutely important for the surveillance system. But the computation complexity rapidly increases when the target number is large. Nowadays, GPU computing enables real-time processing, but they are usually used in ground applications [10, 11]. For airborne or space-based surveillance applications, the hardware is restricted by the carrier, so we still need to develop tracking algorithms that are as less complexity as possible to achieve a real-time performance.

To cope with these challenges, considerable researches have been undertaken in the literature. A majority of early approaches concentrated on dealing with the measurement-track association problem. Bar-Shalom was the pioneer at that time, and his books [12–14] have laid the foundation for MTT. Afterward, various methods have been developed to improve the data association performance. In 2005, Pulford [15] provided a new concise summary of MTT techniques, which classified all methods into more than 35 different algorithmic types, depending on the data association mechanism, processing scheme, complexity scaling and so on. Some other papers attempt to survey one aspect of target tracking technique, such as target motion modeling in [16–20]. But they do not consider the situation of multitarget explicitly.

In recent years, particle filtering (PF) and finite set statistics (FISST) based filtering, have become more and more important in MTT area. The PF filter is a powerful tool for the implementation of Bayesian filtering, and it can tackle the nonlinear/non-Gaussian filtering problem well. The FISST based filters avoid measurement-track association elegantly in multitarget state estimation, and substitute it by a less complex estimation-track association. FISST based methods also have advantage in dense clutter situations. The PF and FISST based methods represent the new generation of MTT techniques, and attract more and more attentions in MTT application. In addiction, the track-before-detect (TBD) scheme is an alternative strategy for MTT, which avoids data association with a pixeled measurement model. The TBD methods are indeed efficient in the situation of low signal-to-noise-ratio (SNR). However, the relative techniques about PF, FISST and TBD are not included in [12–20], hence, the works of [12–20] are of course out of date.

In this paper, we provide a more comprehensive survey to review the state-of-the-art motion-based MTT techniques. Firstly, this study classifies existing methods into detect-before-track (DBT) methods and TBD methods depending on the process strategy. The TBD methods are used at the early phase of the entire MTT process when the SNR is low, and later on, the DBT methods are employed to attain more robust performance as the SNR increases. Depending on the data association mechanism, the DBT methods can be deeply classified into two categories, data association based approaches and finite-set statistics (FISST) based approaches. The TBD methods are also classified into non-Bayesian approaches and Bayesian approaches depending on the basis theory used for tracking. This paper provides detailed descriptions of the representative algorithms of each category, and examines their pros and cons. In the description, we focus on the new techniques such as the PF and FISST methods, which have shown promising use in both DBT strategy and TBD strategy. Finally, the new topics of motion-based MTT, such as extended/group target tracking and multisensor-multitarget tracking, are also identified in this paper.

The rest of the paper is organized as follows. In Section 2 we first formulate the MTT problem and give the taxonomy of motion-based MTT methods. And the detailed description of representative algorithms of the DBT methods and TBD methods are presented in Section 3 and Section 4 respectively. Section 5 presents new trends of MTT in the future works. Finally, we conclude this paper in Section 6.

2. PROBLEM STATEMENT AND TAXONOMY OF MOTION-BASED MTT

According to motion-based tracking, the target states and measurements are firstly modeled via prior knowledge, then the filtering algorithm propagates the target states iteratively, at last the target states estimation and track management is operated in each iteration step to form continual target tracks. It usually employs the dynamic state-space (DSS) modeling approach to describe the target motion.

In the DSS approach, it is usually assumed that the system state is a first-order Markov process [21], and it models the state space by

$$\begin{aligned} X_k &= f_k\left(X_{k-1},\, u_{k-1}\right) \\ Z_k &= h\left(X_k,\, v_k\right) \end{aligned} \tag{1}$$

where $f(\cdot)$ and $h(\cdot)$ are target motion model and sensor measurement model, respectively; X is the multitarget state and has the form $X = \emptyset$, $\{\mathbf{x}_1\}$, $\{\mathbf{x}_1, \mathbf{x}_2\}$, $\ldots$, $\{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$, $\ldots$, where $X = \emptyset$ means that no target is present; $X = \{\mathbf{x}_1\}$ means that one target with state-vector $\mathbf{x}_1$ is present, etc.; X_k is the multitarget state at time k; Z_k is the measurement-set consisting of all measurements collected from all targets at time k; u_k is the uncertain in modeling target motion; v_k is sensor noise.

Target motion model $f(\cdot)$ tries to extract target information via introducing prior knowledge, while the measurements model $h(\cdot)$ explains how the target motions are reflected in the sensor. The task of tracking is to determine X_k from Z_k. In the 3D physical world, a single target state $\mathbf{x}$ can be described by a state vector $\mathbf{x} = [x, \dot{x}, y, \dot{y}, z, \dot{z}]'$, where (x, y, z) are the position coordinates along x, y, and z axes in the Cartesian coordinate system, respectively, and $[\dot{x}, \dot{y}, \dot{z}]'$ is the velocity vector [16].

The description of measurement-set Z_k is relative to the sensor measurement model, and the sensor measurement model depends on the operation strategy of target detection and tracking. Detection and tracking are two interrelated topics in the surveillance systems [22]. The DBT scheme and TBD scheme are two strategies for the operations of detection and tracking, and the sensor measurement model of DBT and TBD scheme is quite different. So in the following, we first classify MTT methods into two groups, the DBT methods and TBD methods.

2.1. Detect-before-Track Scheme

In the DBT scheme, a set of detections is firstly produced from the raw sensor returns and then the detections are fed into the tracking algorithms.

Usually, the traditional *thresholded measurement model* [23] is used to model the detections. At time k, the detections are collected in the measurement-set $Z_k = [z_1^k, \ldots, z_{M^k}^k]'$, where M^k is the number of measurements and changing with the time k. The measurement vector Z^k contains multiple actual detections, clutter and misdetections, due to the system noise and complex environment background. A target with state $\mathbf{x}$ generates measurements according to the density $p_t(\cdot|\mathbf{x})$. Clutter measurements are generated from the density $p_c(\cdot)$. The number of clutter measurements is assumed to be a known distribution, such as Poisson distribution. The measurement origins are specified by the hypothesis vector $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_r]'$ with $\theta_i = 0$ indicating that the ith target has not been detected and $\theta_i = j \in \{1, \ldots, M^k\}$ indicating that the jth measurement is due to the ith target. The set of allowable hypothesis vectors contains all $\boldsymbol{\theta}$ which assign at most one target to each measurement.

In the thresholded measurement model, the hypothesis vector $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_r]'$ is unknown, i.e., the association that a detection is generated by which target is uncertain. To address the data association problem, data association based methods and FISST based methods are two available mechanisms.

(1) Data association based approaches

Data association based methods are the traditional solutions for MTT, they determine the relationship between detections and tracks explicitly, and divide the MTT issue into two sub-problems, data association and state estimation [12–14]. The goal of association is to find optimum techniques that will assign a received set of detections to a set of target tracks. In this context, the association is called measurement-track association. Data association is the core issue in traditional solutions, and it forms the main computation burden in MTT. Once the association is known, standard motion modeling and state filtering techniques in STT can be used to obtain the target state estimates. Though conceptual division is often done in the literature, data association and state estimation are coupled problems such that one cannot be solved without the other. The process scheme for data association based MTT methods is shown in Fig. 1.

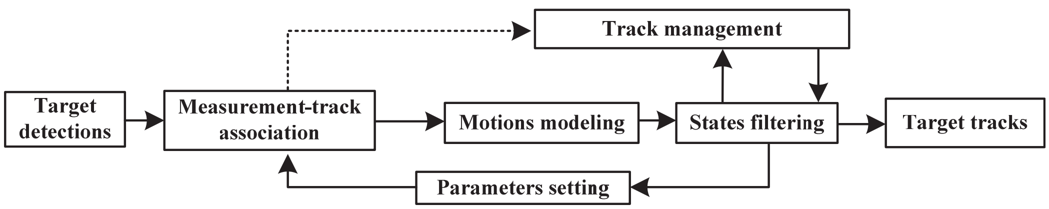

Figure 1. Process scheme for the data association based MTT methods.

