Machine Copy for Proofreading, Vol. x, y–z, 2015

Adaptive Beamforming Algorithms for Cancellation of Multiple Interference Signals

Lay Teen Ong*

Abstract—This paper proposes a fast Minimum-Variance-Distortionless-Response (MVDR) beamforming algorithm for an antenna array for cancellation of multiple interference signals. The proposed algorithm uses Sample-Average Estimate (SAE) of the data covariance matrix and reduces its computational effort by applying the Matrix-Inversion-Lemma (MIL) to its covariance Matrix Inversion (MI) operation. The proposed algorithm is compared to two SAE-based algorithms: the Sample Matrix Inversion (SMI) algorithm that requires an MI operation and the Auxiliary Vector (AV) algorithm that does not need an MI operation. A non-SAE based algorithm using the Least Mean Square (LMS) method is also included for comparison. Simulation results show that the proposed algorithm converges slower than the SMI scheme but outperforms the AV and the LMS scheme during the transient phase. Once convergence is achieved, the proposed algorithm converges to a better Mean Square Error compared to the rest of the algorithms evaluated.

1. INTRODUCTION

Adaptive beamforming applied to antenna arrays has been a popular approach to enhance the desired signal in the presence of interference signals. The technique applied to Radar or Wireless Communications Systems enhances performance [1–5]. Minimum-Variance-Distortionless-Response (MVDR) is a classical method used in adaptive beamforming of an antenna array [6–9]. The array weights of the MVDR beamformer can be adapted through various algorithms. For example, the Sample Matrix Inversion (SMI) based algorithm is a fast adaptive beamforming/nulling technique because it directly calculates the covariance matrix [10–12]. SMI avoids the problem of eigenvalue spread that often limits the convergence rate for close-loop algorithms such as the Least Mean Square (LMS) approach. In practice, the SMI approach makes use of the Sample-Average Estimate (SAE) of the data covariance matrix and the numerical inversion of the covariance matrix to find optimum weight values. The Auxiliary Vector (AV) algorithm is another fast beamforming method that uses the SAE of the covariance matrix to approach the MVDR optimum solution and does not need a numerical inversion operation [13, 14]. The results in [13] have shown that the sequence of AV filter estimators provide favorable bias/variance and have better mean-square estimation error, when compared to LMS, Recursive Least Squares (RLS), SMI and orthogonal multistage decomposition filter estimators. One notes that the LMS algorithm has been widely used because of its simplicity in implementation [7]. However, the trade-offs of the LMS method includes its convergence speed and its dependency on the eigvenvalue spread of the input signals. The RLS method uses a recursive matrix inversion algorithm that is more complicated but efficient than LMS [7, 13]. The above-mentioned algorithms form a class of algorithms that can be implemented effectively using Digital beamforming (DBF) technology. DBF has gained popularity for enabling the flexibility and configurability of a receiving array system [15–18]. For DBF, the array system needs a receiver at each element to receive and process the arriving signals. The receiver down converts the received signal from the array element to an intermediate frequency (IF)

Received 12 June 2015
* Corresponding author: Lay Teen Ong (tslolt@nus.edu.sg).
The author is with Temasek Laboratories, National University of Singapore, Singapore 117411, Singapore.

band and then further converts the analogue signal to digital signals. The digitized signals from all the receivers are then processed by the beamforming algorithm in a fast digital processor. In contrast, in the traditional phased-array beamforming (PAB) technology, the signals received by the array are shifted in phase and/or amplitude by a digital or analogue device at each element, then summed and down converted to an IF band before conversion to digital signals by a single receiver. The traditional PAB technology has retained its popularity because it can be less expensive in terms of hardware but this is at the expense of its design flexibility. Several evolutionary algorithms such as the Genetic algorithms (GA) and the Particle-Swarm Optimization (PSO) have shown to be effective in implementation for the PAB technology [19, 21]. In general, the GA/PSO approach is constrained to making small (or fixed) amplitude and small phase perturbations at each element [19–21].

This paper proposes a fast and computationally efficient beamforming algorithm that uses the SAE approach and applies the Matrix-Inversion-Lemma (MIL) in the MVDR solution for DBF. Some earlier works, such as [3] and [22] had verified the computationally efficient of the MIL implementation for adaptive beamformers but our work has differences compared to these works. The classical work in [3] had showed that a MIL adaptive algorithm, compared to other classical algorithms like the Applebaum adaptive control loop method [1], achieves a rapid convergence by using the system gain of a radar array system. The work in [22] implemented MIL in the linearly constrained minimum variance (LCMV) beamformer. The LCMV method is a generalized form of the MVDR beamformer, allowing for additional constraints to act on known interference and/or additional desired signals [7]. [22] studied the complexity gain of using MIL for various array sizes and various numbers of constraints but did not make comparison to other adaptive algorithms. In our paper, we compare the MIL approach to the more recent and favorable AV algorithm for a MVDR beamformer. We also include the conventional SMI algorithm and a low complexity LMS algorithm for comparison. Our MVDR beamformer is considered for the pre-correlation stage of an antenna array spread spectrum (SS) system and is vulnerable to strong interference signals due to its weak received desired signal. The main benefit of using antenna array based adaptive beamforming in SS system is the extra degree of freedom in spatial and temporal domain for mitigation of interference while maintaining a main antenna beam to the desired signal [23, 24]. However, this approach increases hardware and computational processing complexity. Therefore, improvements to receiver hardware, antenna selection strategies and adaptive beamforming algorithms are imperative [5, 18]. Herein, our paper focuses on the computation processing efficiency of the MVDR beamforming algorithms in terms of their convergence rate analysis and their interference cancellation performance in environments of multiple interference signals. In addition, we analyses their adaption rate in terms of the array's weights-convergence (instead of the system gain parameter). Finally, we show that the Mean Square Error (MSE) of the array weights is a better figure-of-merit (FOM) than the array gain pattern for assessing beamformers, especially in scenarios where there are multiple interference signals.

