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Extrapolation of Transient Electromagnetic Response Using Approximate Prolate Series

Ming-Da Zhu*

Abstract—A novel technique for extrapolation of transient response using early-time and low-frequency data is proposed in this paper. An improved extrapolation scheme using approximate prolate series is presented to obtain a transient electromagnetic response. The approximate prolate series, which has an approximately band-limited and sub-domain nature, is a better choice for extrapolating the time-domain electromagnetic response than orthogonal polynomials, such as Laguerre functions and Hermite functions. A novel regularization method based on truncated generalized SVD is also proposed to solve the ill-posed extrapolation problems, which make the extrapolation technique much less sensitive to noise in the known part of the response. Some numerical results are presented to illustrate the effectiveness and accuracy of the proposed method in extrapolation of transient electromagnetic problems.

1. INTRODUCTION

A transient electromagnetic response for three-dimensional conducting structures can be obtained by computational electromagnetic methods (CEM) in time or frequency domain. CEM methods in time domain require lots of time steps to obtain a complete temporal response, while a broad-band solutions in frequency domain need to conduct computation at a large number of frequency points. In order to reduce the computational intensity of CEM, some researchers have proposed an extrapolation scheme [1–8] that the electromagnetic responses are extrapolated from early-time and low-frequency data by fitting a set of orthogonal functions and its Fourier transform. The missing late-time and high-frequency part is generated by the complementary information in both domains.

In the extrapolation methods [1–5], the electromagnetic response is expressed as linear combination of orthogonal functions, such as Laguerre polynomials, Hermite polynomials, and Bessel-Chebyshev functions. Since it is assumed that the conducting objects are excited by an approximately band-limited incident electromagnetic wave, the temporal response is also an approximately band-limited function. However, the associated orthogonal polynomials used in [1–8] are not band-limited, which has a serious drawback that the extrapolation schemes are highly sensitive to noise in the known part of the electromagnetic response. For above reasons, an optimal choice of basis functions would be approximate prolate (AP) functions which are naturally band-limited and well suited for transient signals with compact support.

On the other hand, it is observed that the system matrix of extrapolation is severely ill-conditioned, and a singular-value decomposition (SVD) which discarding the singular values smaller than a preset tolerance [5], does not guarantee that a meaningful solution of extrapolation can be obtained. Therefore, a suitable regularization method is required to ensure the effectiveness of the extrapolation.

In this paper, we use the AP functions to extrapolate transient electromagnetic responses using limited frequency- and time-domain information. A novel regularization scheme based on truncated generalized SVD (TGSVD) [9] is presented to solve the ill-posed problem. Finally, some numerical results are presented to illustrate the performances of the proposed method in extrapolation of transient electromagnetic problems.

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* Corresponding author: Ming-Da Zhu (mingda.zhu@dhu.edu.cn).
The author is with the Department of Electronics and Communication Engineering, Donghua University, Shanghai 201620, China.

2. FORMULATION

2.1. Approximate Prolate Functions

Strictly speaking, a causal time-domain function cannot be simultaneously band-limited in both time and frequency domains. In this paper, a transient electromagnetic response $y(t)$ is assumed to be approximately band-limited if the responses are effectively time-limited to T and frequency-limited to W. Therefore, the approximate prolate series is a suitable choice of basis functions, as these functions are band-limited and well suited for signals with finite time support. Given the time step size Δt and the width parameter N_{AP}, the AP function is defined as [10]

$$\psi\left(t\right)=\frac{\sin\left(\dfrac{W}{1-\delta}t\right)}{\dfrac{W}{1-\delta}t}\frac{\sin\left[c\sqrt{\left(\dfrac{t}{N_{AP}\Delta t}\right)^{2}-1}\right]}{\sinh\left(c\right)\sqrt{\left(\dfrac{t}{N_{AP}\Delta t}\right)^{2}-1}}\tag{1}$$

where $c=\pi N_{AP}\delta$ and δ is a real number between zero and unity. Generally, the highest frequency present in $\psi\left(t\right)$ is given by