(2) FISST based approaches

Recently, FISST is proposed as a new tool for MTT [24, 25]. The FISST based tracking describes multitarget states by random finite set (RFS), and the MTT can be unified to Bayesian framework and avoids data association in the filtering. The FISST-based filters have been shown to be promising new paradigms in MTT. Though there is no need to operate data association in the target state estimation, it still needs to keep records of the identities of the targets to form continual target tracks. This operation is called estimation-track association, corresponding to measurement-track association in traditional methods. The process scheme for MTT methods based on FISST is shown in Fig. 2.

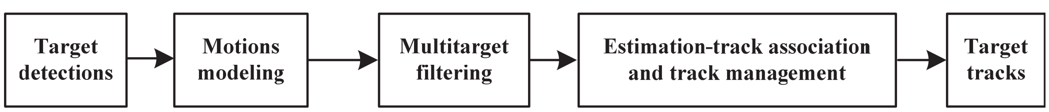

Figure 2. Process scheme for the FISST based MTT methods.

Generally speaking, the DBT algorithms are quite suitable and attain real-time performance in the scenarios of strong SNR and less clutter. In low SNR scenarios, the amplitudes of target signals might not be strong enough to be above the detection threshold. Thus the detection will result in a high number of false alarms. The high density of false alarms caused leads to difficulties in DBT algorithms. Hence, the TBD algorithms are alternative methods to improve the tracking performance.

2.2. Track-before-Detect Scheme

The TBD scheme is also known as joint detection and tracking scheme, and it uses the whole raw sensor data as input data for tacking, and declares that a target is detected after a continual tracking.

In this context, the *pixelized model* [23] is used for raw measurements. According to the pixelized model, the surveillance region is divided into M cells denoted as $V_1, \ldots, V_M$ with $V_j \subset \mathbb{R}^{d/2}$. The measurement data at time k are collected in the measurement-set $Z_k = [z_1^k, \ldots, z_M^k]' \in \mathbb{R}^M$, with z_j^k the intensity measurement obtained in the jth cell. The intensity measurements are assumed to be independently distributed with the measurement in a particular cell depending only on the number of targets occupying that cell. For a multi-target state vector X_k, let $m_j(X_k)$ denote the number of targets in the jth cell. The conditional measurement density can then be written as

$$p\left(\mathbf{Z}^k|\mathbf{X}^k\right) = \prod_{j=1}^{M} l\left(z_j^k;\, m_j(X_k)\right) \tag{2}$$

where $l(\cdot;\, m)$ is the measurement density for a cell occupied by $m \geq 0$ targets. For example, in radar applications a Rayleigh model is adopted. No intermediate detecting takes place in the TBD setup, so there is no need for an explicit mechanism to attach these detections to a track, i.e., no explicit data association algorithm is needed [26].

Following the TBD strategy, considerable methods have been developed, and they can be classified into non-Bayesian approaches and Bayesian approaches based on the basis theory used for tracking.

(1) Non-Bayesian approaches

In non-Bayesian approaches, time energy of the targets is cumulated based on the velocity feature, and then a optimize algorithm is used to find all the target tracks. Suppose we are given a sequence of two-dimensional data frame taken at uniform time intervals by a sensor. Stacking the frames yields a three-dimensional frame that can be described in the Cartesian coordinates *(x, y, t)*. Here (x, y) are the spatial coordinates, while t is the time coordinate. In this approach, relatively many trajectories are estimated in the three dimensional frame, but no commitment is initially made that any of those trajectories actually represents a target. Then, the posterior probability function is computed for each of the tracked trajectories. Finally, each trajectory whose posterior probability is above a certain threshold is declared a target. Fig. 3 shows the process scheme for non-Bayesian TBD methods.

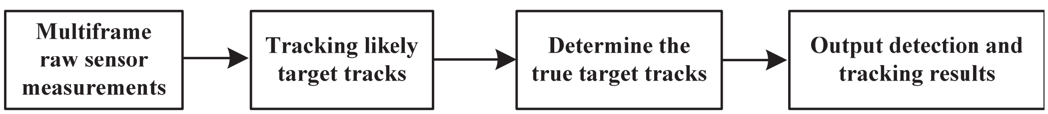

Figure 3. Process scheme for non-Bayesian TBD methods.

(2) Bayesian approaches

In Bayesian TBD approaches, the target state and measurements are described by probability density function (PDF), and recursive Bayesian filtering is operated to obtain the target state estimation. The first step is still to model the target motion, and in order to tackle the problem of multitarget, the birth and death of target are also modeled via prior knowledge. Then an important step in Bayesian TBD approaches is to build the probabilistic entity for the measurements in the surveillance region. Finally, the target state distribution is estimated by Bayesian filtering. Fig. 4 shows the process scheme for Bayesian TBD methods.

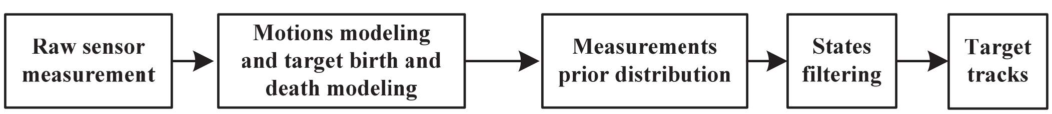

Figure 4. Process scheme for Bayesian TBD methods.

The main idea of TBD algorithms is to search all possible tracks, and select the most possible one as the output. The advantage is that it can accumulate target information by multiframe in low SNR scenarios. But it is at the cost of complex computation, and the track outputs are time-delay.

3. DETECT-BEFORE-TRACK BASED MTT

In this section, the representative algorithms of data association based methods and FISST based methods are presented respectively. The data association methods were proposed earlier. In data association based methods, the relating techniques, such as target motion modeling, filtering algorithms, and tracks management have been investigated extensively. These techniques are inherited and evolve by the FISST based methods, which are proposed more lately. In this section, this evolvement course will be presented.

3.1. Data Association Based Approaches

Traditional MTT includes two steps, measurement-target data association and state estimations. The tasks of them are separate, but the optimum way is to consider them jointly, which is also the goal of many optimum methods.

3.1.1. Data Association Techniques

A good many methods have been proposed to address data association problem. Based on the objective function that they purport to optimize, data association algorithms are often categorized to non-Bayesian approaches and Bayesian approaches [27–29]. Non-Bayesian approaches generally treat the data association as a determined problem, and use a optimize algorithm to find a solution for associating. Bayesian approaches express the problem by statistical theory, and deduce some typical algorithms, including maximum a posteriori (MAP) and Bayesian estimator approaches.

In addition, there are some other criteria to distinguish data association methods [15]: (1) *Recursive process and batch process.* In recursive methods, processing is done at each scan using data received on that scan to update the results of previous processing. On the other hand, batch methods represent the ideal situation where no information is lost due to preprocessing because all measurements are processed together. A further scheme is multi-pass batch in which the entire data set is reprocessed several times to refine the state estimates. (2) *Single-scan and Multi-scan.* Single-scan algorithms estimate the current states of targets based on their previously estimated states and the current scan of measurements. Single-scan algorithms typically use a highly simplified approximate representation of the posterior state estimate. Multi-scan algorithms estimate the current states of targets based on their previously estimated states, multiple past scans and the current scan of measurements. They may revisit past scans when processing each new scan, and can thereby revise previous estimates in the light of new evidence. (3) *Hard association and soft association.* For hard association, namely, it assumes that each measurement may either belong to at most one target or be classified as a false alarm, and at most one measurement may be associated with a given target at a time. The soft association methods ignore the constraint that a point target may only produce a single measurement at a given time, and numerical approximation of the multitarget PDF may be attempted alternatively.

This paper concludes the data association methods by classifying them into non-Bayesian approaches and Bayesian approaches. Meanwhile, the other three criteria are also presented at appropriate places to help the understanding.