2. SIGNAL MODEL & PROBLEM FORMATION

2.1. Signal Model

We consider a Uniform Circular Array (UCA) with N antenna elements positioned equidistant from each other and at radius r from the centre of the array. The array receives one desired signal and $M-1$ interference signals. The received array signal vector $\mathbf{x}(t)$ at time t can be written as [17, 25]

$$\mathbf{x}(t) = \mathbf{A}(\theta, \phi)\, \mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{A}(\theta, \phi) = [\mathbf{a}(\theta_0, \phi_0),\ \mathbf{a}(\theta_1, \phi_1), \ldots, \mathbf{a}(\theta_{M-1}, \phi_{M-1})]$ is an $N \times M$ matrix whose columns, $\mathbf{a}(\theta_i, \phi_i) = \left[1, e^{j\beta r \sin(\theta_i)\cos(\phi_i - \phi_1)}, e^{j\beta r \sin(\theta_i)\cos(\phi_i - \phi_2)}, \ldots, e^{j\beta r \sin(\theta_i)\cos(\phi_i - \phi_{N-1})}\right]^T$ are the steering vectors of the antenna array, and $i = 0, \ldots, M-1$, θ_i and ϕ_i are the respective azimuth and elevation angle of the signal $s_i(t)$. $\beta = 2\pi/\lambda$ is the wave number that corresponds to λ, the wavelength of the signal. $\mathbf{s}(t) = [s_0(t), s_1(t), \cdots, s_{M-1}(t)]^T$ is an $M \times 1$ signal vector whose row components are the uncorrelated desired and interference signal sources and T denotes matrix transpose. $\mathbf{n}(t)$ is an $N \times 1$ noise vector whose row components are independent additive white Gaussian noise with zero mean and variance σ_n^2. The output of the beamformer is $y(t) = \mathbf{w}(t)^H \mathbf{x}(t)$ where

$\mathbf{w}(t) = [w_1(t), w_2(t), \cdots, w_N(t)]$ is a $\times N$ weight vector whose components correspond to the weights of the beamformer and H denotes Hermitian transpose. The exact covariance matrix of the total received signal is $\mathbf{R} = E\left(\mathbf{x}(t)\mathbf{x}^H(t)\right)$ where $E(.)$ denotes the expectation operator.

2.2. MVDR Problem Formation

The MVDR method minimizes the output power of the array subject to a constraint of unity gain in the look direction of the array [1, 7]. The problem formulation is,

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \quad \text{s.t.} \;\; \mathbf{w}^H \mathbf{a}(\theta_0, \phi_0) = 1, \tag{2}$$

where $\mathbf{a}(\theta_0, \phi_0)$ is the steering vector of the desired signal. The solution to the optimization problem can be shown to be

$$\mathbf{w} = \frac{\mathbf{R}^{-1}\mathbf{a}(\theta_0, \phi_0)}{\mathbf{a}(\theta_0, \phi_0)^H \mathbf{R}^{-1}\mathbf{a}(\theta_0, \phi_0)}. \tag{3}$$

Subsequently, for adaptive beamforming, iterative weight values $\mathbf{w}(t)$ that based on the MVDR solution (3) can be computed. t denotes the sample instance. The next section discusses the adaptive beamforming algorithms considered in this work.

3. THE ALGORITHMS

This section describes our proposed MIL-MVDR algorithm and two SAE based algorithms, namely the SMI and the AV algorithm. For comparison to a non-SAE based algorithm, the LMS algorithm is also included. The MIL-MVDR, SMI and AV algorithms are outlined in Table 1. Note that the initial estimate weight, $\hat{\mathbf{w}}(0) = \mathbf{v}/\|\mathbf{v}\|^2$, where $\mathbf{v} = \mathbf{a}(\theta_0, \phi_0)$, is applied to all the algorithms studied here.

3.1. The Sample Matrix Inversion Algorithm

A SMI method uses an SAE approach to estimate its data covariance matrix [10],

$$\hat{\mathbf{R}} = 1/K \sum_{k=1}^{K} \mathbf{x}(k)\mathbf{x}(k)^H, \tag{4}$$

where K is the number of snapshots observed over a data record size. $\hat{\mathbf{R}}$ is then substituted into the MVDR solution (3) to obtain an estimate weight, $\hat{\mathbf{w}}$. The SMI method requires $K > 2N$ samples of data to achieve an average loss of less than 3 dB [10] and a matrix inversion operation, $inv()$, for $\hat{\mathbf{w}}$ computation. To enable meaningful comparison to the MIL-MVDR, AV and LMS algorithms, we adopt a moving average (MA) approach to compute $\hat{\mathbf{R}}$ iteratively. That is, $\hat{\mathbf{R}}$ is computed over a moving, but fixed K-sample of data block, and is denoted as $\hat{\mathbf{R}}(t)_{MA}$. The SMI based algorithm, denoted as SMI, is outlined in Table 1.

