

All-Optical Logic Gates Based on Spatial-Soliton Interaction in Optical Communication Spectral Region

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Abstract—New designs of all-optical logic gates based on spatial-soliton interactions in optical communication spectral regions were proposed. The proposed structures are composed of local nonlinear Mach-Zehnder interferometer (MZI) waveguide structures with multi-input ports and two nonlinear output ports. They can be used to design various all-optical logic gates. The nonlinear MZI waveguide structure with local nonlinear waveguides functions like a phase shifter. It employs angular deflection of spatial solitons controlled by the phase modulation created in the local nonlinear MZI. The light-induced index changes in the local nonlinear MZI waveguide structures break the symmetry of structure and make the output signal beam propagate through different nonlinear output waveguides. By properly choosing the input control power, the spatial solitons will be switched to different output ports. The numerical results show that the proposed local nonlinear MZI waveguide structures could really function as all-optical logic gates in the optical communication spectral region.

1. INTRODUCTION

There has been great interest in the possibility of using nonlinear optical waveguide devices as ultrafast photonic devices for optical signal processing and optical communication systems. All-optical switching devices and logic gates based on the optical Kerr effect in a nonlinear waveguide have been of particular interest for high-bit rate optical communication and optical computing systems. Kerr-like nonlinear waveguide structures containing one or more media, whose refractive index depends on the local intensity, have stimulated a great deal of theoretical and experimental study [1–9]. Several all-optical devices using optical nonlinearity have been proposed and implemented, such as nonlinear directional couplers, nonlinear MZI, and nonlinear symmetric X and Y junctions [10–25]. Most of the conventional all-optical devices are based on uniformly nonlinear waveguide structure. Therefore, the whole of the waveguide has optical nonlinearity uniformly. The nonlinear index changes and interactions between optical beams can be induced at any part of the devices through nonlinear optical effects.

Recently, some studies have been focused on all-optical devices with localized optical nonlinearity, where some part of the waveguide structure is made from nonlinear medium, and the rest are made from linear ones. The application of the spatial soliton in the all-optical devices has been discussed ardently, such as Mach-Zehnder waveguide interferometer device, all-optical logic gates containing a two-mode nonlinear waveguide, all-optical logic devices with localized optical nonlinearity, all-optical switching devices with localized optical nonlinearity and all-optical modulators [26–43]. A. Biswas et al. studied the dynamics of bright and singular optical solitons in quadratic nonlinear media in detail [44–53]. They proposed the variational principle to obtain analytical soliton solutions. In the designing nonlinear waveguide devices, some use the strong nonlinear effect to produce the solitary waves propagating in the devices, but others utilize a weak nonlinearity in the waveguide, whose guiding properties are analogy to linear ones. Because of the particlelike characteristics, solitons are natural candidate for

Received 5 June 2015, Accepted 3 August 2015, Scheduled 12 August 2015

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the implementation of all-optical devices. The nonlinear Kerr-type dielectric medium can lead to the formation of spatial solitons. Two in-phase solitons propagating next to each other will perturb the refractive index of the background and then bend toward the perturbing neighbor. When the two spatial solitons are out of phase, each soliton will ride on an index gradient and then repel away the perturbing neighbor.

In this paper, new designs of all-optical logic gates based on spatial solitons in optical communication spectral regions were proposed. The proposed structures are composed of local nonlinear MZI waveguide structures with two outward control waveguides. They can be used to design various all-optical logic gates. The nonlinear MZI waveguide structure with local nonlinear waveguides functions like a phase shifter. It employs angular deflection of spatial solitons controlled by the phase modulation created in the local nonlinear MZI. The light-induced index changes in the local nonlinear MZI waveguide structures break the symmetry of structure and make the output signal beam propagate through different nonlinear output waveguides. By properly choosing the input control power, the