3.1.1.1 Non-Bayesian Approaches

The non-Bayesian approaches are characterized by hard measurement-track association, such that some cost function is maximized, and they are deterministic algorithms.

Two simple solutions are the strongest neighbor filter (SNF) and the nearest neighbor filter (NNF) [30]. In the SNF, the signal with the highest intensity among the validated measurements is used for track update and the others are discarded. In the NNF, the measurement closest to the predicted measurement is used. The problem with this filter is that it does not consider that the measurement selection could have originated from clutter instead of the target.

To allow for the possibility that the selection of the nearest or strongest measurement is wrong, the track splitting filter, developed by Smith and Buechler [31], creates a new track for each validated measurement received at every update. Unfortunately, the number of measurement histories grows exponentially. To control this exponential growth, the conditional measurement likelihood function is computed for each measurement sequence, and is used to prune the most unlikely tracks.

Additionally, other non-Bayesian approaches include the Hopfield neural network [32], fuzzy data association (FDA) [29, 33], the Viterbi algorithm (VA) using a dynamic programming (DP) technique [34], graph theory [35], genetic algorithm [22], the cross entropy (CE) method [28], game-theoretic framework [36] to solve the data association. Some others turn out to use combined methods, such as the combination of FDA and GA for solving multidimensional assignment problem [37], the combination of GA and neural network based association [38].

The problems of non-Bayesian approaches are two: 1) The computation of heuristic optimization algorithms is time consuming; 2) Non-Bayesian algorithms make hard association decisions at the end of every frame and are characterized by poor performance in the presence of false alarms and in dense target environments [39]. So in practical application, non-Bayesian approaches are restrictedly used.

3.1.1.2 Bayesian Approaches

The optimal data association theory involves enumerating and evaluating all feasible joint assignments for measurements and tracks. The problem is that the number of feasible joint assignments grows combinatorially in the number of measurements and the number of tracks. Instead of optimal method, approximate solution is proposed in practice based on Bayesian theory, including maximum a posteriori (MAP) and Bayesian estimator [27].

(1) MAP approaches

MAP approaches find the most probable association, given the measurements made so far, and estimate tracks given this association.

The most well-known MAP approach is the multiple hypotheses tracking (MHT) algorithm [40]. MHT is a multiscan tracking algorithm that maintains multiple hypotheses associating past measurements with targets. When a new set of measurements arrives, a new set of hypotheses is formed from each previous hypothesis. The algorithm returns a hypothesis with the highest posterior as a solution. MHT is capable of initiating and terminating a varying number of tracks and is suitable for autonomous surveillance applications. The main disadvantage of primal MHT is its computational complexity since the number of hypothesis grows exponentially over time. Various heuristic methods have been developed to control this growth [41]; but these methods are used at expense of the MAP optimality.

Another typical MAP approach is the integer programming algorithm [42] or, more precisely, the multidimensional assignment algorithm [43], where the measurements in the last S scans are associated with the list of tracks (S-dimensional association, denoted as S-D), has been shown to be a practical and feasible alternative to MHT without the limitation of exhaustive enumeration. In the S-D assignment formulation of data association, the association between the lists of measurements and the list of tracks is formulated as a global discrete optimization problem, subject to curtain constraints, where the objective is to minimize the overall cost of association. For the case of S is equal to 2, well-known 2-D assignment methods are the Jonker-Volgenant-Castanon (JVC) algorithm [44]. This method also provides a measure of accuracy for the solution found [45]. While finding the optimal assignment for $S > 2$ is an NP-hard problem [46], a number of near-optimal modifications with polynomial complexity have been proposed [47]. Lagrangian relaxation-based techniques [48] have been proposed to find suboptimal solutions for applications that require realtime performance. In [49], impelled from the success of randomized heuristic methods, Bozdogan and Efe investigated a different stochastic approach, namely, the biologically inspired ant colony optimization to solve the NP hard S-D assignment problem for tracking multiple ground targets. In [50], Walteros et al. considered a variant of the S-D assignment problem with decomposable costs in which the resulting optimal assignment is described as a set of disjoint stars.

There are other attempts to utilize MAP to handling the data association problem. In [51], further association of these highly fragmented tracklets at each level of the hierarchy is formulated as a MAP problem that considers initialization, termination, and transition of tracklets as well as the possibility of them being false alarms, which can be efficiently computed by the Hungarian algorithm. However, the underlying MAP data association problem is NP-hard, so we do not expect to find efficient, exact algorithms.

(2) Bayesian estimator

Bayesian estimator approaches estimate tracks by minimizing the posterior expected value of some risk function. When the mean squared error is used as a risk function, the Bayesian estimator is a minimum mean-square error (MMSE) estimate. The MMSE estimates of tracks are computed by summing over all possible associations, weighted by their posteriors. Optimal Bayesian estimator approaches to solve data association problems are even less tractable than the MAP computation. Otherwise several "pseudo-Bayesian" methods have been proposed.

The best-known one is the joint probabilistic data association (JPDA) filter [52, 53]. JPDA is a suboptimal single-scan approximation to the optimal Bayesian filter; it can also be viewed as an assumed density filter in which the joint posterior distribution is approximated by a product of simpler distributions such as moment matching Gaussian distributions. At each time step, instead of finding a single best association between measurements and tracks, JPDA enumerates all possible associations and computes association probabilities, where is the probability that the measurement extends the track. The exact calculation of association probabilities in JPDA, which requires the summation over all association event probabilities, is NP-hard. Some heuristic approaches to approximate JPDA [54, 55]. A shortcoming of the basic JPDA is its inability to initiate and terminate tracks. Moreover, the original JPDA can only handle targets whose number is fixed over time.

Unlike JPDA, Markov chain Monte Carlo data association (MCMCDA) is a true approximation scheme for the optimal Bayesian filter [56], i.e., when run with unlimited resources, it converges to the Bayesian solution. MCMCDA uses Markov chain Monte Carlo (MCMC) sampling instead of enumerating over all possible associations. And an ability to initiate and terminate tracks, so that the algorithm can be applied to the full range of data association problems. When the target number is fixed, the single-scan version of MCMCDA approximates JPDA. Although the exact computation of association probabilities in JPDA is NP-hard, while the single-scan MCMCDA algorithm provides a fully polynomial randomized approximation scheme for JPDA [27]. Also, a MCMC tracker can solve the problem of one-to-one assumption by propagating both in space and time the association hypothesis [57].

Another method for handling data association in the Bayesian manner is the probabilistic multi-hypothesis tracker (PMHT) [58, 59]. PMHT iteratively computes data association probabilities and track updates, using the expectation maximization (EM) method. The PMHT algorithm employs a more lenient measurement model. It drops the constraint that a target can generate at most one measurement per scan, and posits the measurement/target association process as independent across measurements, so it is a soft association method. Tracks are updated using the forward and backward images in each iteration, so the basic PMHT is a batch process. Similarly to JPDA, the basic PMHT assumes a fixed and known number of targets in the scenario under consideration. The PMHT methods is computational more feasible, but is at the cost of performance loss.

Generally speaking, MHT and JPDA are most widely used methods for data association. But in a dense target and cluttered environment, the number of possible association combinations is explosion and enumeration is computationally expensive, which leads that the association is an NP-hard problem. Moreover, it is even harder when targets come very closely spaced or even cross paths. So it demands further developments for data association in future works.

3.1.2. Target State Estimation

Once the measurement-track association is determined the target state is estimated by a filtering algorithm. Employing the DSS approach to describe the target motion, the key to successful target tracking lies in the effective extraction of useful information about the target's state from measurements. A good motion model of the target will certainly facilitate this information extraction to a great extent. On the other hand, state filtering is another key step that needs to be considered in target state estimation.

3.1.2.1 Target Motion Modeling

Moving targets normally include nonmaneuvers and maneuvers. A nonmaneuvering motion is the straight and level motion at a constant velocity in an inertial reference system. The interested targets, such as aircraft ballistic missiles, submarines, ships, and ground targets, are usually maneuvers. The maneuvers involve the problem of motion uncertainty, and tackling the motion of maneuvering target is really difficult.