3.2. The Auxiliary-vector Filtering Algorithm

The AV algorithm is based on the AV filter which generates a sequence of filter estimators that converges to the SMI unbiased estimator [13]. In contrast to the SMI algorithm, the AV algorithm does not require a matrix inversion operation. However, similar to the SMI algorithm, $\hat{\mathbf{R}}(t)_{MA}$ is computed at every sample instance for the iterative computation of the weight value. The AV algorithm is outlined in Table 1.

3.3. The Proposed MIL-MVDR Algorithm

Our proposed algorithm computes data covariance matrix, $\tilde{\mathbf{R}}_{\text{1st\_K}}$, and its inversion only for the first available K samples of data. After K sample instances, we uses the matrix-inversion-lemma (MIL) function to iteratively compute the inverse value of the data covariance matrix (5). Therefore, the

MIL approach reduces its computational complexity compared to the SMI using a matrix inversion. In addition, the computational load of the MIL-MVDR algorithm is lower than the SMI algorithm as the SMI method computes $\hat{\mathbf{R}}(t)_{MA}$ and its inverse at every sample instance. Our proposed algorithm, denoted as MIL-MVDR, is summarized in Table 1.

Table 1. SMI, AV and MIL-MVDR algorithms.

Step1: Initialization for SMI and AV algorithms

For $t = 0, 1, \ldots K-2$:

$$\hat{\mathbf{w}}(t) = \mathbf{v} \big/ \|\mathbf{v}\|^2.$$

Step 2 for SMI

For $t \geq K-1,\ K,\ K+1,\ \ldots$

$$\hat{\mathbf{R}}(t)_{MA} = 1/K \sum_{k=t-(K-1)}^{t} \mathbf{x}(k)\,\mathbf{x}(k)^H$$

$$\hat{\mathbf{R}}^{-1}(t) = inv\left(\hat{\mathbf{R}}(t)_{MA}\right)$$

Compute $\hat{\mathbf{w}}(t)$ from (3), use $\hat{\mathbf{R}}^{-1}(t)$ in place of $\mathbf{R}^{-1}$ in (3).

Step 2 for AV

For $t \geq K-1, K, K+1, \ldots$

$$\hat{\mathbf{R}}(t)_{MA} = 1/K \sum_{k=t-(K-1)}^{t} \mathbf{x}(k)\,\mathbf{x}(k)^H$$

$$\mathbf{g}(t) = \left(I - \frac{\mathbf{v}\mathbf{v}^H}{\|\mathbf{v}\|^2}\right)\hat{\mathbf{R}}(t)_{MA}\,\hat{\mathbf{w}}(t-1).$$

$$\mu(t) = \frac{\mathbf{g}(t)^H\,\hat{\mathbf{R}}(t)\,\hat{\mathbf{w}}(t-1)}{\mathbf{g}(t)^H\,\hat{\mathbf{R}}(t)\,\mathbf{g}(t)}.$$

$$\hat{\mathbf{w}}(t) = \hat{\mathbf{w}}(t-1) - \mu(t)\,\mathbf{g}(t).$$

Step 2 for MIL-MVDR

For $t \geq K-1,\ K,\ K+1, \ldots$

If $t = K-1$:

$$\tilde{\mathbf{R}}_{\text{1st\_K}} = 1/K \sum_{k=t-(K-1)}^{t} \mathbf{x}(k)\,\mathbf{x}(k)^H$$

$$\tilde{\mathbf{R}}^{-1}(t) = inv\left(\tilde{\mathbf{R}}_{\text{1st\_K}}\right)$$

Else

$$\tilde{\mathbf{R}}^{-1}(t) = \tilde{\mathbf{R}}^{-1}(t-1) - \frac{\tilde{\mathbf{R}}^{-1}(t-1)\mathbf{x}(t)\mathbf{x}(t)^H\tilde{\mathbf{R}}^{-1}(t-1)}{1+\tilde{\mathbf{R}}^{-1}(t-1)\mathbf{x}(t)} \tag{5}$$

end

Compute $\hat{\mathbf{w}}(t)$ from (3), use $\tilde{\mathbf{R}}^{-1}(t)$ in place of $\mathbf{R}^{-1}$ in (3).

3.4. The LMS Algorithm

The LMS [7] algorithm determines its weight vector $\hat{\mathbf{w}}$ iteratively using the following algorithm

Step1: Initialization, $t = 0$

$$\hat{\mathbf{w}}(t) = \mathbf{v} \big/ \|\mathbf{v}\|^2.$$

Step 2 for LMS, for $t \geq 1, 2, \ldots$

$$\hat{\mathbf{w}}(t) = \left(\mathbf{I} - \frac{\mathbf{v}\mathbf{v}^H}{\|\mathbf{v}\|^2}\right)\left[\hat{\mathbf{w}}(t-1) - \mu\mathbf{x}(t)\mathbf{x}(t)^H\hat{\mathbf{w}}(t-1)\right] + \frac{\mathbf{v}}{\|\mathbf{v}\|^2}$$

and $1/2\lambda_{\max} \geq \mu \geq 0$, where $\lambda_{\max}$ is the largest eigenvalue of the data correlation matrix.

