

# Peculiarities of the Spatial Power Spectrum of Scattered Electromagnetic Waves in the Turbulent Collision Magnetized Plasma

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**Abstract**—General dispersion equation is obtained at arbitrary inclination angles of the external magnetic field and wave vector of an incident EM wave. Statistical characteristics of the phase fluctuations of scattered high frequency EM waves in the collision magnetized plasma caused by electron density and external magnetic field fluctuations taking into account polarization coefficients for both ordinary and extraordinary waves are calculated analytically. The influence of collision frequency, anisotropy factor and angle of inclination of prolate irregularities of electron density fluctuations with respect to the geomagnetic field of lines on the broadening of the spatial power spectrum is analyzed. Phase portraits of the phase fluctuations caused by the geomagnetic field fluctuations are constructed at different spatial parameters characterizing magnetic field and electron density fluctuations. Numerical calculations are carried out for the ionospheric  $F$ -region parameters using experimental data.

## 1. INTRODUCTION

The features of EM waves propagation in random media have been rather well studied [1, 2]. Excellent reviews [3–5] related to the EM waves propagation and observations in the ionosphere have been published whereas statistical characteristics of scattered radiation by anisotropic irregularities are less studied.

The geomagnetic field affects as the refractive index as well as the structure of ionospheric irregularities. Based on the dependence of the GNSS (Global Navigation Satellite Systems) phase slip on the angle between the satellite-receiver line of sight (LOS) and the magnetic field line, have revealed an anisotropy effect of ionospheric irregularities [6]. Geomagnetic effects produce anisotropy of ionospheric irregularities — their sizes along magnetic field lines exceed transverse sizes. This, in turn, leads to scattering anisotropy effect, i.e., to the dependence of slips in GNSS measurements on satellite-observer LOS orientation relative to magnetic field line. Thus, observation of this dependence can provide valuable information about anisotropy of ionospheric irregularities.

The analysis of the statistical characteristics of small-amplitude electromagnetic waves passing through the turbulent anisotropic plasma is very important in many practical applications associated with both natural and laboratory plasmas [7, 8]. It has been established that the energy loss due to the collisions between plasma particles can lead not only to a decrease in the amplitude of the electromagnetic waves with distance from the slab boundary but also to a significant distortion of the spatial power spectrum (SPS) of radiation in the events of multiple scattering by random smooth inhomogeneities of the medium.

Oblique incidence of a small-amplitude plane electromagnetic wave on a semi-infinite slab of collisional turbulent plasma in an external uniform magnetic field has been considered in [9].

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Compensation effects of oblique incidence and plasma anisotropy on the statistical properties of multiply scattered radiation in the absorbing medium was analyzed in the small-angle scattering approximation using the geometrical optics method. It was shown that the spectrum width changes non-monotonically versus distance from a plasma boundary. Such an asymmetry occurs not only in the case of oblique incidence of electromagnetic waves on a plasma-vacuum interface, it can also be an intrinsic property of the plasma in an external magnetic field.

The strong scattering anisotropy is revealed at multiple scattering of EM waves on large-scale elongated irregularities. In [10] it was shown that for a fixed collision frequency between plasma particles, the degree to which the absorption influences the SPS of the scattered waves depends strongly on the propagation direction of the original incident wave with respect to the plasma boundary and also on the strength of an external magnetic field using the complex geometrical optics approximation. It was found that the width of the SPS of the received radiation for sufficiently strong absorption in the plasma is greater than in the collisionless plasma; the spectral maximum is substantially displaced with respect to the direction to the source.

Broadening of the SPS of scattered EM waves in turbulent collisional magnetized plasma for both power-law and anisotropic Gaussian correlation function of electron density fluctuations was analyzed [11] in the complex ray (-optics) approximation. Some properties of single scattering of waves by random strong anisotropic inhomogeneities using the stationary phase method have been considered in [12]. It was shown that the angle sensitivity of strong anisotropic scattering, unlike weak anisotropic scattering, strongly depends on the longitudinal statistical homogeneity of the medium.