Various mathematical models have been developed to address the motion modeling problem of maneuvering target Li and Jilkov [16–20] provided comprehensive surveys about the techniques for maneuvering target modeling. In the presence of motion uncertainty a primary solution is the so-called multiple-model (MM) method. The modes in MM approach represent the mixture order of the system PDF, optimal MM approach has a drawback that the number of the modes is exponential growth, and consequently exponentially increase the number of filters [60, 61]. Suboptimal techniques for mode order reduction based on merging and pruning are summarized in [61]. Typical approaches include generalized pseudo-Bayesian (GPB) merging [60, 62] mixture reduction (MR) (such as Gaussian MR in [63]) and interacting multiple models (IMM) methods [60, 64, 65].

The MM techniques can be used in MTT combined with data association. The direct methods for target tracking in the presence of clutter with Gaussian MR method are proposed in [66] through [63]. The MHT and the MR methods [66] employ ad-hoc joining and clustering, which preserves the mean and the covariance of the original distribution. The IMM algorithm has also been extended with well-established data association algorithms exist, such as the IMM version of the JPDA filter (IMMJPDA) [67] the IMM version of the MHT (IMMMHT) [68].

Recently, the third generation several variable structure MM (VSMM) algorithms have been proposed [69], it is potentially much more advanced in the sense of having an open architecture — a variable structure — than its ancestors, which have a closed architecture. General VSMM algorithms [70] select an admissible model set at any time instance [71]. But in order to extend VSMM to practical use, we need to overcome the drawback of their sophistication.

Otherwise, the system DSS modeling would generate two main challenges: nonlinearity and non-Gaussian, which leads to the difficulties in state estimation.

3.1.2.2 Bayesian Filtering

The Bayesian theory provides a general framework for dynamic state estimation. In a Bayesian view, filtering is an iterative updating of conditional probability densities that describe the current target states given the accumulated sensor information and available a priori information. The process equation represents a system evolving with time, where the system is represented by the hidden state, and the prior knowledge of the initial state is given by the probability distribution.

Suppose $Z_{1:k} = \{\mathbf{z}_1, \dots, \mathbf{z}_{k-1}\}$ is the measurement at time $k-1$, and the posterior distribution is $p_{k-1|k-1}(\mathbf{x}|Z_{1:k-1})$, then the predictive distribution at time k is

$$p_{k|k-1}\left(\mathbf{x}|Z_{1:k-1}\right) = \int f_{k|k-1}\left(\mathbf{x}|\mathbf{x}'\right) p_{k-1|k-1}\left(\mathbf{x}'|Z_{1:k-1}\right) d\mathbf{x}' \tag{3}$$

where $f_{k|k-1}(\mathbf{x}_k|\mathbf{x}_{k-1})$ is the Markov transmit model of target state. When $\mathbf{z}_k$ is received at time k, the update of the posterior distribution at time k is

$$p_{k|k}\left(\mathbf{x}|Z_{1:k}\right) = \frac{g_k\left(\mathbf{z}_k|\mathbf{x}\right) p_{k|k-1}\left(\mathbf{x}|Z_{1:k-1}\right)}{\displaystyle\int g_k\left(\mathbf{z}_k|\mathbf{x}\right) p_{k|k-1}\left(\mathbf{x}|Z_{1:k-1}\right) d\mathbf{x}} \tag{4}$$

where $g_k(\mathbf{z}_k|\mathbf{x})$ is the likelihood function at time k.

There are several kinds of methods to implement Bayesian filtering, including Kalman-like filtering, grids based filtering, and particle filtering.

(1) Kalman-like filtering

When the DSS model is linear with Gaussian noise, and the filtering and predictive distributions are Gaussian, then the Kalman filter (KF) provides the mean and covariance sequentially, which is the optimal Bayesian solution [72]. In practice, the DSS model is nonlinear and non-Gaussian. Extended Kalman filter (EKF) [73, 74] and unscented Kalman filter (UKF) [75] are the standard nonlinear filtering techniques to modify the KF to relax some of the linearity assumptions.

There have been other attempts to solve the non-Gaussian problem. One of the methods is the Gaussian sum filters (GSFs) [76] that work by approximating the non-Gaussian target distribution with a mixture of Gaussians. A recursive estimator, denoted as Gaussian mixture KF (GMKF), for linear non-Gaussian problems was derived in [77, 78]. It suffers, however, from the same shortcoming as the EKF in that linear approximations are required. In the GSF methods, the problem of exponential model order growth was solved using a greedy expectation-maximization (EM)-based method in [77, 78]. However, these techniques do not accurately model all of the salient features of the density, which limits their applicability to scenarios where the target state posterior density is well approximated by a multivariate Gaussian density.

Kalman-like filters can be incorporated with existing data association well. MHT and JPDA both use extended Kalman filters (EKFs) for filtering the associated measurements. The Kalman-like filters and GM approximations are generally accurate especially when the nonlinearity and non-Gaussianlity is mildly. However, when the situation is more serious, such methods are inefficient.

(2) Grids based filtering

The grids-based methods evaluate the required densities over grids [79, 80], which utilize a discrete representation of the entire single target density. In this setup, no assumptions on the form of the density are required, so arbitrarily complicated densities may be accommodated. A grids-based method for non-Gaussian models that does not require any linear approximations has been proposed in [81]. It approximates the non-Gaussian state numerically with a fixed grid, and applies numerical integration for the prediction step and Bayesian theory for the filtering step.

Despite their good performance, the grid-based solutions suffer from two shortcomings: 1) They are computationally intensive, fixed grid approaches are computationally intractable except in the case of very low state space dimensionality [82]; 2) A discrete-valued model for target motion may be too simplistic to represent real-world target dynamics.

(3) Particle filtering

The PF, or the sequential Monte Carlo (SMC) method is a Monte Carlo simulation-based method and can be applied to solve nonlinear and non-Gaussian problems [83, 84]. There, a distribution is represented by a weighted set of samples (or particles), which are propagated through the dynamic system using importance sampling to sequentially update the posterior distributions. These methods provide optimal results asymptotically in the number of particles.

The standard method to implement PF deduces a sequential importance sampling resampling (SISR) filter [83]. However, a major disadvantage of SISR is the computational complexity, a large part of which comes from a procedure called resampling [85]. The Gaussian sum particle filter (GSPF) [85] implements the PF assuming Gaussian mixture distributions for the system and measurement noises. The GPF [85] is quite similar to the SISR filter by the fact that importance sampling is used to obtain particles. However, unlike the SISR filters, resampling is not required in the GPF. This results in a reduced complexity of the GPF as compared with the SISR with resampling and is a major advantage.

The main challenge in PF for MTT is the curse of dimensionality, which arises because of the large dimension of the state space. A basic requirement for efficient sampling is the exploitation of posterior independence between target states. Posterior independence refers to the case where the posterior density of the multitarget state can be written as a product of densities of clusters of individual target states. This arises, for instance, when target positions are measured and targets are far apart. But in other situation, this assumption is not satisfied. So the posterior density has to be sampled jointly. Depending on this difference, the PFs for MTT are divided into two groups [23, 86].

1) Independent samplers. Posterior independence was first exploited in the so-called independent partition PF (IPPF) [87], which samples the states of independent target clusters independently. These algorithms have low computational cost, even in the presence of many targets, and are generally applicable. For the sake of convenience it is assumed in the discussion that measurements depend only on target position, as is often the case, so that well-separated targets exhibit posterior independence.

The IPPF of Orton and Fitzgerald [87] and the work of Maskell et al. [88] consider MTT via PF from a purely Bayesian perspective. Measurement-to-target association is not done explicitly; it is implicit within the Bayesian framework. This work has focused on a tractable implementation of ideas in [89].

A majority of independent samplers based PFs for MTT are combined with the well-developed data association algorithms. Once the associations are known, each target can be tracked independently and simultaneously. In [90], Hue and Le Cadre used a PF based on the PMHT. Others have done work which amounts to a blend between JPDA and PF [91–93]. Methods like the Existence JPDAF (E-JPDAF) [94] are reported to be able to perform automatic target detection and tracking using PF.

