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Dispersion Analysis of Double-Sided Open Periodic Media Using Inhomogeneous Plane Wave Excitation

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Abstract—Double-sided open periodic structures are analyzed using inhomogeneous plane wave scattering. The leaky and surface wave modes of several unit cells of different structures are computed using the poles of generalized reflection and transmission coefficients of inhomogeneous plane waves in the spectral domain. It is shown that the reflection and transmission coefficients of the zeroth order Floquet mode contain the poles of the Green's function of the complex stratified periodic structure. The properties of evanescent mode amplification as well as super resolution near field imaging in a wire medium are addressed. A balanced leaky wave antenna unit cell with double-sided radiation feature is introduced and it is shown that, in contrast to grounded structures, total absorption in lossless non-chiral double-sided open unit cells is not feasible as long as the behavior of the unit cell is well described by its fundamental mode.

1. INTRODUCTION

In homogeneous open dielectric structures, the spectral representation of the Green's function leads to a series of propagating discrete surface wave modes, together with a continuous integral representation indicating the branch cut in the Riemann sheet [1]. By introducing possibly inhomogeneous periodicities into these structures, the analysis of the whole configuration can be reduced to the analysis of one period known as unit cell. This unit cell has open boundaries at the unbounded sides, and periodic boundary conditions are assumed at its side walls, see Fig. 1. In homogeneous layered dielectric media, the spectral domain Green's functions have closed form [2]. However, inhomogeneous complex objects do not have any closed form Green's function. Therefore, numerical methods are employed to compute the eigenvalues of these structures.

In the numerical computation of open boundary structures, often perfectly matched layers (PML) are chosen as absorbing boundary condition [3]. As a result, the spectral domain Green's function can be expressed in terms of the summation of three different groups of modes namely Berenger modes, leaky modes and surface wave modes [4]. Among these modes, Berenger modes are complex modes which are more concentrated in the PML layer, while leaky modes are complex modes more concentrated in the guiding structure [5].

For the analysis of complex objects immersed in or placed on top of a layered medium, the computation of leaky complex modes is possible with several methods. The most common approach is based on eigenvalue decomposition of the system matrix resulting from a numerical model [6–8]. Another robust method is trying to find the stationary points of stored energy derived from physical observations. Since eigenvalue decomposition is a numerically expensive procedure, a physics based method for the computation of complex and surface wave modes of planar periodic configurations was addressed in [9, 10]. This method was based on the scattering of inhomogeneous plane waves from grounded unit cells and the eigenmodes were explored by exciting the poles of the reflection coefficient.

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Also, the zeros of the reflection coefficient in these structures were found to be conjugate complex to the poles. In lossless structures, it was shown that in the fast wave region of the dispersion diagram the complex zeros of the reflection coefficient result in complete absorption. Some other methods have also been accomplished to compute eigenmodes and eigenvalues in periodic structures effectively [11].

Periodic electromagnetic devices for which spectral domain analysis can be employed are metamaterial lenses. They are used to reconstruct near field images from evanescent field sources [12]. Most of these structures have been investigated analytically by deriving the circuit model of the structure [13] or by describing effective material properties using certain theoretical relations [14, 15].

In this paper, the eigenmodes and corresponding eigenvalues of the unit cells of double-sided open periodic structures are computed by using inhomogeneous plane wave excitation. Also, by considering the radiation condition, the appropriate eigenvalue on the proper Riemann sheet is selected for left handed or the improper sheet is chosen for right handed structures. Furthermore, the amplification of evanescent waves is explained based on the dispersion properties of surface wave modes in the metamaterial slabs.

The numerical computations in this paper utilize the finite element boundary integral (FEBI) method, in which the open boundary condition of the structure is included in the Green's function of the periodic medium at the open boundaries [16, 17]. The acceleration of the computation of the interaction between moment method basis functions, employs the plane wave decomposition of the Green's function known as fast spectral domain algorithm [16] and its multilevel version [18]. Another alternative would be the fast inhomogeneous plane wave (FIPW) approach [19] where complex wavenumbers and amplitudes of the plane waves are employed based on complex angles between interacting basis functions. The advantage of FIPWA is a reduced interaction complexity when the distance between interacting domains is increased [20]. This advantage becomes in particular important when an interaction is performed between a source at infinity and a scatterer. In this paper, we use the advantage of inhomogeneous plane wave decompositions of the Green's function to compute the scattering of an inhomogeneous plane wave from a single unit cell in periodic stratified structures.