$$W_{\max}=\frac{1+\delta}{1-\delta}W\tag{2}$$

The truncated AP function is also time-limited, where $\psi\left(t\right)\approx 0$ for $t>(N_{AP}+1/2)\Delta t$. Thus, an electromagnetic response $y\left(t\right)$ can be expanded as

$$y\left(t\right)=\sum_{k=0}^{N}x_k\psi\left(t-k\Delta t\right)\tag{3}$$

$$Y\left(f\right)=\sum_{k=0}^{N}x_k e^{-j2\pi k\Delta tf}\Psi\left(f\right)\tag{4}$$

$$x_i=y\left(i\Delta t\right),\quad i=0,1,2,\ldots,N\tag{5}$$

where $\Psi\left(f\right)$ and $Y\left(f\right)$ is Fourier transform of $\psi\left(t\right)$ and $y\left(t\right)$.

2.2. Extrapolation Matrix Formulation

Let M_t^{init} and M_f^{init} be the number of early-time and low-frequency samples that are given for the functions $y(t)$ and $Y(f)$, where Δt^{init} and Δf^{init} are the sampling steps. In order to obtain suitable time and frequency steps for extrapolation, we choose cubic spline interpolation to resample the initial data. The resampled data are denoted by y_1 and Y_1. Let M_t and M_f be the number of samples for y_1 and Y_1, where Δt and Δf are the time- and frequency-domain sampling steps, respectively.

To get an extrapolation solution of the coefficients x_i, $i=0,1,2,\ldots,N$, we construct a matrix equation from (3) and (4), which is shown below

$$Ax=b,\quad A\in\mathbb{R}^{(M_t+M_f)\times(N+1)},\quad x\in\mathbb{R}^{(N+1)},\quad b\in\mathbb{R}^{(M_t+M_f)}\tag{6}$$

$$A=\begin{bmatrix}\begin{matrix}\psi\left(t_1\right) & \psi\left(t_1-\Delta t\right) & \ldots & \psi\left(t_1-N\Delta t\right)\\ \vdots & \vdots & \vdots & \vdots\\ \psi\left(t_{M_t}\right) & \psi\left(t_{M_t}-\Delta t\right) & \ldots & \psi\left(t_{M_t}-N\Delta t\right)\end{matrix}\\ \text{Re}\left\{\begin{matrix}\Psi\left(f_1\right) & e^{-j2\pi\Delta tf_1}\Psi\left(f_1\right) & \ldots & e^{-j2\pi N\Delta tf_1}\Psi\left(f_1\right)\\ \vdots & \vdots & \vdots & \vdots\\ \Psi\left(f_{M_f}\right) & e^{-j2\pi\Delta tf_{M_f}}\Psi\left(f_{M_f}\right) & \ldots & e^{-j2\pi N\Delta tf_{M_f}}\Psi\left(f_{M_f}\right)\end{matrix}\right\}\\ \text{Im}\left\{\begin{matrix}\Psi\left(f_1\right) & e^{-j2\pi\Delta tf_1}\Psi\left(f_1\right) & \ldots & e^{-j2\pi N\Delta tf_1}\Psi\left(f_1\right)\\ \vdots & \vdots & \vdots & \vdots\\ \Psi\left(f_{M_f}\right) & e^{-j2\pi\Delta tf_{M_f}}\Psi\left(f_{M_f}\right) & \ldots & e^{-j2\pi N\Delta tf_{M_f}}\Psi\left(f_{M_f}\right)\end{matrix}\right\}\end{bmatrix}\tag{7}$$

$$x = \begin{bmatrix} x_0 & x_1 & \dots & x_N \end{bmatrix}^T \tag{8}$$

$$b = \begin{bmatrix} y(t_1) & \dots & y(t_{M_t}) & \mathrm{Re}\left\{ Y(f_1) \ \dots \ Y\left(f_{M_f}\right) \right\} & \mathrm{Im}\left\{ Y(f_1) \ \dots \ Y\left(f_{M_f}\right) \right\} \end{bmatrix}^T \tag{9}$$

Since matrix A is severely ill-conditioned due to the cluster of small singular values, the solution x of Equation (6) by SVD [3–5] is potentially sensitive to perturbations. The aim of the next section is to explore a robust solution of Eq. (6) through the application of regularization techniques.