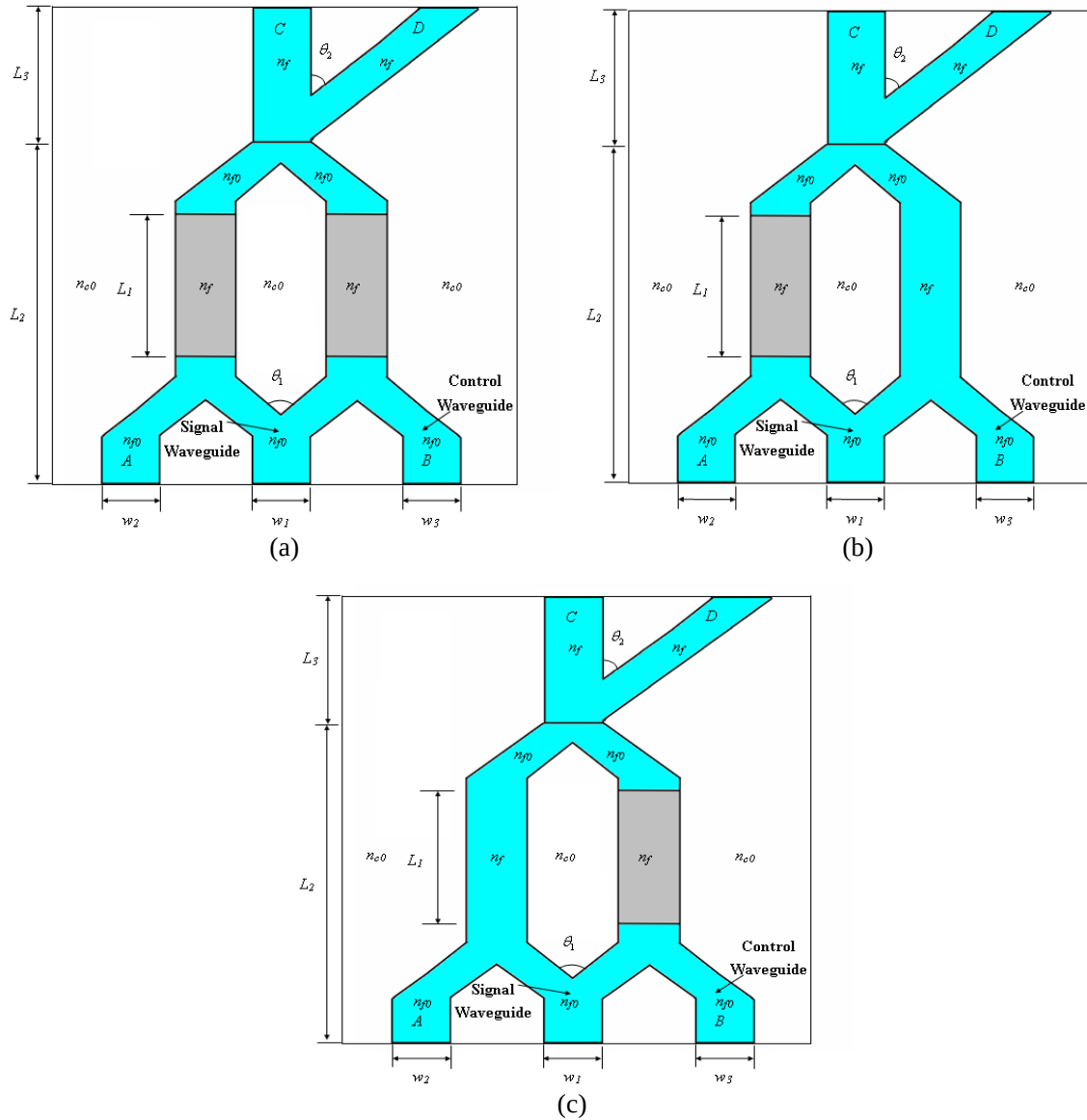


Figure 1. The proposed waveguide structure of all logic gates, (a) XOR/NXOR gate, (b) AND/NAND gate, and (c) OR/NOR gate.

spatial solitons will be switched to different output ports. The numerical results show that the proposed local nonlinear MZI waveguide structures could really function as all-optical logic gates in the optical communication spectral region.

2. ANALYSIS

The proposed waveguide structures of all-optical logic gates are composed of the local nonlinear MZI with three straight input ports (one signal port and two control ports), and two output ports, as shown in Figures 1(a)–(c). It employs angular deflection of spatial solitons controlled by the phase modulation created in the local nonlinear MZI. The light-induced index changes in the local nonlinear MZI waveguide structures break the symmetry of structure and make the output signal beam propagate through different nonlinear output waveguides. By properly choosing the input control power, the spatial solitons will be switched to different output ports. Here three cases of the different local nonlinear distribution in two arms of the nonlinear MZI were considered. For the first case, the local nonlinear waveguides are located on two arms of the nonlinear MZI, as shown in Figure 1(a). For the second case, the local nonlinear waveguide is only located on the left arm of the nonlinear MZI, as shown in Figure 1(b). For the third case, the local nonlinear waveguide is only located on the right arm of the nonlinear MZI, as shown in Figure 1(c). The nonlinear MZI waveguide structure with local nonlinear waveguides functions like a phase shifter. It employs angular deflection of spatial solitons controlled by the phase modulation created in the local nonlinear MZI. The light-induced index changes in the local nonlinear MZI waveguide structures break the symmetry of structure and make the output signal beam propagate through different nonlinear output waveguides. By properly choosing the input control