The structures considered in the current paper are open at both top and bottom sides. Therefore, the poles of the Green's function are obtained as the resonances of the zeroth order Floquet mode reflected from and transmitted through the unit cell. The unit cells are periodic in one or two dimensions. As a one dimensional periodic structure, a leaky wave double-sided radiating balanced unit cell is analyzed. An interesting application of double-sided open periodic structures is their use in perfect metamaterial absorbers [9]. As it was shown for one sided open metasurfaces [21], complete absorption corresponds to a complex zero in the reflection coefficient. By adding losses or chirality, the location of the zero can be changed and the zero of the reflection coefficient may occur in the real spectrum of radiating wavenumbers. In this paper, double-sided open structures are investigated with respect to their complete absorption behavior by analyzing their complex spectrum of transmission and reflection coefficients.

2. EIGENMODE EXCITATION BY INHOMOGENEOUS PLANE WAVES

A complex spectrum plane wave known as inhomogeneous plane wave is utilized to excite open periodic structures. Because of the periodicity, the scattered field is expanded in Floquet modes which are spatial harmonic plane waves in periodic media. The advantage of this approach is that a single plane wave represents an impulse function in the spectral domain, which can be swept to investigate the spectral domain properties of a unit cell.

In periodic structures, as, e.g., represented by a unit cell with geometry shown in Fig. 1, the periodic dyadic Green's function in the spatial domain can be represented as a summation of periodic Floquet modes according to

$$\bar{G}_p(\boldsymbol{\rho}, z, z') = \sum_{mn} \bar{\bar{g}}_{mn}(z, z') e^{-j\mathbf{k}_{tmn}\cdot\boldsymbol{\rho}} \tag{1}$$

with $\boldsymbol{\rho}$ as the transverse distance between source and observation point, z' and z as the vertical locations of the source and observation points, respectively, $\mathbf{k}_{tmn}$ as the transverse wavevector of the mnth Floquet mode according to $\mathbf{k}_{tmn} = \mathbf{k}_{t00} + \frac{2\pi}{A}[m(\boldsymbol{\rho}_b \times \hat{z}) + n(\hat{z} \times \boldsymbol{\rho}_a)]$, and $\bar{\bar{g}}_{mn}(z, z')$ are the dyadic coefficients of the Floquet modes.

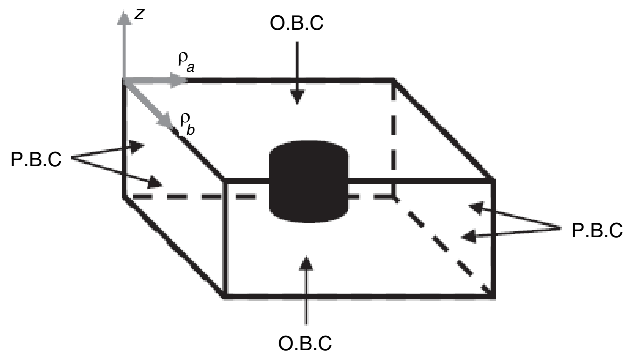

Figure 1. Unit cell of a doubly periodic open structure including an inhomogeneity; O.B.C: open boundary condition, P.B.C: periodic boundary condition.

$\mathbf{k}_{t00} = (\beta - j\alpha)\mathbf{e}_{00}$ is the wavevector of the incident plane wave as well as of the fundamental scattered Floquet mode. Henceforth, in this method the scattered Floquet modes are also complex. According to [22], every component of the spectral dyadic Green's function $\bar{\bar{g}}_{mn}(z, z')$ is a function of reflection and transmission coefficients of the corresponding plane waves.