2.3. A Regularization Scheme Based on TGSVD

In order to obtain a meaningful solution of Equation (6), a novel regularization scheme based on TGSVD [9] is proposed in this section to solve the ill-conditioned problem. Common regularized methods are to compute the solution x to the Tikhonov-regularization problem [11]

$$\min\left\{\|Ax - b\|^2 + \lambda^2 \|Lx\|^2\right\}, \quad x \in \mathbb{R}^{(N+1)} \tag{10}$$

where $\|\cdot\|$ denotes the Euclidean norm, and $L \in \mathbb{R}^{(N+1)\times(N+1)}$ is the regularization matrix. A convenient tool for problem (10) is the generalized SVD (GSVD) of the matrix pair $(A,\ L)$ [9]. The GSVD is a decomposition of A and L in the form

$$A = U{\sum}^A Y^{-1}, \quad {\sum}^A = \mathrm{diag}\left(\sigma_1^A, \sigma_2^A, \dots, \sigma_{N+1}^A\right) \tag{11}$$

$$L = V{\sum}^L Y^{-1}, \quad {\sum}^A = \mathrm{diag}\left(\sigma_1^L, \sigma_2^L, \dots, \sigma_{N+1}^L\right) \tag{12}$$

where $U \in \mathbb{R}^{\left(M_t+M_f\right)\times(N+1)}$ and $V \in \mathbb{R}^{(N+1)\times(N+1)}$ are matrices with orthonormal columns, and $Y \in \mathbb{R}^{(N+1)\times(N+1)}$ is nonsingular. ${\sum}^A$ and ${\sum}^L$ are $(N+1)\times(N+1)$ diagonal matrices.

The regularized solution x_k to (10) by TGSVD is given by neglecting the $(N+1)-k$ smallest σ_1^A, which is defined as

$$x_k = \sum_{i=(N+1)-k+1}^{N+1} \frac{u_i^T b}{\sigma_i^A} y_i \tag{13}$$

where k is the truncation index, and u_i and y_i are columns of matrices U and Y, respectively.

In order to compute a meaningful solution of Equation (6), a novel four-step algorithm based on TGSVD is described as follows:

(1) In solving the original problem (6), the L-curve criterion [12] and SVD is applied to seek an approximated solution x^{SVD}, $x^{SVD} = \left[x_1^{SVD}, x_2^{SVD}, \dots, x_{N+1}^{SVD}\right]^T$.

(2) Construct the regularization matrix L by $L = \mathrm{diag}\left[\frac{1}{x_1^{SVD}}, \frac{1}{x_2^{SVD}}, \dots, \frac{1}{x_{N+1}^{SVD}}\right] \in \mathbb{R}^{(N+1)\times(N+1)}$.

(3) After selecting the regularization matrix L, the regularization problem (10) is formed, and GSVD is then used to get a decomposition of the matrix pair $(A,\ L)$.

(4) The L-curve criterion is utilized again to determine the truncation index k of Equation (13). After determining the parameter k, the regularized solution $\bar{x}$ can be computed by TGSVD, where $\bar{x}$ is a good approximation to x.

3. NUMERICAL RESULT AND DISCUSSION

Based on above mathematical treatment, computer codes are developed for verifying the effectiveness of the proposed method. In each of them given below, we use a temporal pulse with a modulated Gaussian shape as the excitation, and described by

$$\vec{E}_i(\vec{r}, t) = -\vec{E}_0 \frac{4}{T\sqrt{\pi}} \cos(2\pi f_0 t)\, e^{-\gamma^2} \tag{14}$$

$$\gamma = \frac{4}{cT}\left(ct - ct_0 - \vec{r}\cdot\hat{k}\right) \tag{15}$$

where f_0 is the central frequency of the pulse, $\hat{k}$ the direction of the wave propagation, $\hat{p}$ the polarization vector, t the time variable, T the pulse width, and ct_0 the time delay.