In contrast to the SMI and AV algorithm, the covariance matrix of the signal is computed at every sample instance instead of over a block of K samples.

4. SIMULATIONS AND RESULTS ANALYSIS

We evaluate the performance of the beamformer implemented using the MIL-MVDR, SMI, AV and LMS algorithms in an environment with multiple interference signals. For reference, an ideal MVDR algorithm is included. The algorithms are simulated using Matlab software running on an Intel i7 CPU-3.4 GHz General Purpose Computer. We use the MSE and the Array Pattern as two different Figure-of-Merits to measure the performance of the beamformer. We also evaluate the computational complexity of the algorithm based on the CPU time.

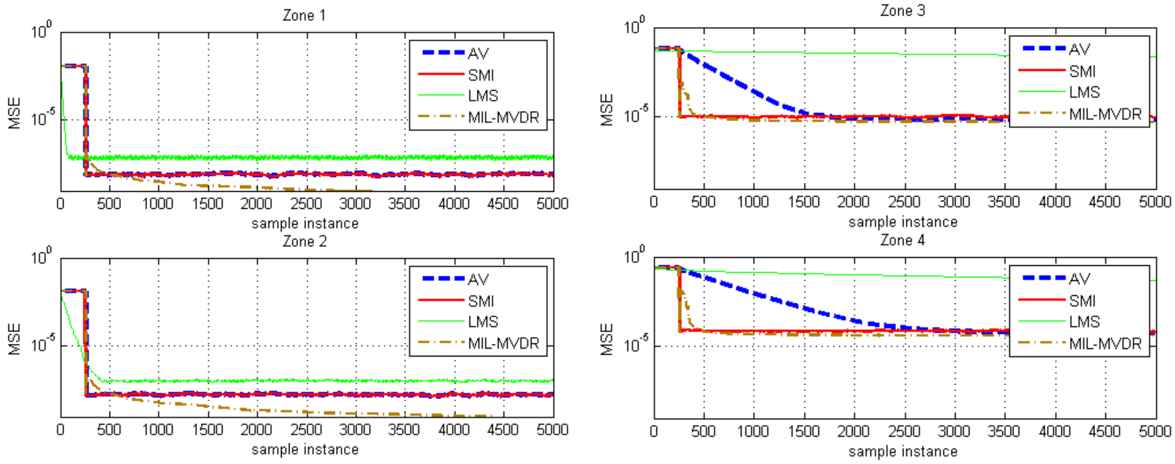
A UCA antenna with 8 elements and $r = 0.5\lambda$ is used. We consider an 8-element array for its practical realization in terms of compactness and interference mitigation performance when used in SS systems for GNSS applications [26–28]. The desired signal arrives from the broadside direction (0°, 0°) of the array and its signal power is 20 dB below the noise level. We consider a total of four interference signals and each signal has an interference-to-noise ratio of 40 dB. We evaluate four different scenarios where there is 1, 2, 3 or 4 interference signals corresponding to scenario Zone 1, 2, 3 and 4 respectively. Table 2 defines the angle setting of the scenarios. The data record size, K, is set to 256. The simulation is repeated 1000 trials for average weight analysis.

Table 2. Interference bearing setting for Zone 1 to 4.

Zone #	(Elevation, Azimuth): (θ, ϕ)
1	(80°, 0°)
2	(80°, 0°), (70°, 0°)
3	(80°, 0°), (70°, 0°), (−60°,0°)
4	(80°, 0°), (70°, 0°), (−60°, 0°), (−85°, 0°)

