# Planar Dual-Band Power Divider with Short-Circuited Stub

## Xin-Huai Wang<sup>\*</sup>, Yang Bing Xu, Le Kang, Xiao Shuang Li, Wei Jian He, and Xiao Wei Shi

Abstract—A novel planar dual-band microstrip power divider is proposed in this paper. The circuit is composed of two sections of transmission line, short-circuited stub line and planar resistor, which can provide high isolation and good amplitude balance simultaneously at two frequencies. The closed-form equations are derived, and the design procedures of dual-band power divider are given. To certify the validity, a proposed power divider was fabricated and measured at 950 MHz and 2200 MHz which might be applied to GSM and CDMA systems. Both theoretical and simulated results are given, which are in good agreement with the measured results.

### 1. INTRODUCTION

Power divider is one of the most useful fundamental components in microwave circuits and systems such as mixers, power amplifiers, phase shifters and antenna arrays. In 1960, Ernest J. Wilkinson developed the conventional Wilkinson power divider which could achieve the equivalent amplitude and in-phase outputs [1]. It has been widely utilized in microwave and millimeterwave designs. Subsequently, some new types of power dividers have been proposed which operate at one design frequency  $f_1$  and at all its odd harmonics [2,3]. However, they are not suitable for some dual-band operations. Nowadays, there has been increased interest in dual-band or multi-band microcircuits for multi-band application [4–22]. Recent years have seen a worldwide effort to develop dual-band or multi-band power dividers. According to Monzon's theory, a dual-band 3 dB power divider utilizing two-sector transmission lines cascaded was proposed with good isolation and output return loss [4]. Some modified power dividers were presented by putting in parallel RLC, series RLC or its combination, which might cause parasitic effect especially working at high frequency [5–9]. Some improved topologies applying extended ports or stub lines were reported in [10–15]. The artificial transmission lines were also used to realize arbitrary dual-frequency operations [16–20]. In 2008, a simpler dual-band scheme was reported based on the input stub and the cascaded transmission line sections [21]. In 2011, a compact dual-band power divider using non-uniform transmission line was proposed in [22].

In this paper, a dual-band power divider using two sections of transmission linen and short-circuit stub line is presented. The structure of the dual-band has been given in this paper. Closed-form equations containing all parameters of this structure are derived based on circuit theory and transmission line theory. Since the power divider is symmetric, it is convenient to analyze it by using the even-mode and odd-mode method. The equivalent even-mode and odd-mode circuits are analyzed, and design equations are derived. In order to achieve high isolation, an isolation resistor is inserted in the middle of the two stubs. Finally, a prototype of dual-band power divider working at 950 MHz and 2200 MHz which can be applied in GSM and CDMA with an equal power split ratio is designed, fabricated, and measured. Simulation and measurement results are in good agreement and validate the proposed approach.

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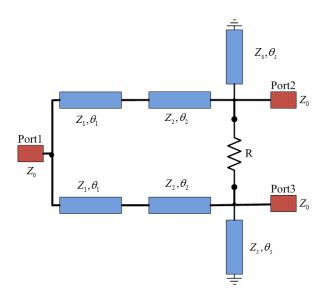


Figure 1. Structure of the proposed power divider.

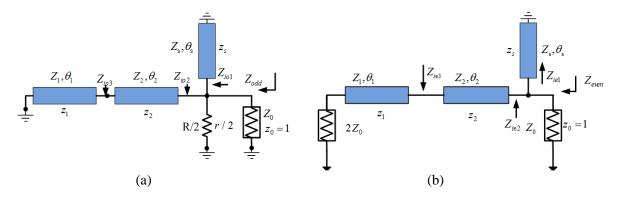


Figure 2. Equivalent circuits. (a) Odd-mode circuit. (b) Even-mode circuit.