In periodic stratified media, the computation of the reflection and transmission coefficients in general inhomogeneous unit cells requires the utilization of numerical techniques. In this paper, the hybrid finite element boundary integral (FEBI) method has been implemented to compute the magnetic currents at the open boundaries [16]. The FEBI formulation for one unit cell is written as

$$\begin{aligned} F(\mathbf{E}_{ad}, \mathbf{E}) =& \iiint\limits_V \left[\frac{1}{\mu_r}(\nabla \times \mathbf{E}_{ad}) \cdot (\nabla \times \mathbf{E}) - k_0^2 \epsilon_r \mathbf{E}_{ad} \cdot \mathbf{E} + jk_0 Z_0 \mathbf{E}_{ad} \cdot \mathbf{J}^{int}\right] dv \\ & - 2k_0^2 \iint\limits_S \iint\limits_S G_p(\mathbf{r}, \mathbf{r}_s)[(\hat{\mathbf{n}} \times \mathbf{E}_{ad}(\mathbf{r})) \cdot (\hat{\mathbf{n}} \times \mathbf{E}(\mathbf{r}_s)) - \nabla_s \cdot (\hat{\mathbf{n}} \times \mathbf{E}_{ad}(\mathbf{r})) \nabla_s \cdot (\hat{\mathbf{n}} \times \mathbf{E}(\mathbf{r}_s))]\, ds_s ds \\ & + jk_0 Z_0 \iint\limits_S (\hat{\mathbf{n}} \times \mathbf{E}_{ad}(\mathbf{r})) \cdot \mathbf{H}^{exc}(\mathbf{r}) ds \end{aligned} \tag{2}$$

where for non PEC boundaries over the open boundary surfaces, the electric field over a surface is related to the magnetic current as $\mathbf{E}(\mathbf{r}') \times \hat{\mathbf{n}} = \mathbf{M}(\mathbf{r}')$. Equation (2) is solved by minimizing the functional F where the integration over the volume V of the unit cell is the FE part and over the surface S is the BI part. $\mathbf{E}_{ad}$ is the adjoint of the electric field. The magnetic currents over the open boundaries are then used as secondary sources of the object to produce the scattered fields. This makes it possible to use an integral equation system based on the periodic Green's function G_p of free space [16]. Also, an appropriate multilayered media dyadic Green's function can be used [16–18, 23]. The general periodic boundary conditions at the open boundary and inside the finite element part are [17]

$$\mathbf{E}(\mathbf{r} + m\boldsymbol{\rho}_a + n\boldsymbol{\rho}_b) = \mathbf{E}(\mathbf{r})e^{-j\mathbf{k}_{t00}\cdot(m\boldsymbol{\rho}_a + n\boldsymbol{\rho}_b)} \tag{3}$$

$$\mathbf{H}(\mathbf{r} + m\boldsymbol{\rho}_a + n\boldsymbol{\rho}_b) = \mathbf{H}(\mathbf{r})e^{-j\mathbf{k}_{t00}\cdot(m\boldsymbol{\rho}_a + n\boldsymbol{\rho}_b)}. \tag{4}$$

After solving the coupled FEBI equation system by numerical iterative methods, the magnetic currents and, thus, the scattered fields are known in the BI surfaces. From the scattered field, the reflection coefficient of the zeroth order Floquet mode at the top open boundary is obtained by projection of the scattered field on the zeroth order Floquet mode. At the other side of the object, the transmitted field is projected on the zeroth order Floquet mode to compute the transmission coefficient [9, 16].

2.1. Complex Eigenvalues

$\mathbf{k}_{t00} = (\beta - j\alpha)\mathbf{e}_{00}$ can be on the proper or the improper region of the Riemann sheet. As it was discussed in [9], these complex wavenumbers, which satisfy the dispersion equation, exist in conjugate quadruples as long as lossless structures are considered. In homogeneous grounded slab waveguides, additional zeros exist in the same locations as the poles resulting in complete absorption in the fast wave region.

In homogeneous slab waveguides in free space, the reflection and transmission coefficients are

$$R = \frac{r\left(1 - e^{-2jk_{z_1}d}\right)}{1 - r^2 e^{-2jk_{z_1}d}}, \quad T = \frac{\left(1 - r^2\right) e^{-j2k_{z_1}d}}{1 - r^2 e^{-2jk_{z_1}d}}. \tag{5}$$

$r = (k_{z_1}/\epsilon_r - k_{z_0})/(k_{z_1}/\epsilon_r + k_{z_0})$, k_{z_0} and k_{z_1} are the wavenumbers in free space and inside the slab. Also, ϵ_r is the permittivity inside the slab and d is the thickness of it.