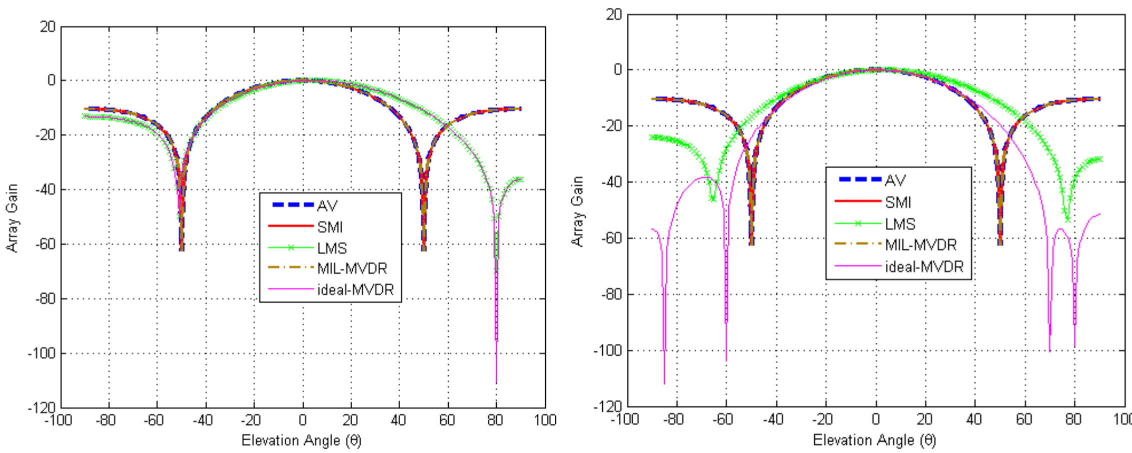
Figure 1 shows the MSE of the beamformers for the four different interference zones. The expression $\mathrm{MSE} = E(\|\hat{\mathbf{w}}(t) - \mathbf{w}_{\mathrm{MVDR}}\|^2)$ is computed; $\mathbf{w}_{\mathrm{MVDR}}$ denotes the weight vector of the ideal MVDR beamformer. Herein, we examine the convergence rate of the beamformers, that is, how fast a beamformer converges to its converged MSE We note that the LMS beamformer computes its weight iteratively from $t = 0$ while the other SAE-based beamformers start their weights adaption after $t = 256$ (after K-sample of data record is available). Therefore, the LMS beamformer should have an advantage over the other beamformers in converging to an optimal solution. However, the results show that the LMS beamformer converges fast in Zone 1 and 2, and converges very slow in Zone 3 and 4. This is because the larger eigenvalue spreads of the covariance data in Zone 3 and Zone 4 have affected the convergence rate of the LMS scheme. Therefore, the LMS beamformer performs better in the transient phase for limited interference scenarios. Next, we assess the convergence rates among the SAE-based beamformers, which start their adaption from $t \geq 256$. The SMI beamformer converges almost instantly for all the interference zones. The AV scheme converges almost as fast as the SMI scheme in Zone 1 and 2 but becomes the worst in Zone 3 and 4. In contrast, our proposed MIL-MVDR beamformer converges much faster than the AV scheme in Zone 3 and 4. Next, in terms of the converged MSE value (as $t > 600$), the MIL-MVDR scheme achieves the lowest error while the LMS scheme has the highest error.

Next, we examine the performance of the beamformer through its Array Pattern. We will also correlate the nulling performance of the beamformer from its Array Pattern to its MSE, and explain why the MSE is a better FOM in the assessment of the beamformers in interference cancellation. We



(a) (b)

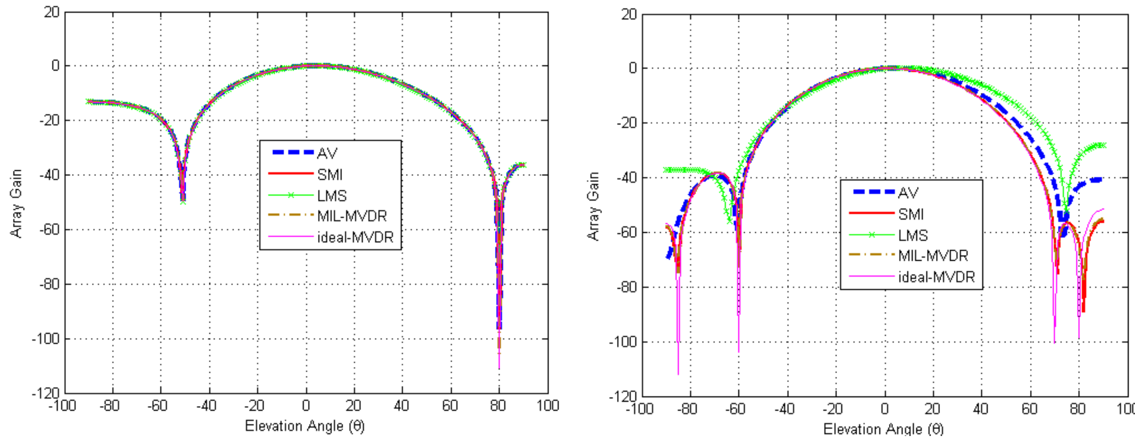
Figure 1. Mean-square error versus sample instances for scenarios of one to four uncorrelated interference signals.



(a) (b)

Figure 2. Array gain (dB) patterns of the beamformers at $\phi = 0°$, $t = 100$. (a) Left plot for Zone 1. (b) Right plot for Zone 4.

present only array pattern plots of Zone 1 and Zone 4 for clarity in presentation. Fig. 2 shows the average pattern plots at the transient phase. In Fig. 2(a), the array pattern plot at $t = 100$ of Zone-1 shows that the LMS beamformer performs the best because it achieves a nulling depth approaching to that of the ideal MVDR beamformer while the SAE beamformers remain in their quiescent pattern. In Fig. 2(b), the array pattern at $t = 100$ of Zone-4 shows that the LMS beamformer cannot fully null all the four interference signals and the SAE beamformers remain in their quiescent state. Indeed, these assessments correlate well to the previous analysis made using the MSE parameter. Next, Fig. 3 shows the average pattern plot at the converged phase. In Fig. 3(a), at $t = 1000$ of Zone 1, all beamformers are shown to achieve a good nulling depth at the interference angle of 80°. However, it is difficult to do an in-depth assessment of these beamformers' performance from this macro plot. In Fig. 3(b), at $t = 1000$ of Zone 4, the array pattern plot shows that the LMS beamformer has the worst nulling performance. However, it is even harder to assess the performance of the other beamformers, especially when nulling depths of multiple interference signals need to be evaluated. In contrast, if one uses the MSE results (Fig. 1) as the FOM, a clear assessment on the beamformers performance is possible. Therefore, we conclude that the Array Pattern plots provide a first-cut assessment of the various beamformer's performance while the MSE enables an in-depth assessment.