#### 2. THE STRUCTURE AND ANALYTICAL EQUATIONS

The topology of the proposed dual-band power divider with short-circuited stub is shown in Figure 1. As shown in Figure 1,  $Z_0$  is the reference impedance. The circuit consists of two sections of transmission lines with the characteristic impedance of  $Z_1$ ,  $Z_2$ , length of  $\theta_1$ ,  $\theta_2$ , and stub transmissions with equivalent impedance  $Z_s$  and length of  $\theta_s$ . We assume that it works at both frequencies  $f_1$  and  $f_2 = kf_1$  ( $k \ge 1$ , and k is the frequencies ratio), that all ports are matched and that the two output ports are isolated by the resistor R which is inserted in the middle of the two stubs. To simplify the analysis, we assume  $\theta_1 = \theta_2 = \theta$ . The power divider is symmetric, and it is convenient to analyze it by using the even-mode and odd-mode method. All the impedance values are normalized to the input reference impedance. The equivalent even-mode and odd-mode circuits of the proposed construction are shows in Figure 2(a) and Figure 2(b).

### 2.1. Odd-Mode Analysis

In odd-mode analysis, the voltage of the middle of the circuit is zero. Therefore, the circuit can be bisected by grounding it at two points on its mid-plane, and Figure 1 is simplified to Figure 2(a). According to odd-mode circuit, we can obtain the expressions as follows:

$$\frac{1}{z_{odd}} = \frac{1}{z_{io1}} + \frac{1}{z_{io2}} + \frac{2}{r} = 1 \tag{1}$$

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$$z_{io3} = j z_1 \tan \theta \tag{2}$$

$$z_{io2} = z_2 \frac{z_{io3} + j z_2 \tan \theta}{z_2 + j z_{io3} \tan \theta}$$
(3)

$$z_{io1} = j z_s \tan \theta_s \tag{4}$$

According to equations of (1), (2), (3) and (4), we can obtain:

$$r = 2 \tag{5}$$

$$\frac{z_2 - z_1 \tan^2 \theta}{(z_1 + z_2) \tan \theta} = \frac{-z_2}{z_s \tan \theta_s} \tag{6}$$

#### 2.2. Even-Mode Analysis

In even-mode analysis, the symmetrical plane is seen as a magnetic wall. No current flows through the isolation element. We can bisect the whole structure symmetrically, as shown in Figure 2(b). According to transmission line theory, we can obtain:

$$\frac{1}{z_{even}} = \frac{1}{z_{ie1}} + \frac{1}{z_{ie2}} = 1 \tag{7}$$

$$z_{ie3} = z_1 \frac{2 + jz_1 \tan \theta}{z_1 + 2j \tan \theta} \tag{8}$$

$$z_{ie2} = z_2 \frac{z_{ie3} + jz_2 \tan \theta}{z_2 + jz_{ie3} \tan \theta}$$
(9)

$$z_{ie1} = \frac{z_s}{j\tan\theta_s} \tag{10}$$

Using Equations (7)–(10) and separating the real and imaginary, we can obtain:

$$2\left(z_{2}^{2}-z_{1}^{2}\right)\tan^{4}\theta+\left[z_{1}^{2}(z_{1}+z_{2})^{2}-4z_{1}\left(z_{1}+2z_{2}\right)\right]\tan^{2}\theta+2z_{1}^{2}=0$$

$$\left[\left(4z_{1}-z_{2}^{3}\right)\tan^{2}\theta+z_{2}z_{1}\left(z_{1}-z_{2}\right)\tan^{2}\theta+2z_{1}^{2}=0\right]$$
(11)

$$\frac{\left[(4z_2 - z_1)\tan^2\theta + z_1z_2(z_1 - 4)\right](z_1 + z_2)\tan^2\theta}{4\left(z_1^2 + z_2^2\tan^4\theta\right) + z_1\left[z_1\left(z_1 + z_2\right)^2 - 8z_2\tan^2\theta\right]} = \frac{-z_2}{z_s\tan\theta_s}$$
(12)

Using Equations (6), (11) and (12) and separating the real and imaginary, we can obtain:

$$z_1 = z_2 \tan^2 \theta \tag{13}$$

By solving the above Equations (1)–(10), we can obtain the final solutions

$$z_1 = \sqrt{2} \tan \theta \tag{14}$$

$$z_2 = \frac{\sqrt{2}}{\tan \theta} \tag{15}$$

$$z_s = \frac{\sqrt{2}}{(\tan^2 \theta - 1) \tan \theta_s} \tag{16}$$

$$r = 2 \tag{17}$$

where,

$$\theta = \beta_1 l = \beta_2 l \tag{18}$$

$$\theta_s = \beta_s l_s \tag{19}$$

#### 2.3. Analysis and Relationship of the Parameters

To satisfy simultaneously at both the frequencies  $f_1$  and  $f_2 = kf_1$  (k is the frequencies ratio and  $k \ge 1$ ) and the size of the proposed power divider is compact, according to Equation (13), the value of the