At first we consider a thin wire centrally placed in front of a square plate. In Figure 1, the wire length is 0.5 m; the side length of the PEC plate is 3 m; the distance between the wire and the plate is 1 m. The incident field, polarized along the direction of $\hat{y}$, propagates along the direction of $-\hat{z}$. Other parameters are chosen to be $T = 26.67$ ns, $V_0 = 1.0$ KV, and $f_0 = 200$ MHz.

The time-domain initial data are obtained from 0 to 39.6 ns by time-domain integral equation (TDIE). Figure 2(a) illustrates the transient current at the centre of the thin wire, obtained by Laguerre extrapolation method [3] and the proposed extrapolation technique based on AP functions and the TGSVD regularization in this paper, respectively. The extrapolation results in Figure 2(b) show that the extrapolated data with the proposed regularization method match the original data better, compared with the one obtained by original SVD with preset tolerance 10^{-3} [5].

Secondly, we consider a PEC cone, as shown in Figure 3, with the base radius 0.5 m and height 1.5 m. The extrapolated current across edge 1 is shown in Figure 4(a) in time-domain and in Figure 4(b) for frequency-domain, which is obtained by Laguerre extrapolation method [3] and the proposed extrapolation technique with AP functions and the TGSVD regularization in this paper, respectively. Compared with the method [3], the proposed scheme, based on AP functions and TGSVD regularization, obtains better extrapolation accuracy in both time- and frequency-domains.

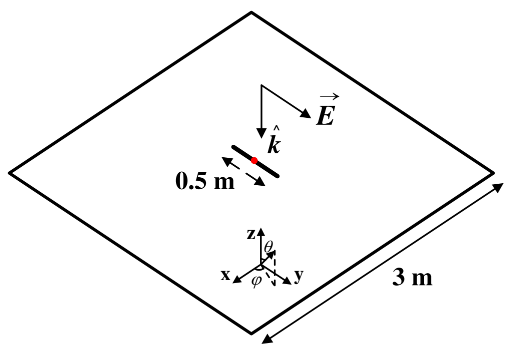

Figure 1. Both wire and PEC plate are illuminated by by a modulated Gaussian pulse.