In this paper the discussion is divided into five sections. The first section gives an introduction into the basic concepts that are discussed in the paper. The second section covers formulation of the problem. Generalized dispersion equation is obtained at arbitrary inclination angles of both external magnetic field and wave vector of an incident EM waves in the turbulent collisional magnetized plasma. Set of stochastic differential equations is solved using the modify perturbation method taking into account both electron density and external magnetic field fluctuations. Second-order statistical moments of the phase fluctuations of scattered ordinary waves are calculated in the third section. The forth section is devoted to the numerical calculation of the influence of collision frequency on the broadening of the spatial power spectrum. Evaluation of the phase portraits caused by the external magnetic field fluctuations are constructed for the ordinary EM waves scattered in the collisional magnetized plasma. Numerical calculations are carried out for the ionospheric *F*-region parameters using the experimental data. Finally, the paper is concluded with a review of the results obtained and its significance.

## 2. FORMULATION OF THE PROBLEM

In the vertical sounding of the ionosphere by radio pulses small differences can be important or sometimes necessary when high accuracy is required to allow for the effect of collision on the statistical characteristics of scattered radio waves. Therefore more detailed theoretical investigation of the EM waves in the collisional ionosphere is needed.

At high frequency the effect of ions can be neglected. Maxwell's equations and equation of motion of electron in the collisional magnetized plasma  $ig\omega\mathbf{w} = (e/m)(\mathbf{E} + [\mathbf{w} \cdot \mathbf{H}_0]/c)$  lead to the wave equation for the electric field coinciding with [13] at  $g = 1$

$$\text{grad div } \mathbf{E} - \Delta \mathbf{E} - k_0^2 \mathbf{E} = -\frac{k_0^2 v g}{g^2 - u} \left\{ \mathbf{E} - \frac{i\sqrt{u}}{g} [\mathbf{E} \cdot \mathbf{m}] - \frac{u}{g^2} (\mathbf{E} \cdot \mathbf{m}) \mathbf{m} \right\} \quad (1)$$

It is assumed that the velocity and fields vary as  $\exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$ ,  $\mathbf{w}$  is the velocity of an electron,  $g = 1 - is$ ,  $s = \nu_{\text{eff}}/\omega$ ,  $\nu_{\text{eff}} = \nu_{ei} + \nu_{en}$  is the effective collision frequency of an electron with other plasma particles in which

$$\nu_{ei} = N \left[ 59 + 4.18 \log \left( \frac{T_e^3}{N} \right) \right] \times 10^{-6} T_e^{-3/2} [\text{m.k.s}] \quad \text{and} \quad \nu_{en} = 5.4 \times 10^{-16} N_n T_e^{1/2} [\text{m.k.s}]$$

are the electron-ion and electron-neutral collision frequencies, respectively [14];  $k_0 = \omega/c$ ,  $c$  is a light speed in the free space,  $\omega = 2\pi f$ ,  $f$  is an operating frequency,  $u = \omega_H^2/\omega^2$  and  $v = \omega_p^2/\omega^2$  are non-dimensional magneto-ionic parameters,  $\omega_H = eH_0/mc$  and  $\omega_p = 4\pi Ne^2/m\omega^2$  are the electron

gyrofrequency and plasma frequency, respectively,  $N$  is an electron density,  $e$  and  $m$  are the charge and mass of an electron,  $\mathbf{m} = \mathbf{H}_0/H_0$  is the unite vector along the direction of an external magnetic field locating in the  $YOZ$  plane. Complex refractive index of the collisional magnetized plasma is [7]:

$$N^2 \equiv (n - i\kappa)^2 = 1 - \frac{2v(g - v)}{2g(g - v) - u_T \pm \sqrt{u_T^2 + 4u_L(g - v)^2}}, \quad (2)$$

where:  $u_T = u \sin^2 \alpha$ ,  $u_L = u \cos^2 \alpha$ ,  $\alpha$  is the angle between vectors  $\mathbf{k}$  and  $\mathbf{H}_0$ , upper sign corresponds to the ordinary wave, lower sign to the extraordinary wave;  $n$  is refractive index,  $\kappa$  is an absorption coefficient.