(26)

corresponding electrical lengths of each transmission-line section at the frequencies  $f_1$  and  $f_2$  can be obtained as:

$$\tan^2 \beta_1 l = \tan^2 k \beta_1 l \tag{20}$$

That is:

$$\beta_1 l = n\pi \pm k\beta_1 l \tag{21}$$

We assume n = 1 and then further simplify,

$$\beta_1 l = \frac{\pi}{1+k} \tag{22}$$

which means,

$$\theta_{f1} = \frac{\pi}{1+k}, \quad \theta_{f2} = \frac{k\pi}{1+k}$$
(23)

From Equation (23), it can be obtained that  $\theta_{f1} + \theta_{f2} = \pi$  is fulfilled, which leads to Equation (13) being unchanged at the frequencies  $f_1$  and  $f_2$ .

The same as above, Equation (16) should also be satisfied at  $f_1$  and  $f_2$ , and we can obtain,

$$\theta_s = \beta_s l_s = \frac{m\pi}{1+k} \tag{24}$$

where m is integer.

Here, we discuss the relationships between the frequency-ratio k and integer m of the electrical length  $\theta_s$ .

(a). When

$$1 \le k < 3 \tag{25}$$

which leads to

$$\tan \theta = \beta_1 l > 1 \tag{27}$$

To ensure that the impedance  $z_3$  is positive in Equation (16), the  $\tan \theta_s$  must be positive. And considering the size of the proposed power divider is compact,

 $\frac{\pi}{4} < \theta_{f1} \le \frac{\pi}{2}$ 

$$0 < \theta_s = \beta_s l_s \le \frac{\pi}{2} \tag{28}$$

The constraint relationship can be obtained,

- (29)m = 1
- (b). When (30)

 $k \geq 3$ 

$$0 < \theta_{f1} \le \frac{\pi}{4} \tag{31}$$

we can obtain,

which leads to

 $0 < \tan \theta_1 = \beta_1 l \le 1$ (32)

To ensure that the impedance  $z_3$  is positive in Equation (16),  $\tan \theta_s$  must be negative. And considering the size of the proposed power divider is compact,

$$\frac{\pi}{2} \le \theta_{f1} < \pi \tag{33}$$

The constraint relationship can be obtained,

$$(k+1)/2 \le m < k+1 \tag{34}$$

Therefore, the procedure to design the quad-frequency transformer is as follows. Firstly. determining the required dual-frequencies, once  $f_1$  and  $f_2$  are determined, the values of k can be obtained. Secondly, we should calculate the physical length of the transmission lines section using Equations (22) and (24). Thirdly, we need to get the characteristic impedance of  $Z_1$ ,  $Z_2$ ,  $Z_s$  using Equations (15)–(17). Finally, we can get the suitable design parameters which could be used to establish the circuit model.

#### 3. IMPLEMENTATION AND RESULTS

In this section, the proposed power divider operating at 0.95 and 2.2 GHz is designed and implemented to verify the above method. It is designed on a PTFE substrate with relative dielectric constant 2.65 and thickness h = 1 mm. As shown in Figure 1, based on the real microstrip line, resistor and substrate,