In [95, 96], a Rao-blackwellized Monte Carlo data association is proposed to reduce the computational cost of a multitarget PF. In [97] and [98], William et al. presented an online approach for joint detection and tracking for multiple targets with multiple sensors using PF methods. To cope with the data association problem, an efficient 2-D assignment algorithm is adopted. In [36], Chavali et al. developed and employs a game-theoretic formulation to solve the data association in a deterministic fashion. In the MCMC-based PF of [35], the Viterbi algorithm (VA) is used for data association.

Other PFs belong to this group include the methods in [99–101]. In the independent samplers based PFs for MTT, the performance of PFs degrades unduly if several targets move in proximity for extended durations. In this situation, the posterior independence assumption is invalid, and sampling the states of nearby targets independently is also inefficient.

2) Joint samplers. The states of targets which are not sufficiently well-separated for posterior independence to be assumed are sampled jointly. These PFs, referred to as joint samplers, can deal with the target coupling and measurement ambiguity via joint sampling but have a much heavier computational complexity than independent samplers. The representative works are the sequential sampling PF (SSPF) in [93] and the methods in [102–104]. In [105], a novel algorithm named evidence theory-based mixture particle filter is proposed for joint detection and tracking for a varying number of targets. The posterior distribution of multiple target state considered in single target state space is a multi-modal distribution with each mode corresponding to either a target or clutter. An evidence theory-based framework is utilized to determine the structure of the global posterior distribution. In [8, 9], an enhanced version of PF is employed and is called Mixture PF, which samples from both the prior and the observation likelihood, as compared to the majority of PF variants that sample from only a single importance density. In order to be able to track an unknown time varying number of multiple targets, two Mixture PFs are used, one for target detection and the other for tracking multiple targets, and a density-based clustering technique is used after the first filter.

Table 1 shows representative algorithms and performance analysis for state filtering. Indeed, PFs based approaches have been used successfully in areas where Kalman like filters or grid-based filters have previously been employed. Also, PF can be used as a tool for the implementation other algorithms, such as the PHD/CPHD filtering and joint multitarget probability density (JMPD) filtering in following sections. So PFs have been a hot topic for researchers.

Table 1. Representative algorithms and performance analysis for state filtering.

Approaches	Linear/nonlinear	Gaussian/non-Gaussian
KF	Linear	Gaussian
EKF	nonlinear	Gaussian
UKF	nonlinear	Gaussian
grid-based filtering	nonlinear	Gaussian
PF	nonlinear	non-Gaussian

3.2. FISST Based Approaches

The random finite set (RFS) framework developed by Mahler [24] using finite set statistics (FISST) offers a distinct alternative to the traditional approach for MTT, by treating the collection of individual targets as a set-valued state and the collection of individual measurements as a set-valued measurement. It propagates the posterior intensity of the RFS of targets in time and does not require any data association computations. FISST is a system-level, "top-down," direct generalization of ordinary single-target engineering statistics to the realm of multitarget detection and tracking [106, 107].

3.2.1. Multitarget Bayesian Filtering

Firstly, the RFSs are used to redefine the DSS for multitarget [24]. Suppose at time k that there are $N(k)$ targets with states $x_{k,1}, \ldots, x_{k,N(k)}$, each taking values in a state space $\mathcal{X} \subseteq \mathbb{R}^{n_x}$. Suppose also at time k that $M(k)$ measurements $z_{k,1}, \ldots, z_{k,M(k)}$ are received and each taking values in a measurement space $\mathcal{Z} \subseteq \mathbb{R}^{n_Z}$. Then, the multitarget state X_k and multitarget measurement Z_k, at time k, are defined as $X_k = \{x_{k,1}, \ldots, x_{k,N(k)}\} \in \mathcal{F}(\mathcal{X})$, $Z_k = \{z_{k,1}, \ldots, z_{k,M(k)}\} \in \mathcal{F}(\mathcal{Z})$, where $\mathcal{F}(\mathcal{X})$ and $\mathcal{F}(\mathcal{Z})$ denote the respective collections of all finite subsets of $\mathcal{X}$ and $\mathcal{Z}$. By modeling the multitarget state and multitarget measurement as RFSs, the multitarget filtering problem can be posed as a Bayesian filtering problem with state space $\mathcal{F}(\mathcal{X})$ and measurement space $\mathcal{F}(\mathcal{Z})$. The number variety of multitarget is decided by target survival, birth and spawn. So in prediction step of Bayesian filtering, multitarget dynamics can be modeled by

$$X_k = \left[\bigcup_{\zeta \in X_{k-1}} S_{k|k-1}(\zeta)\right] \cup \left[\bigcup_{\zeta \in X_{k-1}} B_{k|k-1}(\zeta)\right] \cup \Gamma_k \tag{5}$$

where X_{k-1} is the multitarget state at time $k-1$, $S_{k|k-1}(\zeta)$ the surviving RFS of target at time k evolved from a target with previous state ζ, $B_{k|k-1}(\zeta)$ the spawning RFS of target at time k that evolved from a target with previous state ζ, and Γ_k the RFS of spontaneous births at time k. Within the area of target tracking, spawning means that appearing targets can be modeled as generated by existing targets. Examples of such situations can be when airplanes take off from a carrier boat or when a sensor resolves new features on an extended object. Similarly, the multitarget sensor measurements are modeled by

$$Z_k = \left[\bigcup_{x \in X_k} \Theta_k(x)\right] \cup K_k \tag{6}$$

where $\Theta_k(x)$ is the RFS of measurements generated by the single-target state x at time k and K_k the RFS of clutter measurements or false alarms at time k.

Also divide the process of filtering to prediction step and update step. Suppose that at time $k-1$, the multitarget posterior density $\pi_{k-1|k-1}(X|Z_{1:k-1})$ and cumulative measurements $Z_{1:k-1} = (Z_1, \ldots, Z_{k-1})$ is known, then the predicted multitarget posterior density $\pi_{k|k-1}(X|Z_{1:k-1})$ is

$$\pi_{k|k-1}\left(X|Z_{1:k-1}\right) = \int f_{k|k-1}\left(X|X'\right)\pi_{k-1|k-1}\left(X'|Z_{1:k-1}\right)\mu_s(dX') \tag{7}$$

where $f_{k|k-1}(X|X')$ is the Markov state transition density function and $\mu_s(\cdot)$ an appropriate reference measure on $\mathcal{F}(\mathcal{X})$. Received measurement Z_k at time k, then the updating of posterior probability density is

$$\pi_{k|k}\left(X|Z_{1:k}\right) = \frac{g_k\left(Z_k|X\right)\pi_{k|k-1}\left(X|Z_{1:k-1}\right)}{\int g_{k|k-1}\left(Z_k|X\right)\pi_{k|k-1}\left(X|Z_{1:k-1}\right)\mu_s(dX')} \tag{8}$$

where $g_k(Z_k|X)$ is multitarget likelihood function.

In the formulation to use RFS to describe multitarget state and measurement state with varying number, the targets can appear and disappear anywhere and anytime, while target motion can be described by a nonlinear stochastic dynamic model. The multitarget posterior can be estimated using a generalization of the single-target Bayesian filtering equations to a multiple-target scenario.

The complexity of computing the recursion of (7) and (8) grows exponentially with the number of targets, due to the state of the transfer function and the measurements likelihood function, modeling and measuring the state of the transfer function of the likelihood function is very difficult, the corresponding integral operation is difficult to achieve. Therefore, the need to find approximate solutions can be realized works.