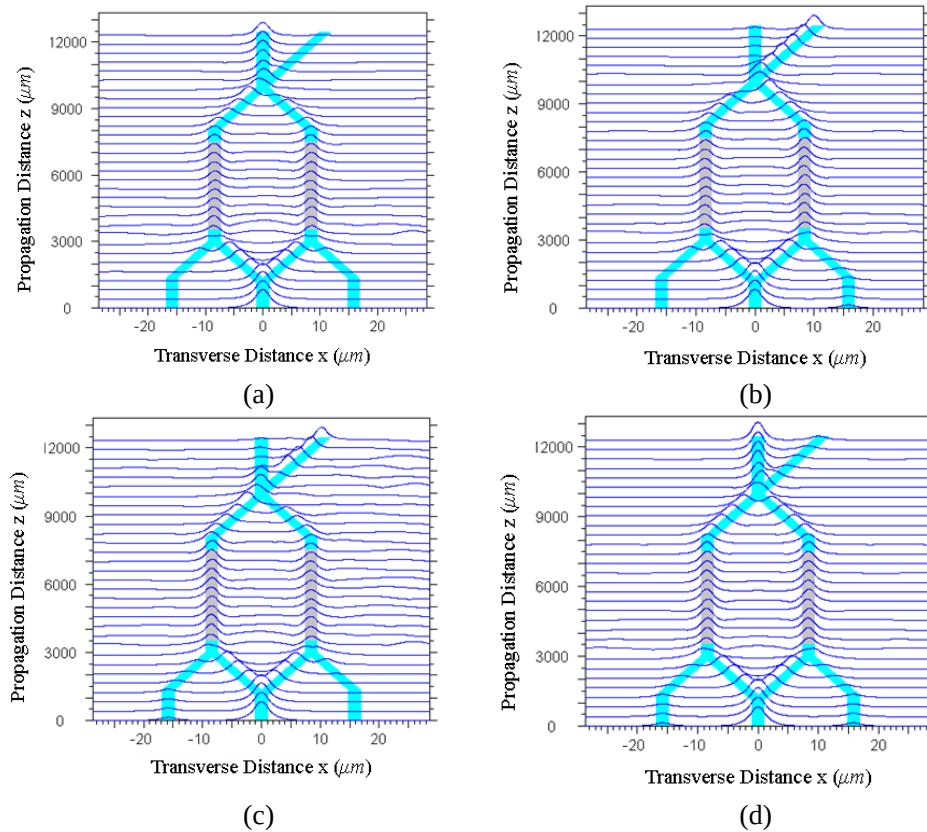


Figure 2. The XOR/NXOR logic functions with the input signal power $P_s = 65 \text{ W/m}$ and the input control power $P_c = 0.02858P_s$, (a) $A = 0$, $B = 0$, (b) $A = 0$, $B = 1$, (c) $A = 1$, $B = 0$, and (d) $A = 1$, $B = 1$.

power, the spatial solitons will be switched to different output ports. For the local nonlinear MZI, the branching angle is θ_1 , w the width of MZI waveguide, L_1 the length of local nonlinear waveguides, L_2 the length of MZI waveguide, and L_3 the length of nonlinear output waveguides. The branching angle between the output guides is θ_2 . Considering the transverse electric polarized waves propagating along the z direction and taking the field to be homogeneous in the y direction, the wave equation can be reduced to

$$\nabla^2 \psi_{yi} = \frac{n_i^2}{c^2} \frac{\partial^2 \psi_{yi}}{\partial t^2} \quad (1)$$

with solutions of the form

$$\psi_{yi}(x, z, t) = E_i(x) \exp[j(\omega t - \beta k_0 z)] \quad (2)$$

where ω is the angular frequency, k_0 the wave number in the free space, and β the effective refractive index. The entire waveguide structure, assumed to consist of Kerr-like nonlinear material, is embedded in a homogenous linear medium. For the Kerr-type nonlinear medium, the square of the refractive index of each layer can be expressed as [54]

$$n_i^2 = n_{i0}^2 + \alpha_i |E_i(x)|^2 \quad (3)$$

where n_{i0} , α_i , and $|E_i(x)|^2$ are the linear refractive index, the nonlinear coefficient, and the intensity of the electric fields in the i -th layer, respectively. A general method for analyzing the multilayer and multibranch optical waveguide structures with arbitrary Kerr-type nonlinear layers has been proposed [54]. The transverse electric field in each layer can be rewritten as

$$E_i(x) = A_i \frac{cn[q_i(x - x_i) + B_i|m_i|]}{cn[B_i|m_i|]} = A_i cn[q_i(x - x_i)|m_i|] \frac{\{1 - fn[q_i(x - x_i)|m_i|]fn[B_i|m_i|]\}}{1 - m_i sn^2[q_i(x - x_i)|m_i|]sn^2[B_i|m_i|]} \quad (4)$$