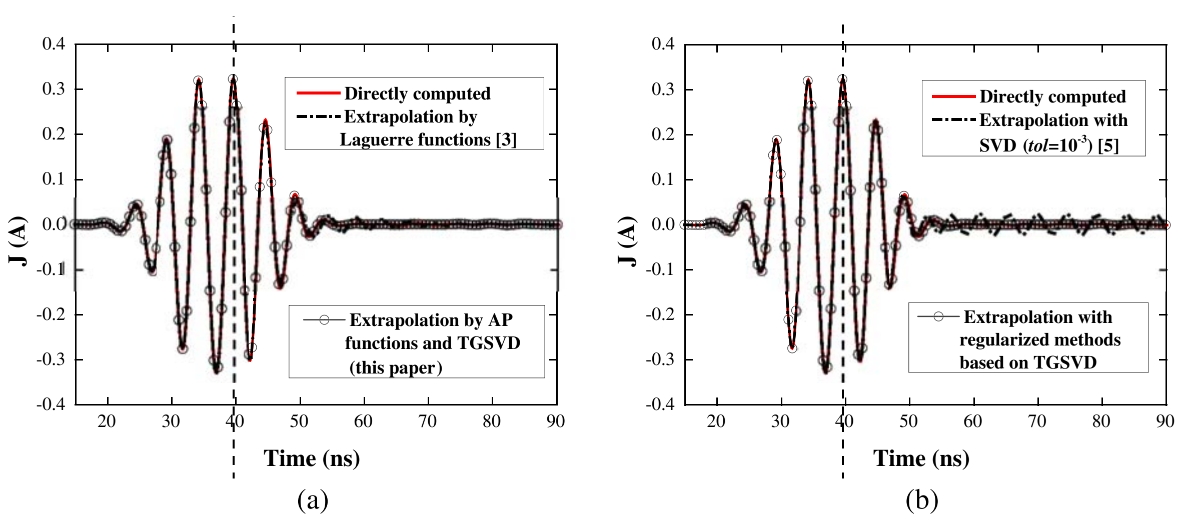

Figure 2. Transient current at the centre of the thin wire. (a) Extrapolation results obtained by Laguerre extrapolation method [3] and the proposed extrapolation technique based on AP functions and TGSVD regularization, respectively. (b) Extrapolated response obtained by original SVD with preset tolerance 10^{-3} [5] and the proposed regularization scheme based on TGSVD, respectively.

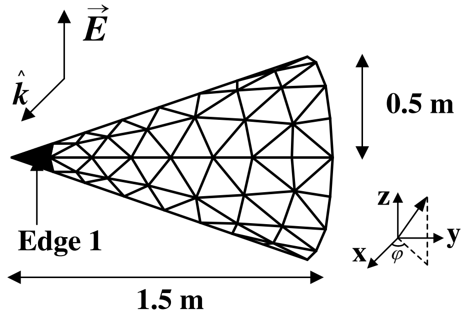

Figure 3. A PEC cone with base radius 0.5 m and height 1.5 m.

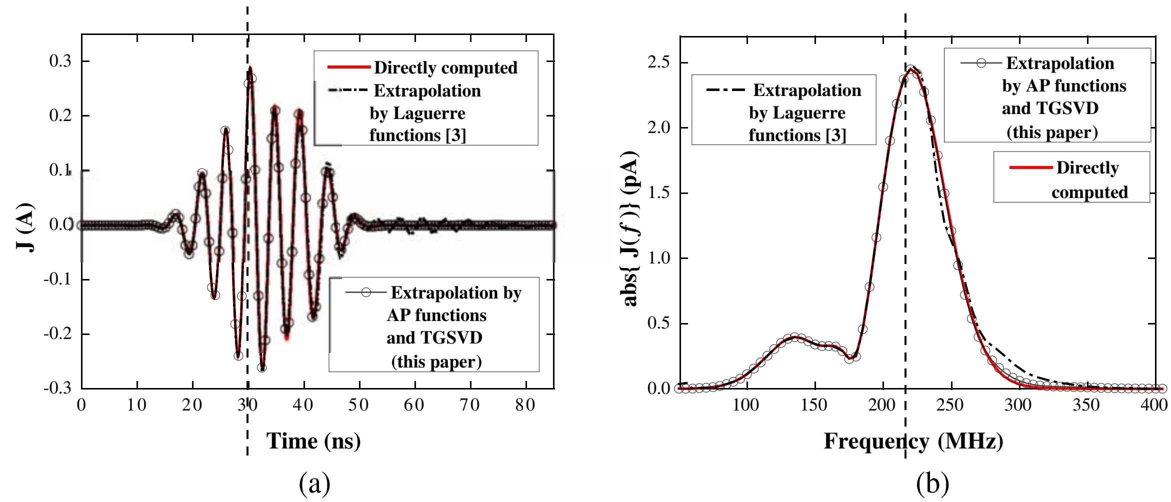

Figure 4. (a) Time-domain extrapolated response across edge 1 on the cone, obtained by Laguerre extrapolation method [3] and the proposed extrapolation technique based on AP functions and TGSVD regularization, respectively. (b) Frequency-domain extrapolated response across edge 1 on the cone, obtained by Laguerre extrapolation method [3] and the proposed extrapolation technique, respectively.

4. CONCLUSION

In this paper, we have utilized the AP functions to extrapolate the transient electromagnetic response using limited frequency- and time-domain information. The ill-posed extrapolation problems are treated by a novel regularization scheme based on TGSVD. The missing late-time and high-frequency data can be accurately extrapolated without direct CEM computations. Finally, some numerical results are presented to illustrate the performances of the proposed method in extrapolation of transient electromagnetic problems.

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