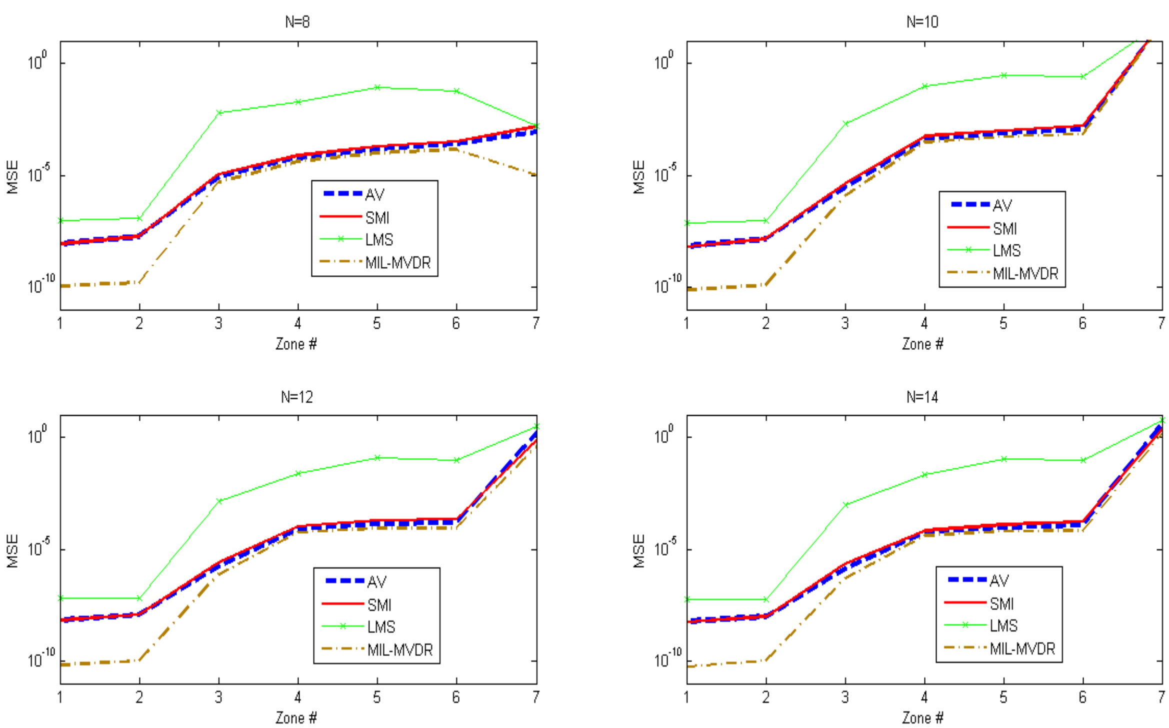


(a) (b)

Figure 3. Array gain (dB) patterns of the beamformers at $\phi = 0°$, $t = 1000$. (a) Left plot for Zone 1. (b) Right plot for Zone 4.

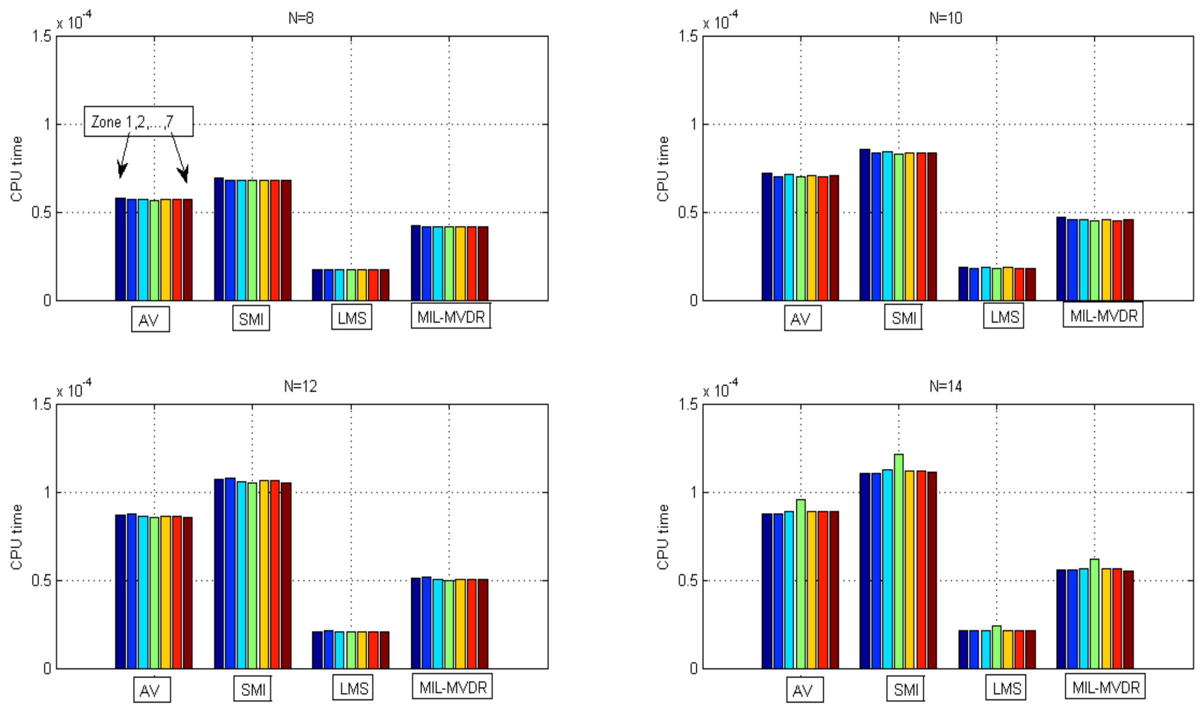
Table 3. Interference bearing setting for Zone 5 to 7.