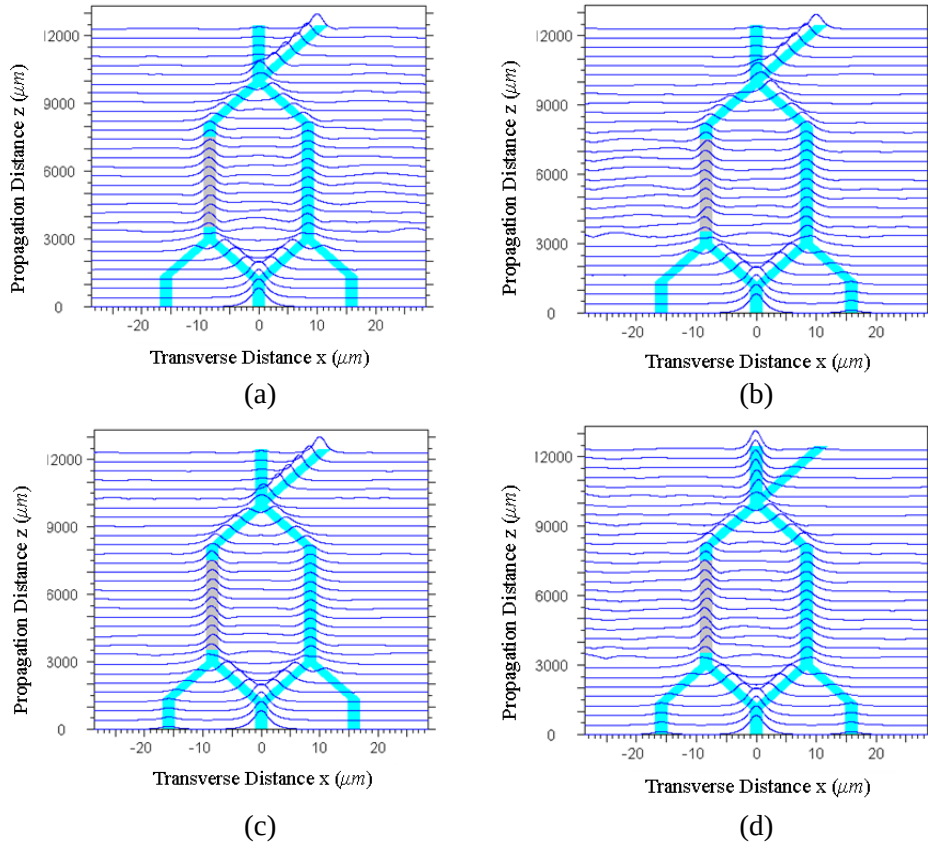


Figure 3. The AND/NAND logic functions with the input signal power $P_s = 77 \text{ W/m}$ and the input control power $P_c = 0.0267P_s$, (a) $A = 0$, $B = 0$, (b) $A = 0$, $B = 1$, (c) $A = 1$, $B = 0$, and (d) $A = 1$, $B = 1$.

where A_i is the amplitude of the electric field at the lower boundary in each layer. Constant x_{ci} can be replaced by the additional unknown variable B_i . sn and fn are the Jacobi elliptic functions, respectively. $sn[B_i|m_i]$ and $fn[B_i|m_i]$ can be given at the lower boundary $x = x_i$.

3. NUMERICAL RESULTS AND DISCUSSIONS

In this paper, the finite difference beam propagation method (FD-BPM) [55] with 4096 transverse sampling points and a longitudinal step length $\Delta z = 0.05 \mu\text{m}$ was used to simulate the proposed structures. The numerical data have been calculated with the value: the total propagation distance $z = 12500 \mu\text{m}$, $\theta_1 = 0.5^\circ$, $\theta_2 = 0.25^\circ$, $w = 2 \mu\text{m}$, $L_1 = 4000 \mu\text{m}$, $L_2 = 10000 \mu\text{m}$, $L_3 = 2500 \mu\text{m}$, $\alpha = 6.3786 \mu\text{m}^2/\text{V}^2$ [56], $n_{c0} = 1.55$, $n_{f0} = 1.57$, the free space wavelength $\lambda = 1.55 \mu\text{m}$. The XOR/NXOR logic functions were examined first, as shown in Figures 2(a)–(d). Figure 2 shows the typical evolutions of the input light waves propagating along the structure with the input signal power $P_s = 65 \text{ W/m}$ and the input control power $P_c = 0.02858 P_s$. When there is no control beam, the output signal beam will propagate straight through the central output guide C, as shown in Figure 2(a). When only the right control guide B is excited, the output signal beam will propagate through the right output guide D, as shown in Figure 2(b). When only the left control guide A is excited, the output signal beam will propagate through the right output guide D, as shown in Figure 2(c). When both of the control guides A and B are excited simultaneously, the output signal beam will propagate straight through the central output guide C, as shown in Figure 2(d). As the results shown above, the output port C functions as a NXOR gate and the output port D functions as an XOR gate.