It is observed from (5) that the reflection coefficient has both real (surface wave mode) and complex (leaky wave mode) poles similar to grounded slabs [9], while zeros exist only for real values of the wavenumbers with $k_{z_1}d = n\pi$. Therefore, there are only real-valued zeros. On the other hand, these real-valued zeros do not yield any zero in the transmission coefficient. Consequently, there is neither complete absorption in ungrounded lossless homogeneous slabs nor zero scattered field.

The homogeneous structure and its behavior is studied in more detail in [24, 25]. In this paper the leaky waves in periodic structures are computed and it is shown that complete absorption in metamaterial double-sided open structures [21] and therefore zero scattering of the fields cannot be achieved in non-chiral lossless structures. It should be noted that the chiral behavior of metamaterial unit cells is the consequence of polarization conversion in the scattered field which in turn is not resulting into zero field scattering in these structures. Therefore, the only possibility to construct a complete absorber is adding dissipation loss into the unit cell.

2.2. Periodic Structures

Periodic stratified metamaterial slabs can be treated as homogeneous slab waveguides, when the dimension of the unit cells is much smaller than the operating wavelength. The method of excitation of eigenmodes by external excitation has already been used for the computation of the dispersion diagram of grounded periodic structures [9]. In the current paper, complex eigenvalues for double-sided open structures are investigated. Both reflection and transmission coefficients of the fundamental Floquet mode are computed at the open boundary from the surface magnetic currents and the eigenvalues are sought to maximize the reflection and transmission coefficients.

Another group of microwave and millimeter wave devices, which can be effectively analyzed in the spectral domain, are metamaterial lenses used for near field imaging. The image is reconstructed from evanescent components of the incident wave over the slab. These evanescent modes are amplified passing through the metamaterial slab which has been shown in [24] for the array of 1D periodic metallic plates. One of the most investigated lens structures is a slab made up of wires closely placed in a periodic arrangement [26]. Instead of extracting the effective material properties and modeling the periodic medium with these properties, the dispersion curve of the unit cell is calculated and it is shown that this 2D structure is the general form of a 1D structure of plates for imaging.

3. NUMERICAL RESULTS

3.1. Wire Medium

The first unit cell analyzed is a slab made up of thin wires to realize a metamaterial lens [26]. The unit cell is one wire at the center with radius of 1 mm and 15 cm length with period of 1 cm in transverse direction.

The periodic arrangement of the cell creates a slab in free space. This slab supports surface wave modes and to excite the eigenmode, an evanescent plane wave with a tangential wavenumber in x direction is considered with $k_x > k_0$. The reflection and transmission coefficients of the zeroth order Floquet mode are computed and the eigenvalue is derived from the singularities. Transmission and reflection coefficients of the plane wave impinging on the unit cell at the frequency 761 MHz are shown in Fig. 2, which confirm the surface wave mode excitation. For the detected eigenvalue, the computed electric field is shown in Fig. 3. As it is observed, the electric field has a high value at both ends of the wire while the excitation is an evanescent wave. For the 1D structure analyzed in [24], the field distribution is more clearly explaining the excited mode and the near field imaging phenomenon. However, the wire medium is 2D and it is more suitable for general near field imaging applications.