Zone #	(Elevation, Azimuth): (θ, ϕ)
5	(80°, 0°), (70°, 0°), (−60°, 0°), (−85°, 0°), (75°, 90°)
6	(80°, 0°), (70°, 0°), (−60°, 0°), (−85°, 0°), (75°, 90°), (50°, 90°)
7	(80°, 0°), (70°, 0°), (−60°, 0°), (−85°, 0°), (75°, 90°), (50°, 90°), (−50°, 90°)



(a) (b)

Figure 4. MSE of the beamformers for the 7 interference zones and different number of antenna elements.



(a) (b)

Figure 5. CPU time of the beamformers for the 7 interference zones and different number of antenna elements.

Subsequently we compare the MSE performance of the four beamformers for a higher number of interference and the number of antenna elements $N = 8, 10, 12, 14$. A maximum of 7 interference signals is considered to agree with the degree of freedom limit for the $N = 8$ element array. Table 2 and Table 3 show the bearing setting for the 7 interference zones considered. Fig. 4 shows the computed MSE at the steady state of t =256000, where the LMS beamformer performs the worst while the MIL-MVDR beamformer achieves the lowest MSE The MIL-MVDR achieves a best MSE ratio of $\sim 1/1000$ when compared to the LMS and a MSE ratio of $\sim 1/100$ when compared to the AV and SMI.

Finally, we compare the complexity of the four beamformers by computing the CPU time required to process one estimate of $\mathbf{w}\,(.)$, $\hat{\mathbf{R}}\,(.)$ and their associate functions used in the algorithms (see Section 3). The CPU time is an empirical result that relates to the computational complexity of the algorithm. Fig. 5 shows the CPU time for different N and interference zones. The results verify that the LMS algorithm has the lowest computational complexity while the SMI has the highest. Among the SAE-based algorithm (SMI, AV and MIL-MVDR), the MIL-MVDR has a lower complexity requirement than the SMI and the AV. The results also show that the number of antenna elements impact more on the complexity of the SAE-based algorithms than the LMS algorithm. On the other hand, the number of interference signals is less likely to impact the computational complexity of the algorithms.

5. CONCLUSIONS

We proposed a fast beamforming algorithm, which is a matrix inversion lemma based MVDR approach (MIL-MVDR), for an antenna array for cancellation of multiple interference signals. We used simulation to analyze the performance of our proposed algorithm and made comparison to alternative algorithms, namely the SMI, AV and LMS algorithms. The proposed algorithm converges slower than the SMI algorithm during the transient phase but outperforms the AV and the LMS algorithms, especially for scenarios with a high number of interference signals. In the converged phase, our proposed algorithm achieved the lowest MSE compared to the other three algorithms.

Next, we compare the nulling performance of the beamformers by looking at the Array Pattern and the array weight MSE. It demonstrated that the MSE is a better figure-of-merit for assessment of the beamformers in a multiple interference signals environment. For $N = 8$ to 14 antenna elements and up to 7 interference signals, the MIL-MVDR has shown to achieve the lowest converged MSE. The MIL-MVDR achieves a best MSE ratio of 1/1000 as compared to the LMS and a MSE ratio of 1/100 as compared to the AV and SMI. We also evaluate the computational complexity of the beamforming algorithms and verify that the LMS has the lowest computational complexity (but at the expense of a higher MSE) while the MIL-MVDR is the next lower in complexity with its computational load about half that of the AV and SMI algorithms.