Then the AND/NAND logic functions were examined next, as shown in Figures 3(a)–(d). Figure 3

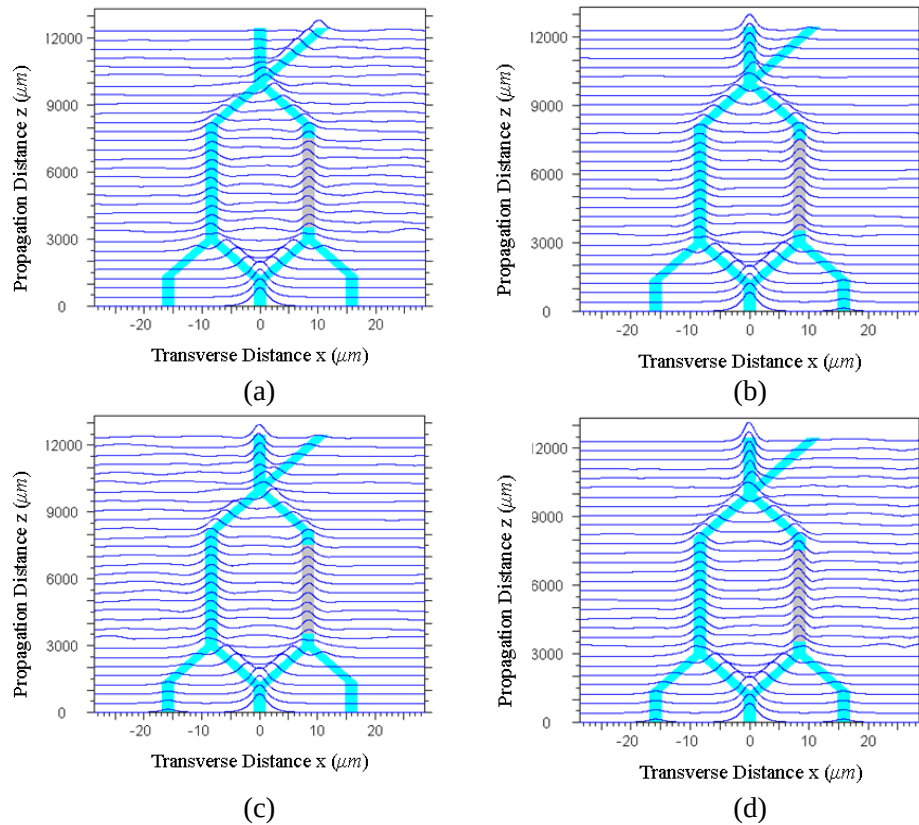


Figure 4. The OR/NOR logic functions with the input signal power $P_s = 79 \text{ W/m}$ and the input control power $P_c = 0.02875 P_s$, (a) $A = 0$, $B = 0$, (b) $A = 0$, $B = 1$, (c) $A = 1$, $B = 0$, and (d) $A = 1$, $B = 1$.

shows the typical evolutions of the input light waves propagating along the structure with the input signal power $P_s = 77 \text{ W/m}$ and the input control power $P_c = 0.0267P_s$. When there is no control beam, the output signal beam will propagate through the right output guide D, as shown in Figure 3(a). When only the right control guide B is excited, the output signal beam will propagate through the right output guide D, as shown in Figure 3(b). When only the left control guide A is excited, the output signal beam will propagate through the right output guide D, as shown in Figure 3(c). When both of the control guides A and B are excited simultaneously, the output signal beam will propagate straight through the central output guide C, as shown in Figure 3(d). As the results shown above, the output port C functions as an AND gate and the output port D functions as a NAND gate.

Finally, the OR/NOR logic functions were examined, as shown in Figures 4(a)–(d). Figure 4 shows the typical evolutions of the input light waves propagating along the structure with the input signal power $P_s = 79 \text{ W/m}$ and the input control power $P_c = 0.02875P_s$. When there is no control beam, the output signal beam will propagate through the right output guide D, as shown in Figure 4(a). When only the right control guide B is excited, the output signal beam will propagate straight through the central output guide C, as shown in Figure 4(b). When only the left control guide A is excited, the output signal beam will propagate straight through the central output guide C, as shown in Figure 4(c). When both of the control guides A and B are excited simultaneously, the output signal beam will propagate straight through the central output guide C, as shown in Figure 4(d). As the results shown above, the output port C functions as an OR gate and the output port D functions as a NOR gate.