To alleviate the complexity of computing the multitarget posterior, a recursion was derived for the first-order moment of the multitarget posterior distribution, known as the PHD filter [24], based on FISST. FISST provides the definition of set measure, set integral, set derivative for RFS. The derivation of PHD is based on the assumptions that the clutter RFS is Poisson and independent of the measurement RFSs, and the predicted multitarget RFS is Poisson. The PHD filter use intensity function to replace the postier PDF $f_{k|k}(X|Z_{1:k})$. The intensity function is denoted as $v_k(x)$, which is defined as a function whose integral in the state space S is the expect target number $\hat{N}$ in this area [24]. Since the PHD recursion is a first-order approximation, it propagates cardinality distribution (the probability distribution of the number of targets) with only a single parameter and effectively approximates the cardinality distribution by a Poisson distribution with matching mean. Since the mean and variance of a Poisson distribution are equal, when the number of targets present is high, the PHD filter estimates the cardinality with a correspondingly high variance.

Furthermore, the CPHD recursion was proposed by Mahler in [25] to address the limitations of the PHD recursion. In essence, the strategy behind the CPHD recursion is to jointly propagate the intensity function and the cardinality distribution. The CPHD filter is a PHD filter with an extra level of complexity added to its underlying hidden Markov model (HMM), i.e., the cardinality of the Poisson RFS is not restricted to be Poisson distributed and can be arbitrary. The CPHD filter can attain more accurate estimation of target number, but it is at the cost of complexity increase. The computational complexity of CPHD is $O(m^3n)$, while the computational complexity of PHD is $O(mn)$, where n is the target number and m the measurement number.

Moreover, Erdinc et al. deduced the prediction and update equations by a "bin-occupancy" model in [108], which is connected to PHD/CPHD filtering. A method for deriving the PHD and CPHD recursions, without using FISST, is presented in [109, 110]. In addition to the PHD/CPHD filters, Mahler also proposed the multitarget multi-Bernoulli (MeMBer) filter [111] and cardinality balanced MeMBer (CBMeMBer) filter [112] as a tractable approximation to the Bayes multitarget recursion under low clutter density scenarios. Unlike the PHD/CPHD recursions, which propagate moments and cardinality distributions, the MeMBer/CBMeMBer recursion propagates (approximately) the multitarget posterior density. Specifically, the parameters of a multi-Bernoulli RFS that approximates the posterior multitarget RFS are propagated.

3.2.2. FISST Based Filters for MTT

3.2.2.1 Implementation of FISST Based Filters

In order to have a practical implementation of the PHD/CPHD filters, it is necessary to have a tractable representation of the intensity function. Two alternatives are the Gaussian mixture (GM) versions [113, 114] and the sequential Monte Carlo (SMC)/PF versions [115, 116] of the filters.

With the assumption that target state and measurement obey a linear Gaussian model, an effective approach based on GM can be used to implement PHD/CPHD, which denoted as GM-PHD/GM-CPHD. Clark [117] proved uniform convergence of the errors in the algorithm and provides error bounds for the pruning and merging stages. GM approach can also extend to non-linear application by non-linear techniques such as EKF and UKF [117]. In the situation of mildly non-linear, the GM implementation is more commonly used, but when the non-linearity of target dynamics and/or the measurement process is severe, such method fails.

The SMC method can solve the problems of non-linear and non-Gaussian. However, the SMC implementation is computational time-consuming and the state estimation is expensive and unreliable. So a lot of researches have been developed to improve the performance of the SMC method. For SMC implementation of PHD/CPHD (SMC-PHD/SMC-CPHD), several improvements have been developed. In [86], the data-driven importance functions and corresponding weight functions of the SMC-PHD filter are proposed for survival targets and spontaneous birth targets, instead of use the dynamic model of system as the importance function simply [115]. In [118], a novel particle resampling strategy and adapts dynamic and measurement models to cope with varying object scales is proposed. The auxiliary SMC-PHD filter [119] and measurement-oriented particle labeling technique [120], are also developed to partially solve the clustering problem in the extraction of state estimates from the particle population. In [121], a novel ant clustering filtering algorithm, under the guidance of SMC-PHD, is investigated and applied to estimate the time-varying number of targets and their individual states in a cluttered environment. In [122], the distribution of the particles is fitted using finite mixture models (FMMs), whose parameters can be derived using a MCMC sampling scheme, then the states can be extracted according to the fitted mixture distribution. In [123], a flexible modularized structure for SMC-PHD filter is introduced, and the particles with the same class label and their corresponding weights represent the estimated class-conditioned PHD distribution. The mathematical proofs of convergence for the SMC algorithm and gives bounds for the mean-square error is presented in [124].

In addition, for the MeMBer recursions, [111, 112] also propose a GM implementation for linear Gaussian multitarget models and extend this technique to mildly nonlinear multitarget models via linearization and the unscented transform. In [112], the authors also proposed a generic SMC implementation of the MeMBer/CBMeMBer recursion that accommodates nonlinear dynamic and measurement models with state-dependent sensor field of view, the key advantage of this approach over the SMC-PHD/SMC-CPHD filters is that the multi-Bernoulli representation allows reliable and inexpensive extraction of state estimates.

3.2.2.2 Improved Problems of FISST Based Filters

In order to apply FISST-based filters for practical application, we still need to deal with numerous practical problems. These problems mainly include maneuvering target motion modeling, unknown detection probability and clutter rate accommodation, target birth and target spawning modeling, and track management.

(1) Maneuvering target motion modeling

In tracking maneuvering targets, the MM methods have been extended to PHD/CPHD filters. Such as linear jump Markov system (JMS) for GM-PHD filter [125, 126] and SMC-PHD [127, 128], nonlinear JMS models for PHD filter [129]. Also, the IMM method for GM-PHD filter [130], and the MM method for GM-CPHD filter [131], are also presented in the literature.

(2) Unknown detection probability and clutter rate

Conventional PHD/CPHD filters assume that the parameters of clutter rate and detection profile are known a priori. For the clutter density, it is commonly assumed to be uniformly distributed over the measurement region. But in some practical applications, such an assumption is not realistic. Both the average clutter number per scan and the clutter spatial distribution might become unknown due to the complicated background and other disturbances. Thus, the ability of the PHD and CPHD filters to accommodate unknown clutter rate and detection profile is very important in practice.

Preliminary researches about this problem have been reported in [132–135] by Mahler, which outline a general formulation for estimating both the detection profile and the clutter intensity function. However, the proposed method is intractable and no practical implementation method was given. In [136], closed-form solutions to these filtering recursions are derived using Beta and GM. Otherwise, in [137], nonhomogeneous Poisson point processes, whose intensity function are assumed to be a mixture of Gaussian functions, are used to model clutter points in PHD filter to relax the Poisson point process assumption. In [138], the clutter density is estimated as finite mixture models (FMM) for the PHD filter.

(3) Target birth and spawning model

The standard formulation of the PHD/CPHD filters assumes that the target birth intensity is known a priori. In situations where the targets can appear anywhere in the surveillance volume this is clearly inefficient, since the target birth intensity needs to cover the entire state space.

In [139], Ristic et al. presented a new extension of the PHD/CPHD filters, which enables us to adaptively design the target birth intensity at each scan using the received measurements. Maggio and Cavallaro [140] proposed a framework that the spatial distributions of birth and clutter events are incrementally learned based on the learned contextual information. Clever uses of the PHD filter with measurement-driven birth intensity are independently proposed in [120] and [141] to improve tracking performance as well as obviating exact knowledge of the birth intensity. A track initiation technique is proposed to detect the position unknown birth targets and is hybridized with PHD and CPHD filter in [142]. In [143], a multitarget visual tracking system that combines object detection with the GM-PHD is developed, in which a new birth intensity estimation method based on entropy distribution and coverage rate is proposed. In [144], the Doppler information (DI) is incorporated for more precise birth target selection and improving the accuracy of predicted states of GM-PHD filtering.

Moreover, in [114], target spawning is incorporated into the original PHD filter but not into the CPHD filter where new targets are modeled by spontaneous birth only. Lundgren et al. [145] derived the necessary equations for a CPHD filter for the case when the process model also includes target spawning, he also derived expressions that are practical and computationally efficient when the cardinality distribution of the spawning targets is either Bernoulli or Poisson.

(4) Track management

Another major drawback of the PHD/CPHD filter is that it cannot identify the trajectories of different targets. A couple of different methods for solving the problem have been reported in the literature. These methods can be classified to two kinds, the peak-to-track method, and the labeling method.