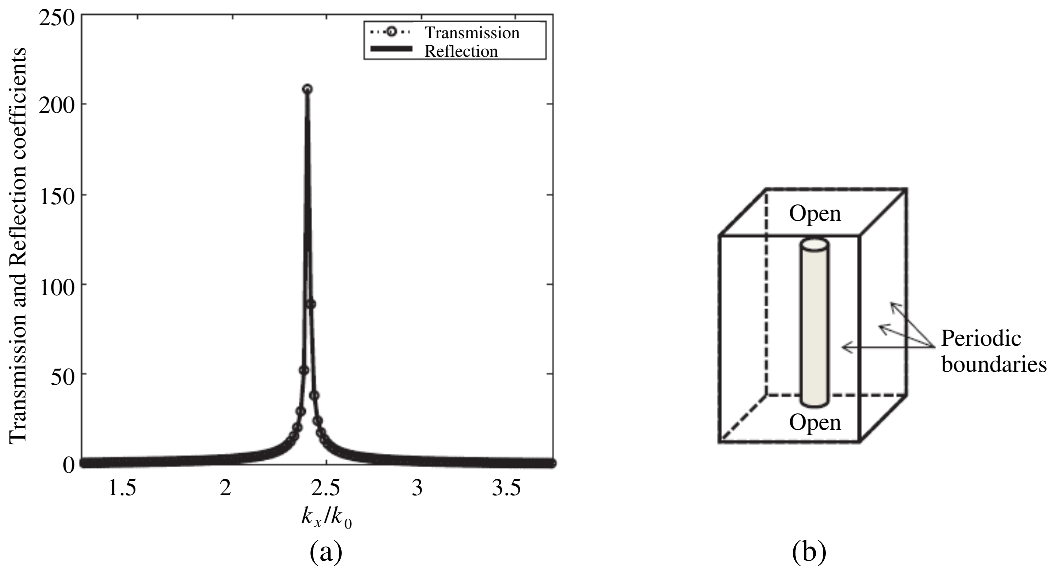

Figure 2. (a) Transmission and reflection coefficients of incident inhomogeneous plane wave for a wire medium unit cell with $r_{\text{wire}} = 1\,\text{mm}$, length $= 15\,\text{cm}$ and period $= 1\,\text{cm}$ at frequency 761 MHz. (b) The unit cell of a wire with corresponding boundary conditions.

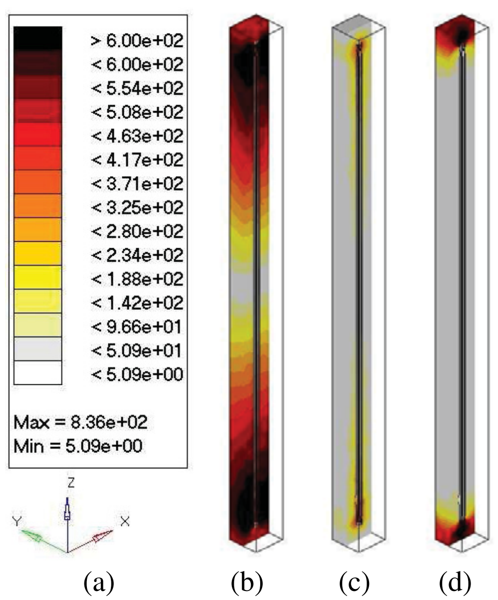

Figure 3. Distribution of electric field (in V/m) in a unit cell of the wire medium in free space at 761 MHz and $k_x/k_0 = 2.4$. (a) $|E|$ (V/m), (b) $|E_z|$, (c) $|E_y|$, (d) $|E_z|$.

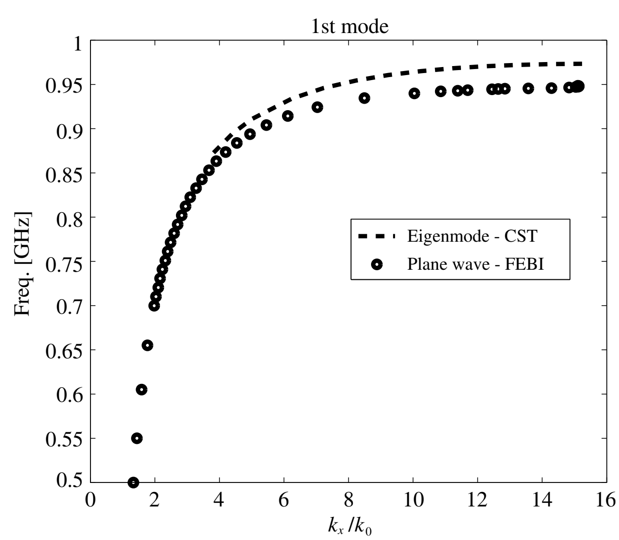

Figure 4. 1st mode of the wire medium computed by proposed method and CST Eigenmode solver.