4. CONCLUSIONS

In this paper, new all-optical logic gates based on spatial solitons in the optical communication spectral region were proposed. The proposed waveguide structures of all-optical logic gates are composed of the local nonlinear MZI waveguide structures with multi-input ports and two nonlinear output ports. They can be used to design various all-optical logic gates. The nonlinear MZI waveguide structure with local nonlinear waveguides functions like a phase shifter. It employs angular deflection of spatial solitons controlled by the phase modulation created in the local nonlinear MZI. The light-induced index changes in the local nonlinear MZI waveguide structures break the symmetry of structure and make the output signal beam propagate through different nonlinear output waveguides. By properly choosing the input control power, the spatial solitons will be switched to different output ports. The numerical results show that the proposed local nonlinear MZI waveguide structures could really function as all-optical logic gates in the optical communication spectral region. They would be potential key components in the application of the optical signal processing and optical computing systems.

ACKNOWLEDGMENT

The author would like to thank Jheng-Hong Ciou for his constructive discussion and help. This work was partly supported by Ministry of Science and Technology Taiwan under Grant No. MOST 104-2221-E-151-033.

REFERENCES

1. Yariv, A., Y. Xu, R. K. Lee, and A. Scherer, "Coupled-resonator optical waveguide: A proposal and analysis," *Opt. Lett.*, Vol. 24, 711–713, 1999.
2. Shi, T. T. and S. Chi, "Nonlinear photonic switching by using the spatial soliton collision," *Opt. Lett.*, Vol. 15, 1123–1125, 1990.
3. Wu, Y. D., "New all-optical wavelength auto-router based on spatial solitons," *Optics Express*, Vol. 12, 4172–4177, 2004.
4. Villeneuve, A., K. A. Hemyari, J. U. Kang, C. N. Ironside, J. S. Aitchison, and G. I. Stegeman, "Demonstration of all-optical demultiplexing at 1555 nm with an AlGaAs directional coupler," *Electronics Lett.*, Vol. 29, 721–722, 1993.

5. Wu, Y. D., "Coupled-soliton all-optical logic device with two parallel tapered waveguides," *Fiber and Integrated Optics*, Vol. 23, 405–414, 2004.
6. Ironside, N. and M. O'Neill, "Guided wave all-optical logic devices," *IEE Colloquium on Non-Linear Optical Waveguides*, 15/1–15/4, 1988.
7. Jensen, S. M., "The nonlinear coherent coupler," *IEEE Journal of Quantum Electronics*, Vol. 18, 1580–1583, 1982.
8. Thylen, L., "Beam-propagation method analysis of a nonlinear directional coupler," *Opt. Lett.*, Vol. 11, 739–741, 1986.
9. Pertsch, T., U. Peschel, and F. Lederer, "All-optical switching in quadratically nonlinear waveguide arrays," *Opt. Lett.*, Vol. 28, 102–104, 2003.
10. Jensen, S. M., "The nonlinear coherent coupler," *IEEE J. Quantum Electron.*, Vol. 18, 1580–1583, 1982.
11. Lattes, A., H. Haus, F. J. Leonberger, and E. P. Ippen, "An ultrafast all-optical gate," *IEEE J. Quantum Electron.*, Vol. 19, 1718–1723, 1983.
12. Kawaguchi, H., "Proposal for a new all-optical waveguide functional device," *Opt. Lett.*, 411–413, 1985.
13. Shi, T. T. and S. Chi, "Nonlinear TE-wave propagation in a symmetric, converging, single-mode Y-junction waveguide," *J. Opt. Soc. Amer. B*, Vol. 9, 1338–1340, 1992.
14. Wa, P. L. K., J. E. Sitch, N. J. Mason, J. S. Roberts, and P. N. Robson, "All-optical multiple-quantum-well wave-guide switch," *Electronics Lett.*, Vol. 21, 26–28, 1985.
15. Finlayson, N., W. C. Banyai, E. M. Wright, C. T. Seaton, G. I. Stegeman, T. J. Cullen, and C. N. Ironside, "Picosecond switching induced by saturable absorption in a nonlinear directional coupler," *Appl. Phys. Lett.*, Vol. 53, 1144–1146, 1988.
16. Villeneuve, A., C. C. Yang, P. G. J. Wigley, G. I. Stegeman, J. S. Aitchinson, and C. N. Ironside, "Ultrafast all-optical switching in semiconductor nonlinear directional coupler at half band gap," *Appl. Phys. Lett.*, Vol. 61, 147–149, 1992.
17. Al-hemyai, K., J. S. Aitchison, C. N. Ironside, G. T. Kennedy, R. S. Grant, and W. Sibbett, "Ultrafast all-optical switching in GaAlAs integrated interferometer in 1.55 μm spectral region," *Electronics Lett.*, Vol. 28, 1090–1092, 1992.
18. Nakamura, S., K. Tajima, and Y. Sugimoto, "Experimental investigation on high-speed switching characteristics of a novel symmetric Mach-Zehnder all-optical switch," *Appl. Phys. Lett.*, Vol. 65, 283–285, 1994.
19. Silberberg, Y. and B. G. Sfez, "All-optical phase- and power-controlled switching in nonlinear waveguide junctions," *Opt. Lett.*, Vol. 13, 1132–1134, 1988.
20. Fouckhardt, H. and Y. Silberberg, "All-optical switching in waveguide X junctions," *J. Opt. Soc. Amer. B*, Vol. 7, 803–809, 1990.
21. Sabini, J. P., N. Finlayson, and G. I. Stegeman, "All-optical switching in nonlinear X-junctions," *Appl. Phys. Lett.*, Vol. 55, 1176–1178, 1989.
22. Aitchison, J. S., A. Villeneuve, and G. I. Stegeman, "All-optical switching in a nonlinear GaAlAs X junction," *Opt. Lett.*, Vol. 18, 1153–1155, 1993.
23. Murata, H., M. Izutsu, and T. Sueta, "All-optical switching in new nonlinear X-junctions," *Proc. Nonlinear Optics*, Vol. 90, 63–64, 1990.
24. Yokota, H., K. Kimura, and S. Kurazono, "Numerical analysis of an optical X coupler with a nonlinear dielectric region," *IEICE Trans. Electron.*, Vol. E78-C, 61–66, 1995.
25. Pramono, Y. H., M. Geshiro, T. Kitamura, and S. Sawa, "Self-switching in crossing waveguides with three channels consisting of nonlinear material," *IEICE Trans. Electron.*, Vol. E82-C, 111–118, 1999.
26. Wu, Y. D., M. H. Chen, and C. H. Chu, "All-optical logic device using bent nonlinear tapered Y-junction waveguide structure," *Fiber Integrated Opt.*, Vol. 20, 517–524, 2001.
27. Pramono, Y. H., M. Geshiro, T. Kitamura, and S. Sawa, "Optical logic OR-AND-NOT and NOR gates in waveguides consisting of nonlinear material," *IEICE Trans. Electron.*, Vol. E83-C, 1755–