The peak-to-track methods also use PHD/CPHD filtering to estimate the peaks of the intensity, which are deemed as target positions. Then the peaks are association to target tracks by traditional data association methods, such as PHD/CPHD filtering with MHT [146, 147], PHD/CPHD filtering with the connectivity graph and cross entropy (CE) technique [148], CPHD filtering with fuzzy logy [149], 2-D assignment for PHD filtering [150, 151] and CPHD filtering [152, 153], and PHD/CPHD filtering with graph matching [118, 140].

In the labeling methods, the essential task of linking target state estimates along time to form tracks can be done if a unique label is added to each single target state in both approaches. The labeling methods are operated in the implementation process of PHD/CPHD filtering. In the implementation, the elements used for representing target are particles in SMC implementation [154] or Gaussian element in GM implementation [131, 155–158]. The labeling methods partition the elements in the position domain and give each element in the same partition with the same label. In subsequent iterations, when resampling (or pruning and merging), we give the children of an element the same label as its parent. After resampling (or pruning and merging), repartition the data, and if the majority of the particles in one partition have the same label, then associate these partitions to the label. So we can keep the records of the target identities. The advantage of labeling method is that it can be combined with the filtering implementation directly, with little complexity increases.

4. TRACK-BEFORE-DETECT BASED MTT

In low SNR scenarios, the TBD algorithms are the alternative methods to improve the tracking performance. Early approaches to TBD are non-Bayesian approaches. They perform well in the scenarios for which the target velocity is known a prior. But these approaches are not recursive and have difficulty modeling complicated target motion. More promising approaches are to adopt a Bayesian perspective where the quantity of interest is a Markov process. The Bayesian approaches have the advantages of providing a recursive solution with arbitrary target dynamic models. This section presents both the non-Bayesian TBD approaches and Bayesian TBD approaches.

In the context of TBD, the pixelized model is used for raw measurements. Apparently, the imaging sensor match pixelized measurement model naturally. Otherwise, for radar and sonar, their sensor data is similarly processed to that in image data using the pixelized model.

4.1. Non-Bayesian Approaches

In the following, the non-Bayesian approaches include projective transforms methods, multistage hypothesis testing method, 3-D matched filter, higher-order correlation method, and dynamic programming algorithm.

(1) Projective Transforms

Since the sensor collects each frame periodically and the target is moving, the target track appears in a set of spatiotemporal 3-D data as a straight line or other curve. One method is to project the 3-D data along the temporal axis onto a plane defined by two spatial axes (x and y) to create the so-called track map. This enables us to analyze the 3-D problem in terms of 2-D projections and allows us to reconstruct the estimated 3-D target trajectory in the original spatiotemporal space using back projection. Strictly speaking, the projective transforms approaches do not deem that the target tracking as a stochastic estimation problem, but the tracking still depends on the motion measurements, so this paper presents them here for integrity.

Hough Transforms [159, 160] or Radon transform [161] is the most typical algorithm. In this method, except for the time required to compute the forward and backward projections, the amount of data to be processed can be reduced significantly. Since the projections are basically orthogonal projections, the technique further enables 3-D target trajectory estimation and reconstruction in real time. But the drawback is that it can only track target with determinate trajectory. The multi-dimensional Hough transform technique presented in [162] provides a computationally-efficient approach to address random clutter backgrounds. Also, when the clutter is dense or the target movement between two frames is significant, the detection performance deteriorates.

(2) Multistage Hypothesis Testing

Blostein and Huang [163] proposed a multistage hypothesis testing algorithm (MSHT) for target tacking and detection. The MSHT algorithm uses a truncated sequential probability ratio test (TSPRT), to efficiently evaluate a dense tree of linear, constant velocity candidate trajectory segments [164]. A large number of candidate trajectories, organized into a tree structure, are hypothesized at each pixel in the sequence and tested sequentially for a shift in mean intensity.

But under the condition of low SNR, due to dense clutter, the candidate target trajectories are numerous, leads to the increase of tree branches, the composite exposure leads the computation intractable.

(3) 3-D matched filters

A localization technique using 3-D matched filtering (3-DMF) has been developed by Reed [165–167] for detecting multiple weak, moving targets in clutter. Such methods synthesize the 2-D matched filter (2-DMF) and the 1-D time energy, where the 2-DMF is used to depress the clutter, and the 1-D of time energy is cumulated based on the velocity feature.

Target velocity is a parameter that completely characterizes the matched filter. In the situation of unknown target velocity, Chen [168] designated this type of matched filters to be the assumed velocity filters (AVF) to emphasize the velocity parameter. But AVF must be implemented by partitioning the velocity space, so for target with drastic velocity variety, the needs of velocity filters are too many, leads to intractable in practice. In [169], Zhang et al. proposed a novel 3D wide-to-exact search directional filtering (3-DWESDF) to decrease the 3-DMF computational requirements and increase the target energy accumulation ability further.

The 3-DMF is suitable at the situation of target with constant velocity. However, target velocity is usually unknown in practice. Besides, the strategy of 3-DMF is exhaustive search with high complexity, so it is intractable in the application of real time.

(4) Higher-order correlation

A method referred to as high order correlation (HOC) was developed which does not make any a priori assumption about the targets and background clutter [170]. In [171], a track scoring mechanism is proposed and a new method is developed to perform data association and track identification in the presence of heavy clutter using the modified HOC. This method was modified by imposing velocity and curvature constraints in order to reject false tracks even at a greater degree and improve clutter rejection performance.

Both the original and modified HOC methods exploit the temporal and spatial dependencies of consecutive data points on a target track to discriminate them from background clutter. The real-time implementation of these methods using connectionist networks was also presented [171] which showed the potential of these schemes for parallel implementation.

(5) Dynamic Programming Algorithm

The dynamic programming (DP) approach originally proposed in [172] avoids problems with velocity mismatch and can handle slowly maneuvering targets. The algorithm of [172] is modified in [173] and extended to improve efficiency, enhance performance in non-Gaussian noise and allow velocity transitions. The whole performance analysis of DP techniques was presented in [174, 175]. In IR image tracking [176, 177], DP algorithm gives each pixel in the image a score based on the current frame and the previous one. By doing so, the temporal behavior difference between targets, clutter and noise is utilized to distinguish between them; the algorithm gives scores accordingly. An efficient Viterbi-like algorithm based on generalized likelihood ratio test (GLRT) is presented [178] for tracking a radar target. Sequentially it is extended for multi-target scenarios via DP or via an equivalent minimum network flow optimization when the target number is a priori known. When the target number is unknown, the more challenging problem of tracking and detecting multitarget can be solved by composite MSHT [179]. However, the algorithms presented in [178, 179] do not incorporate the target kinematics, but simply consider a maximum target velocity to define the admissible target transitions.

The DP method is operated based on pixel process, so it is easy for hardware implementation, and it can detect and track maneuvering target with straightaway velocity under low SNR. The drawbacks of DP method are 1) the requirement of exhaustive matched filtering for target searching, when the search widow is large, the computation complex increases, 2) large memory is required when tracking target with inverse velocity.

4.2. Bayesian Approaches

The non-Bayesian based TBD technique is implemented in limited scenarios. On the other hand, a recursive Bayesian filter is more commonly used, which incorporates the complete data as a highly non-linear measurement.

(1) Particle filtering

A recursive Bayesian TBD implemented using PF techniques, was first introduced by Salmond and Birch [180], in parallel with work by Boers and Driessen [181]. The extension from a single target filter to two targets can be found in [182–184]. An extension of particle based TBD algorithm for MTT is given in [26].