Other modes of this medium are also surface wave modes and can be excited by evanescent plane waves. The dispersion diagram of the first mode in the Γ-X path in the Brillouin zone of the dispersion diagram in Fig. 4 is compared with the result achieved from a CST Microwave Studio Eigenmode solution [27]. The slight difference between the FEBI and the CST results is due to the fact that the realization of open boundary conditions in the CST Eigenmode solution is not possible. In the computations, we have used a PEC boundary with a quarter wavelength distance away from the wire ends what would be justified by the assumption of a bound mode.

Every source placed very close to the surface of a slab of wires will excite evanescent waves as found in the dispersion diagram. These evanescent waves will be amplified according to their eigenvalues. The best resolution of the image reconstruction will be achieved at the end of the Brillouin zone in the dispersion diagram in Fig. 4. At this frequency, the wavenumber reaches $k_x a = \pi$ and the dispersion diagram is becoming flat, so that the group velocity is zero while the phase velocity is nonzero. Consequently, there is no surface wave mode but the mode can be considered as a standing wave and the highest resolution is achieved [28].

3.2. Double-Sided Radiating Leaky Wave Antenna

The next unit cell considered is a double-sided leaky wave antenna, periodic in y direction, in which the eigenvalues are complex because of the radiation loss of the structure, Fig. 5.

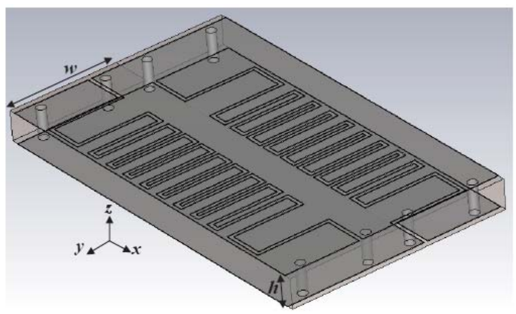

Figure 5. Double-sided leaky wave antenna unit cell, $w = 5.9\,\text{mm}$, $h = 1.524\,\text{mm}$, $\epsilon_r = 3.88$, unit cell length $= 2w$. The unit cell is periodic in y direction and open in other directions. Along the cell, two interdigital slots one in the top conductor and one in the bottom conductor exist. At every side, four vias connect the top and bottom conductors.

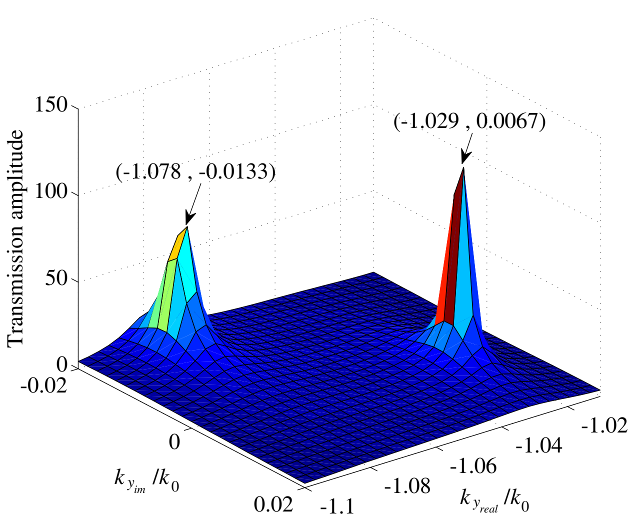

Figure 6. Transmission coefficient of leaky wave antenna unit cell at 3.05 GHz.

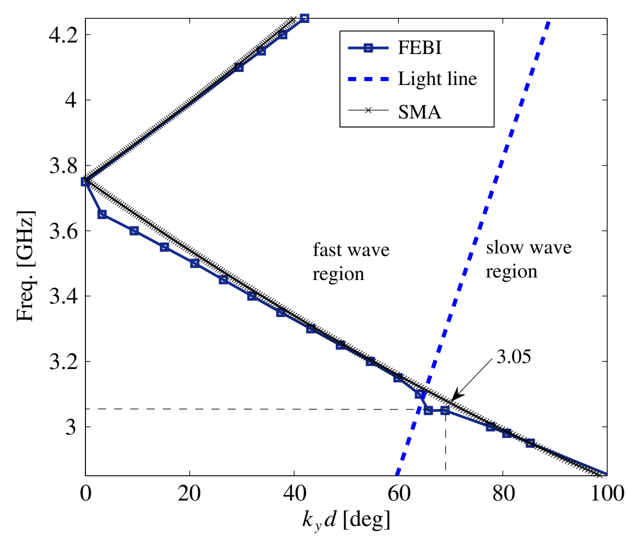

Figure 7. Dispersion diagram of double-sided leaky wave antenna.