- 1761, 2000.
28. Pramono, Y. H., "Nonlinear waveguides for optical logic and computation," *J. Nonlinear Opt. Phys. Mater.*, Vol. 10, 209–222, 2001.
29. Wu, Y. D., "Nonlinear all-optical switching device by using the spatial soliton collision," *Fiber Integr. Opt.*, Vol. 23, 387, 2004.
30. Wu, Y. D., " $1 \times N$ all-optical switching device by using the phase modulation of spatial solitons," *Appl. Opt.*, Vol. 44, 4114, 2005.
31. Wu, Y. D., "New all-optical switch based on the spatial soliton repulsion," *Optics Express*, Vol. 14, 4005, 2006.
32. Wu, Y. D., M. L. Whang, M. H. Chen, and R. Z. Tasy, "All-optical switch based on the local nonlinear Mach-Zehnder interferometer," *Optics Express*, Vol. 15, 9883, 2007.
33. Radwell, N., C. McIntyre, A. J. Scroggie, G. L. Oppo, W. J. Firth and T. Ackemann, "Switching spatial dissipative solitons in a VCSEL with frequency selective feedback," *Eur. Phys. J. D*, Vol. 59, 121, 2010.
34. Sarma, K., "Vector soliton switching in a fiber nonlinear directional coupler," *Opt. Comm.*, Vol. 284, 186, 2011.
35. Hatami, M., R. Attarzadeh, and A. Gharaati, "Design of an ultra-fast all-optical dark soliton switch in a three-core nonlinear directional coupler (TNLDC) made of chalcogenide glasses," *J. Nonlinear Optic. Phys. Mat.*, Vol. 21, 1250038, 2012.
36. Karimi, S., M. E. Heidari, and F. Forootan, "Design and modelling of a $1 \times N$ all-optical nonlinear Mach-Zehnder switch controlled by wavelength and input power," *Progress In Electromagnetics Research M*, Vol. 28, 101–113, 2013.
37. Liu, W.-J. and M. Lei, "All-optical soliton switching for the asymmetric fiber couplers," *Journal of Electromagnetic Waves and Applications*, Vol. 27, No. 18, 2288–2297, 2013.
38. Zhong, H., B. Tian, Y. Jiang, M. Li, P. Wang, and W.-J. Liu, "All-optical soliton switching for the asymmetric fiber couplers," *Eur. Phys. J. D*, Vol. 67, 1, 2013.
39. Wu, Y. D., "All-optical logic gates by using multibranch waveguide structure with localized optical nonlinearity," *IEEE J. Sel. Top. Quantum. Electron.*, Vol. 11, 307, 2005.
40. Serak, S. V., N. V. Tabiryan, M. Peccianti, and G. Assanto, "Spatial soliton all-optical logic gates," *IEEE Photon. Techn. Lett.*, Vol. 18, 1287, 2006.
41. Wu, Y. D., T. T. Shih, and M. H. Chen, "New all-optical logic gates based on the local nonlinear Mach-Zehnder interferometer," *Optics Express*, Vol. 16, 248, 2008.
42. Corbelli, M. M., F. Garzia, and R. Cusani, "All-optical EXOR for cryptographic application based on spatial solitons," *J. of Info. Security*, Vol. 4, 180, 2013.
43. Kubota, Y. and T. Odagaki, "Logic gates based on soliton transmission in the Toda lattice," *Adv. in Appl. Phys.*, Vol. 1, 29, 2013.
44. Bhrawy, A. H., A. A. Alshaery, E. M. Hilal, W. Manrakhan, M. Savescu, and A. Biswas, "Dispersive optical solitons with Schrödinger-Hirota equation," *J. of Nonlinear Opt. Phys. and Mater.*, Vol. 23, 1450014, 2014.
45. Bhrawy, A. H., A. A. Alshaery, E. M. Hilal, K. R. Khan, M. F. Mahmood, and A. Biswas, "Optical soliton in nonlinear directional couplers with spatio-temporal dispersion," *J. of Modern Opt.*, Vol. 61, 442–459, 2014.
46. Savescu, M., S. Johnson, A. H. Kara, S. H. Crutcher, R. Kohl, and A. Biswas, "Convention laws for optical solitons with spatio-temporal dispersion," *Journal of Electromagnetic Waves and Applications*, Vol. 28, 242–252, 2014.
47. Alshaery, A. A., A. H. Bhrawy, A. E. M. Hilal, and A. Biswas, "Bright and singular solitons in quadratic nonlinear media," *Journal of Electromagnetic Waves and Applications*, Vol. 28, No. 3, 275–280, 2014.
48. Savescu, M., K. R. Khan, R. W. Kohl, L. Moraru, A. Yildirim, and A. Biswas, "Optical soliton perturbation with improved nonlinear Schrödinger's equation in nanofibers," *J. of Nanoelectron. and Optoelectron.*, Vol. 8, 208–220, 2013.