The problem of PF-TBD trackers are their intense computation. For instance, the PF of [26], which uses the kinematic prior to propose particles, requires an exponentially increasing sample size to maintain performance as the number of targets increases. So similar to the PF trackers in data association based MTT in Section 3.1.1.2 of this paper, existing methods of PF-TBD trackers can also be divided into independent samplers and joint samplers:

1) Independent samplers. Ristic [185] designed a PF-TBD tracker that used an electrical optical sensor with Gaussian likelihood function. Rutten et al. [186] built upon and improved Ristic's algorithm. His PF-TBD algorithm modeled a radar sensor that used a Ricean-Rayleigh measurement model. Rutten's algorithm differed from Ristic's in that it explicitly included the track existence probability in the target state vector and used separate particle filters to compute the newborn and existing densities. In [23], the focus is on sampling the states of persisting targets. This is equivalent to performing tracking with a fixed and known number of targets. The method developed in [23] can easily be used as part of the framework of [187] for tracking a varying and unknown number of targets. In [188], the detection and tracking of an unknown number of targets using a Bayesian hierarchical model with target labels is presented. To approximate the posterior PDF, the authors developed a two-layer PF. One deals with track initiation, and the other deals with track maintenance. In addition the parallel partition method is proposed to sample the states of the surviving targets. In [189], the proposed tracker is based on PF and automatically initializes tracks. The main novelty is the inclusion of the target ID in the particle state, enabling the algorithm to deal with unknown and large number of targets. Other works belong to independent samplers include the methods in [190]. These independent samplers are generally feasible, but their performance degrades unduly if several targets move in proximity for extended durations.

2) Joint samplers. The PFs belonging to the second group assume posterior independence between well-separated targets rather than between individual targets. The representative works are the joint optimal importance density (JOID) method in [191]. The adaptive systems approach of [192] applies the idea of joint sampling to subsets of the measurement record. These approaches can deal with the target coupling and measurement ambiguity via joint sampling, but have a much heavier computational complexity than independent samplers.

(2) Joint multitarget probability density filtering

In recent years, the approaches based on joint multitarget probability density (JMPD), which captures uncertainty about the number of targets as well as their individual states, are used widely in the joint detection and tracking. The JMPD is a vector-based approach in which the quantity of interest is a hybrid state formed by the target number and a multitarget state, which is a vector formed by concatenating the single target state vectors. This hybrid state does not include identifying labels. As discussed in [193], JMPD can be traced back to the event-averaged maximum-likelihood estimation (EAMLE) work of [194, 195]. The JMPD method can also be derived using the mathematics of RFS and expressed in the FISST framework [106]. In [191] and [196], the JMPD is found by the usual Bayesian filtering recursion involving evaluation of the Chapman-Kolmogorov equation followed by the multiplication of the result by the likelihood. This simplifies the calculation of the posterior in comparison with the RFS framework mainly due to the use of vector integrals rather than set integrals.

Since the JMPD is a high-dimensional entity that cannot be computed in closed form, sophisticated numerical techniques are required to obtain a tractable approximation. Early work used a deterministic grid approximation, which is practical only for simple problems involving a small number of targets moving in one dimension [197]. PFs have also been used to approximate the JMPD in realistic scenarios involving tracking multitarget [196, 198]. PFs provide a recursive stochastic grid approximation to the exact solution of Bayesian state estimation problems. Based on [191, 196] considers targets of varying number. A model for the time evolution of the extended multitarget state is developed that allows the rates at which targets arrive and depart to vary both spatially and temporally. Sampling directly from this transition model does not lead to a tractable algorithm since, so sampling is therefore performed from an importance density, which uses a measurement-directed method to decide which areas of the surveillance region are more likely to have had a target arrive or leave.

However, the JMPD approach is only sound if the posterior remains unchanged under target permutations. This leads to a theoretical inconsistency since the Bayesian filtering recursion for vector-valued variables does not generally provide the required symmetry [199]. Rather, this symmetry must be imposed. The resulting JMPD actually ends up being equivalent to the RFS posterior but is not satisfying theoretically. As such the RFS framework should be used if individual target states are not labeled. In addition, the PF-based JMPD approach is theoretically sound, it demands intense computation, with a huge number of particles required to explore different dimensional state spaces for target detection and tracking.

(3) FISST-based filtering

A PHD filter for recursive TBD algorithm is proposed in [200] for multiple-input-multiple-output (MIMO) radars. With the PHD filter implementation it is possible to estimate the number of targets in each time step together with their associated states. But in [200], it is assumed that the targets are resolved in the sense that no two targets fall with in the same resolution cell. Hence, there is no measurement origin uncertainty with respect to targets. In [201], Long et al. presented a novel and efficient TBD algorithm based on MM-PHD for tracking IR maneuvering dim multitarget. Also the mechanism to integrate a TBD approach to the CPHD filter should be explored. The applications of MeMBer filters have been also developed in the literature [202–204].

5. NEW TOPICAL FOR FUTURE WORKS

The motion-based MTT problem in this paper concerns the single-sensor, automatic tracking of multiple point targets. If we relax the constraint of point targets, a distinct problem of extended/group target tracking arises in the MTT area; if we relax the constraint of single-sensor, another distinct problem of multisensor-multitarget tracking also arises in the MTT area.

5.1. Extended/Group Target Tracking

In most target tracking applications it is assumed that each target produces at most one measurement per time step. This is true for the cases when the distance between the target and the sensor is large in comparison to the target's size. In other cases, however, the target size may be such that multiple resolution cells of the sensor are occupied by the target. Targets that potentially give rise to more than one measurement per time step are categorized as extended. Related problems arise if a group of target is to be tracked. Practically important examples are aircraft formations or ground moving convoys. Under these circumstances it seems to be reasonable to treat the group as an individual object and to estimate and track its current extension from the sensor data.

The method to track extended target and group target are in the same way, they teat multiple measurements as a whole entire. In extended/group target tracking, the target extension is thus a part of the target state and has to be estimated jointly with the kinematical properties involved. It makes the assumption that the measurements are well-separated from each other, so the inter-cluster data association problem can be neglected.

Typical methods for extended/group target tracking include random matrix framework [205, 206], PHD/CPHD filter [207–209], Bernoulli filter [210, 211]. More researches need to be undertaken in future works, especially that the estimated shape of the target is an important piece of information that has to be incorporated into the entire tracking procedure.

5.2. Multisensor-Multitarget Tracking

Combining the results of multiple sensors can provide more accurate information than using a single sensor [212]; this allows either improved accuracy from existing sensors or the same performance from smaller or cheaper sensors.

In multisensor-multitarget tracking, we turn our attention to the related problem of track-to-track association and fusion. Track association is defined as the process of deciding if two tracks belong to the same target. Track fusion deals with methods to combine the state estimates, given that the measurements come from the same target. In a multisensor-multitarget environment the thorny problem is that of track segment association. Approaches for dealing with the track association and fusion problem include the use of the GLRT, statistical distances, fuzzy logic, neural nets, un-supervised learning, clustering techniques, Bayesian inference networks, MHT techniques, and so on. Other problems in multisensor-multitarget tracking, such as sensor management [213, 214], data register [215] and so on. Multisensor-multitarget tracking is really an area needs tremendous efforts.

6. CONCLUSION

MTT confronts several challenges in practical applications, such as data association uncertainty, dense clutter disturbance, maneuverable motion, and real-time processing requirements. Considerable researches have been undertaken in the literature. In this paper, we classify existing MTT methods into two strategies, the DBT methods and TBD methods. Furthermore, the DBT approaches are classified into data association based methods and FISST based methods, while the TBD approaches are classified into non-Bayesian approaches and Bayesian approaches. For each category, this paper provides detailed descriptions of the representative algorithms, and examines their pros and cons.

In this paper, we can see that Bayesian theory is the most important basis theory for target tracking and plays important role in both target state filtering and data association. In the implements of Bayesian filtering, PF is a powerful tool to address the nonlinear/non-Gaussian problem. Also, the recent theoretical developments of sequential Bayesian estimation in the framework of FISST theory have dramatically widened the scope of application: from single to multiple appearing/disappearing objects, from precise to imprecise measurements and measurement models.

Lastly, this paper cites many opinions in the literature. We endeavor to organize them systematically and denote them to the reference papers where they appear. Also, it includes our personal ideas through the whole paper. So we would appreciate very much receiving comments from the readers that have different opinions.

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