REFERENCES

1. Applebaum, S., "Adaptive arrays," *IEEE Trans. Antennas Propaga.*, Vol. 24, No. 5, 585–599, 1976.
2. Hudson, J. E., *Adaptive Array Principles*, Peter Peregrinus Ltd-IET, 1981.
3. Brennan, L. E., J. D. Mallett, and I. S. Reed, "Adaptive arrays in airborne MTI radar," *IEEE Trans. Antennas Propaga.*, Vol. 24, 607–615, Sep. 1976.
4. Choi, S., J. Choi, H.-J. Im, and B. Choi, "A novel adaptive beamforming algorithm for antenna array CDMA systems with strong interferers," *IEEE Trans. Veh. Technol.*, Vol. 51, No. 5, 808–816, Sep. 2002.
5. Singh, H. and R. Jha, "Trends in adaptive array processing," *Int. J. Antennas Progpag.*, Vol. 2012, 2012.
6. Capon, J., "High-resolution Frequency-wavenumber spectrum analysis," *Proceedings of the IEEE*, Vol. 57, No. 8, 1408–1418, 1969.
7. Haykin, S., *Adaptive Filter Theory*, 4th Edition, Prentice Hall, New Jersey, 2002.
8. Vorobyov, S. A., "Principles of minimum variance robust adaptive beamforming design," *Elsevier Signal Processing*, Vol. 93, No. 12, 3264–3277, Dec. 2013.
9. Liu, F., J. Wang, C. Y. Sun, and R. Du, "Robust MVDR beamformer for nulling level control via multi-parametric quadratic programming," *Progress In Electromagnetics Research C*, Vol. 20, 239–254, 2011.
10. Reed, I. S., J. D. Mallett, and L. E. Brennan, "Rapid convergence rate in adaptive arrays," *IEEE Trans. Aerosp. Electron. Syst.*, Vol. 10, No. 6, 853–863, Nov. 1974.
11. Horowitz, L. L., H. Blatt, W. G. Brodsky, and D. K. Senne, "Controlling adaptive arrays with the sample matrix inversion algorithm," *IEEE Trans. Aerosp. Electron. Syst.*, Vol. 15, 840–847, Nov. 1979.
12. Hara, Y., "Weight-convergence analysis of adaptive antenna arrays based on SMI algorithm," *IEEE Trans. Wireless Communications*, Vol. 2, No. 4, 749–757, July 2003.
13. Pados, D. A. and G. N. Karystinos, "An iterative algorithm for the computation of the MVDR filter," *IEEE Trans. Signal Process.*, Vol. 49, No. 2, 290–300, Feb. 2001.
14. Qian, H. and S. N. Batalama, "Data record-based criteria for the selection of an auxiliary vector estimator of the MMSE/MVDF filter," *IEEE Trans. on Commu.*, Vol. 51, No. 10, 1700–1708, Oct. 2003.
15. Seguin, E., R. Tessier, E. Knapp, and R. W. Jackson, "A dynamically reconfigurable phased array radar processing system," *Proc. Int. Conference on Field-Programmable Logic and Applications*, 258–26, 20113.
16. Weedon, W. H., "Phased array digital beamforming hardware development at applied radar," *Proc. IEEE Int. Symp. Phased Array Systems and Technology*, 854–859, 2010.
17. Farina, A., *Antenna-based Signal Processing Techniques for Radar Systems*, Artech House, Norwood, Massachusetts, 1992.
18. Wang, X., E. Aboutanios, M. Trinkle, and M. G. Amin, "Reconfigurable adaptive array beamforming by antenna selection," *IEEE Trans. Signal Process.*, Vol. 62, No. 9, May 2014.
19. Haupt, R. L., "Phase-only adaptive nulling with a genetic algorithm," *IEEE Trans. Antennas Propaga.*, Vol. 45, 1009–1015, 1997.

20. Massa, A., M. Donelli, F. G. B. De Natale, S. Caorsi, and A. Lommi, "Planar antenna array control with Genetic algorithms and adaptive array theory," *IEEE Trans. Antennas Propaga.*, Vol. 52, 2919–2924, 2004.
21. Mahmoud, K. R., M. I. Eladawy, R. Bansal, S. H. Zainud-Deen, and S. M. M. Ibrahem, "Analysis of uniform circular arrays for adaptive beamforming applications using particle swarm optimization algorithm," *International Journal of RF and Microwave Computer-aided Engineering*, Vol. 18, No. 1, 42–52, Jan. 2008.
22. Jakobsson, A., S. R. Alty, and S. Lambotharan, "On the implementation of the linearly constrained minimum variance beamformer," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, Vol. 53, No. 10, 1059–1062, Oct. 2006.
23. Amin, M. and W. Sun, "A novel interference suppression scheme for global navigation satellite systems using antenna array," *IEEE J. Sel. Areas Commun.*, Vol. 23, No. 5, 999–1012, 2005.
24. Fante, R. L. and J. J. Vaccaro, "Wideband cancellation of interference in a GPS receive array," *IEEE Trans. Aerosp. Electron. Syst.*, Vol. 36, No. 2, 549–564, Apr. 2000.
25. Balanis, C. A., *Antenna Theory*, 2nd Edition, John Wiley, New Jersey, 1997.
26. Ong, L. T., "An adaptive beamformer based on adaptive covariance estimator," *Progress In Electromagnetics Research M*, Vol. 36, 149–160, 2014.
27. Loecker, C., P. Knott, R. Sekora, and S. Algermissen, "Antenna design for a conformal antenna array demonstrator," *2012 6th European Conference on Antennas and Propagation (EUCAP)*, 151–153, 2012.
28. Griffith, K. A. and I. J. Gupta, "Effect of mutual coupling on the performance of GPS AJ antennas," *IEEE/ION Position, Location and Navigation Symposium*, 871–877, 2008.