49. Kohl, R., A. Biswas, D. Milovic, and E. Zerrad, "Optical soliton perturbation in a non-Kerr law media," *Opt. and Laser Tech.*, Vol. 40, 647–662, 2008.
50. Biswas, A., M. Fessak, S. Johnson, S. Beatrice, D. Milovic, Z. Jovanoski, R. Kohl, and F. Majid, "Optical soliton perturbation in non-Kerr law media: Traveling wave solution," *Opt. and Laser Tech.*, Vol. 44, 1775–1780, 2012.
51. Biswas, A., A. J. M. Jawad, W. N. Manrakhan, A. K. Sarma, and K. R. Khan, "Optical solitons and complexions of the Schrödinger-Hirota equation," *Opt. and Laser Tech.*, Vol. 44, 2265–2269, 2012.
52. Biswas, A., D. Milovic, M. Savescu, M. F. Mahmood, K. R. Khan, and R. Kohl, "Optical soliton perturbation in nanofibers with improved nonlinear Schrödinger's equation by semi-inverse variational principle," *J. of Nonlinear Opt. Phys. and Mater.*, Vol. 12, 1250054, 2012.
53. Xu, Y., Z. Jovanoski, A. Bouasla, H. Triki, L. Moraru, and A. Biswas, "Optical solitons in multi-dimensions with spatio-temporal dispersion and non-Kerr law nonlinearity," *J. of Nonlinear Opt. Phys. and Mater.*, Vol. 22, 1350035, 2013.
54. Kuo, C. W., S. Y. Chen, M. H. Chen, C. F. Chang, and Y. D. Wu, "Analyzing multilayer optical waveguide with all nonlinear layers," *Optics Express*, Vol. 15, 2499, 2007.
55. Chung, Y. and N. Dagli, "An assessment of finite difference beam propagation method," *IEEE J. Quantum Electron.*, Vol. 26, 1335–1339, 1990.
56. Stegeman, G. I., E. M. Wright, N. Finlayson, R. Zanon, and C. T. Seaton, "Third order nonlinear integrated optics," *J. Lightwave Technol.*, Vol. 6, 953–990, 1988.