The antenna structure has a composite right/left handed behavior between 3 and 6 GHz in the radiation region. The transition frequency is around 3.7 GHz. For the analysis of the unit cell, the same procedure as described for the homogeneous slab is employed [24]. The difference is that for complex eigenvalues, the excitation is an inhomogeneous plane wave with complex wavenumber.

The reflection and transmission coefficients of the excited plane wave are computed in the complex wavenumber plane at the frequency 3.05 GHz. As it is depicted in Fig. 6, the transmission coefficient shows two maxima in the complex wavenumber plane, one with negative imaginary part and one with positive imaginary part. In the slow wave region, both of these poles are proper [9]. The computed reflection coefficient shows the same maxima as the transmission coefficient. The time harmonic function is $e^{j\omega t}$ and the antenna operates in the left handed region of the working frequency band. Moving to the fast wave region from 3.05 GHz on the dispersion curve, the proper mode with respect to the radiation condition is the one with positive imaginary part. The improper mode is removed from the response and only the proper left handed eigenmode is observable in the fast wave region.

The complete dispersion diagram can be obtained by sweeping the proper mode in the left handed region and the improper mode in the right handed region. The resulting dispersion diagram in Fig. 7 compares our results with results computed by CST MWS [27] using the scattering matrix (SMA)

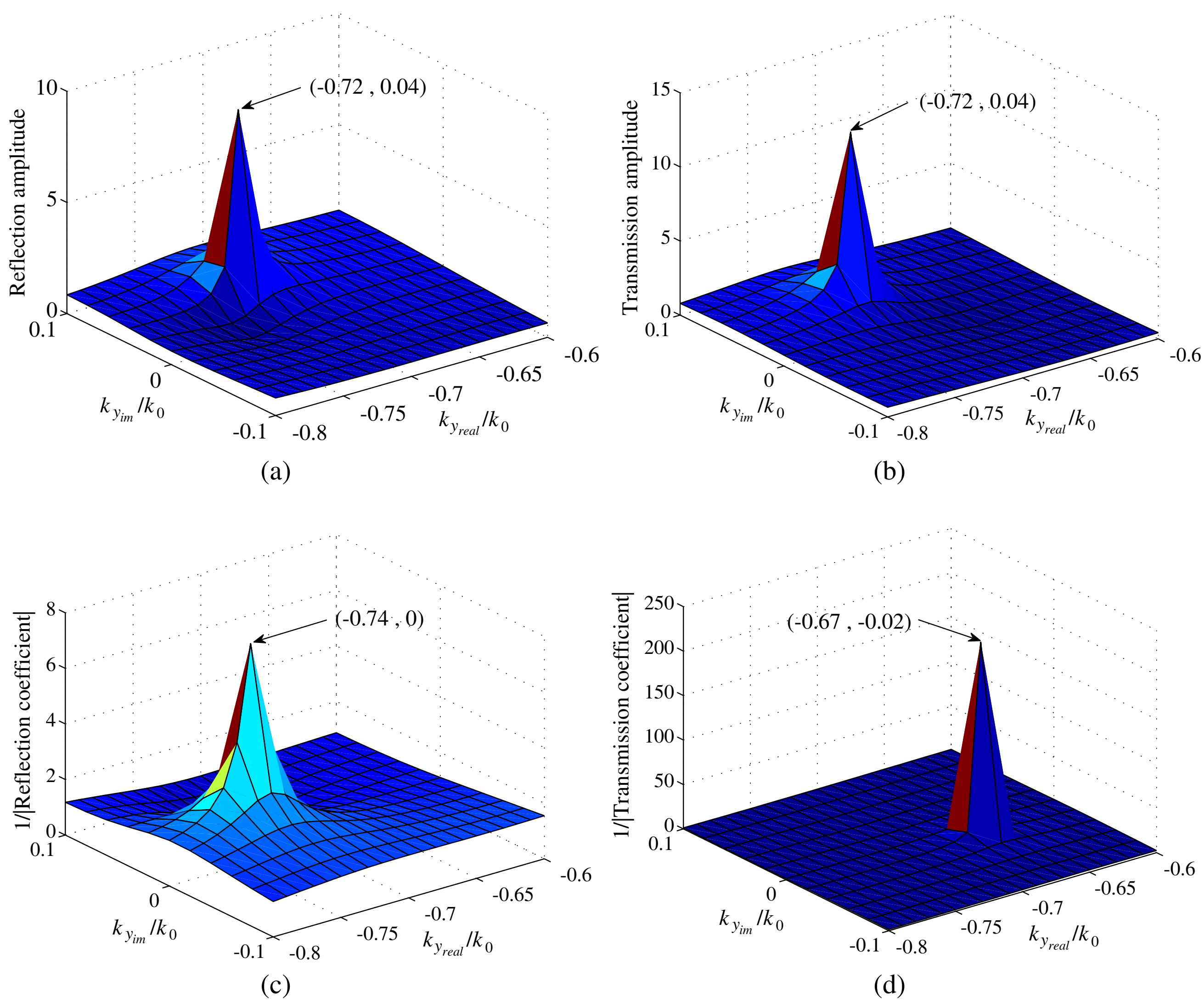

Figure 8. The location of poles and zeros in reflection and transmission coefficients from leaky antenna unit cell at 3.25 GHz. (a) |Reflection|, (b) |Transmission|, (c) 1/|Reflection|, (d) 1/|Transmission|.

translated into the transmission matrix [29], where the Bloch theory is employed to extract the dispersion diagram of the unit cell. The results are overall in good agreement. The slight differences between the two results is due to the weak chiral behavior of the unit cell which also prevents the two complex pole pairs in Fig. 6 from being exactly conjugate complex to each other. To find the transition frequency in the SMA method, the phase reference for computing scattering parameters is important, while in the proposed method the right handed and left handed bands next to the transition frequency or band gap can be computed without any prior knowledge about the unit cell.

3.3. Absorption in Metamaterial Unit Cells