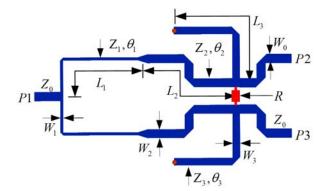
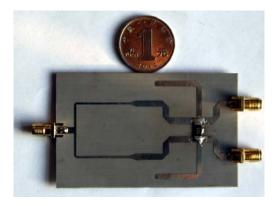
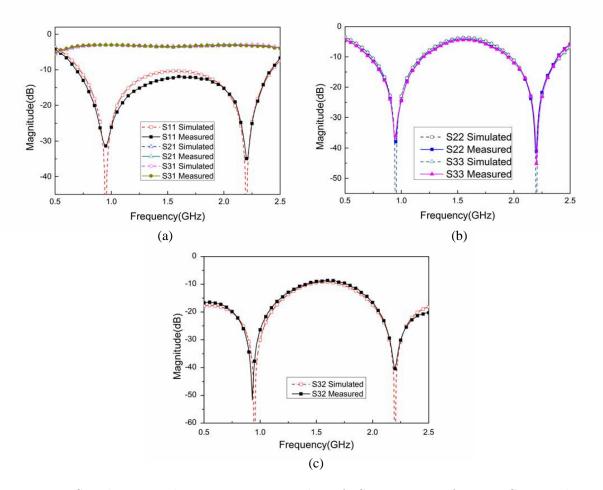


Figure 3. Dimensions of the fabricated dual-frequency power divider.



**Figure 4.** Photograph of the fabricated dual-frequency power divider.



**Figure 5.** Simulation and measurement results of S-parameters for 0.95 GHz and 2.2 GHz. (a) Magnitude of  $S_{11}$ ,  $S_{21}$ , and  $S_{31}$ . (b) Magnitude of  $S_{22}$ , and  $S_{33}$ . (c) Magnitude of  $S_{32}$ .

the model is built up with the dimension marked, and simulation has been carried out by the EM simulation software Microwave Office 2004 and the full-wave EM simulator (Ansoft HFSS 13.0). All the measured results are taken within the Agilent N5230A network analyzer.

The center frequencies of the two bands are 0.95 GHz and 2.2 GHz. When these two frequencies are used to design a dual-band Wilkinson power divider on a common substrate, the parameters of the power divider are calculated by using (2)–(5). The input impedance of each transformer section can be obtained as  $Z_1 = 98.34 \Omega$ ,  $Z_2 = 50.84 \Omega$  and  $Z_3 = 54.38 \Omega$ , and the length is calculated as  $\theta_1 = \theta_2 = \theta_3 = 54.29^\circ$ . The final optimum dimensions shown in Figure 3 are listed as follows:  $L_1 = 34.98 \text{ mm}$ ,  $L_2 = 33.78 \text{ mm}$ ,  $L_3 = 34 \text{ mm}$ ,  $W_0 = 2.73 \text{ mm}$ ,  $W_1 = 0.8 \text{ mm}$ ,  $W_2 = 2.62 \text{ mm}$ ,  $W_3 = 2.2 \text{ mm}$ . A photograph of the fabricated power divider is shown in Figure 4. The measured and simulated S-parameters are presented in Figure 5.

It is observed that the measured return losses  $S_{11}$  are about -34.3 dB at 0.95 GHz and -35.2 dB at 2.2 GHz, and the measured insertion losses  $S_{21}$ , and  $S_{31}$  are about  $-3.04 \pm 0.02 \text{ dB}$  at 0.95 GHz and  $-3.21 \pm 0.12 \text{ dB}$  at 2.2 GHz. The isolations between port3 and port2 ( $S_{32}$ ) are better than -37.4 dB at 0.95 GHz and -40.3 dB at 2.2 GHz. The measured  $S_{22}$  and  $S_{33}$  are also better than -35.8 dB at 0.95 GHz and -41.0 dB at 2.2 GHz. Simulation and measurement both confirm that the proposed power divider has good characteristics. There are some slight differences between simulated and measured data, which might result in dielectric loss and limited precision of fabrication.

### 4. CONCLUSION

This paper presents the analysis and design of a novel dual-band power divider using two sections of transmission line and short-circuit stub line, which enable input signal to be separated into equal power split ratio at two frequencies. The structure of the dual-band has been given in this paper. Practical and simple design equations are derived by using the even-mode and odd-mode method and transmission line theory. To certify the validity, a compact dual-band power divider working at 0.95 GHz and 2.2 GHz for GSM and CDMA is designed, simulated and realized on a PTFE substrate. The measured results of the proposed dual-band power divider are in good agreement with the simulation. The power divider also shows better return loss and isolation than  $-30 \, \text{dB}$ . The simulated and measured results with good agreement are presented to confirm the proposed method. With the good character of equal power split, low insert loss and good isolation, the proposed power divider can be widely applied to dual-band applications.

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