As it was explained in Section 2.1, complete absorption is not feasible in lossless non-chiral structures. Considering the unit cell of the leaky wave antenna (Fig. 5), the transmission and reflection of the incident TM plane wave are swept for complex wavenumbers at frequency 3.25 GHz. The swept wavenumbers are in the fast wave region of the left handed part of the dispersion diagram and the proper poles are obtained as the maxima of transmission and reflection coefficients. As depicted in Fig. 8, the poles of reflection and transmission coefficients are at the same location. To have better appearance of the zeros of the reflection and transmission coefficients, the reverse of these coefficients are shown in Figs. 8(c) and 8(d). As it is observed, the zero of the reflection coefficient is on the real wavenumber axis which indicates the unit cell is lossless and non-chiral. The zero of the transmission coefficient is found for complex wavenumbers as it is expected for metamaterial lossless structures. Consequently, in order to have complete absorption, these zeros must become coincident which needs to move them on the complex wavenumber plane by adding loss or chirality.

4. CONCLUSION

Double-sided open periodic structures have been studied by excitation of eigenmodes and extraction of eigenvalues using space domain inhomogeneous plane wave incidence. It was shown by considering the physical properties of stratified planar periodic structures that the discrete surface wave and leaky modes of the structure can be computed efficiently by utilizing an integral equation based method. The moment method part of the utilized hybrid numerical finite element boundaries integral technique employs the periodic Green's function of a layered medium at the open boundary, where the reflection and transmission coefficients of the Floquet modes are computed. Also, the excitation of eigenmodes was found to be the basic functional principle of evanescent wave amplification, happening in periodic structures, which is used for near field imaging metamaterial slabs and other applications. An interdigital periodic leaky wave antenna and a wire medium unit cell were analyzed and the dispersion diagram as well as the eigenmodes were computed by the proposed method. For double-sided open structures, it was shown that complete absorption and therefore zero scattered fields cannot be achieved in lossless non-chiral materials. By introducing chirality into these structures, the zeros of the reflection and transmission coefficients may become identical, which can, however, not be regarded as complete absorption due to the nonzero scattered fields carried by other Floquet modes.

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