Using the perturbation method each parameter in Equation (1) we submit as sum of the constant mean and small fluctuating terms which are random functions of the spatial coordinates:

$$\mathbf{E} = \langle \mathbf{E} \rangle + \mathbf{e}, \quad \mathbf{H}_0 = \langle \mathbf{H}_0 \rangle + \mathbf{h}_0, \quad N = \langle N \rangle + n. \quad (3)$$

The angular brackets indicate the statistical average. Moreover electron density and magnetic field fluctuations are statistically independent as far as isotropic scalar field not correlated with solenoidal vector field [15]. Substitution (2) into (1) gives linearized set of equations for the mean field:

$$(\text{rot rot} \langle \mathbf{E} \rangle)_i - k_0^2 \sum_{j=1}^3 \varepsilon_{ij} \langle E_j \rangle = 0, \quad (4)$$

where components of the dielectric permittivity for the collisional magnetized plasma are [7]:

$$\begin{aligned} \varepsilon_{xx} &= 1 - \frac{vg}{g^2 - u}, \quad \varepsilon_{yy} = 1 - \frac{v(g^2 - u_T)}{g(g^2 - u)}, \quad \varepsilon_{zz} = 1 - \frac{v(g^2 - u_L)}{g(g^2 - u)}, \\ \varepsilon_{xy} &= -\varepsilon_{yx} = -i \frac{v\sqrt{u_L}}{g^2 - u}, \quad \varepsilon_{xz} = -\varepsilon_{zx} = i \frac{v\sqrt{u_T}}{g^2 - u}, \quad \varepsilon_{yz} = \varepsilon_{zy} = \frac{v\sqrt{u_L u_T}}{g(g^2 - u)}, \end{aligned} \quad (5)$$

Components of the wave vector in homogeneous medium are determined as:

$$k_x = k_0 N \sin \theta \sin \varphi \equiv k_0 \tau_1, \quad k_y = k_0 N \sin \theta \cos \varphi \equiv k_0 \tau_2, \quad k_z = k_0 N \cos \theta \equiv k_0 q, \quad \tau_1^2 + \tau_2^2 + q^2 = N^2, \quad (6)$$

where:  $\theta$  is the angle between wave vector  $\mathbf{k}$  and  $Z$  axis,  $\varphi$  is the angle between projection of vector  $\mathbf{k}$  on the  $XOY$  plane and  $X$  axis.

Substituting (3) and (6) into (1),  $\langle E_j \rangle = E_{0j} \exp[ik_0(\tau_1 x + \tau_2 y + qz)]$  for the mean field in a homogeneous medium we obtain set of equations:

$$\begin{aligned} (-\tau_2^2 - q^2 + \varepsilon_{xx}) \langle E_x \rangle + (\tau_1 \tau_2 + \varepsilon_{xy}) \langle E_y \rangle + (\tau_1 q + \varepsilon_{xz}) \langle E_z \rangle &= 0, \\ (\tau_1 \tau_2 + \varepsilon_{yx}) \langle E_x \rangle + (-\tau_1^2 - q^2 + \varepsilon_{yy}) \langle E_y \rangle + (\tau_2 q + \varepsilon_{yz}) \langle E_z \rangle &= 0 \\ (\tau_1 q + \varepsilon_{zx}) \langle E_x \rangle + (\tau_2 q + \varepsilon_{zy}) \langle E_y \rangle + (-\tau_1^2 - \tau_2^2 + \varepsilon_{zz}) \langle E_z \rangle &= 0, \end{aligned} \quad (7)$$

the solution of which in general case, at  $\theta \neq 0$ ,  $\varphi \neq 0$  and  $\alpha \neq 0$  leads to the dispersion relation

$$\Delta \equiv a_4 q^4 + a_3 q^3 + a_2 q^2 + a_1 q + a_0 = 0, \quad (8)$$

here:  $a_4 = \varepsilon_{zz}$ ,  $a_3 = 2\tau_2 \varepsilon_{yz}$ ,  $a_2 = \varepsilon_{yz}^2 - \varepsilon_{xz}^2 - \varepsilon_{zz}(\varepsilon_{xx} + \varepsilon_{yy}) + \tau_1^2(\varepsilon_{xx} + \varepsilon_{zz}) + \tau_2^2(\varepsilon_{yy} + \varepsilon_{zz})$ ,  $a_1 = 2\tau_2[\varepsilon_{yz}(\tau_1^2 + \tau_2^2) - \varepsilon_{xx}\varepsilon_{yz} - \varepsilon_{xy}\varepsilon_{xz}]$ ,  $a_0 = (\tau_1^2 + \tau_2^2 - \varepsilon_{zz})(\tau_1^2 \varepsilon_{xx} + \tau_2^2 \varepsilon_{yy} - \varepsilon_{xx}\varepsilon_{yy} - \varepsilon_{xy}^2) + \varepsilon_{yz}^2(\tau_2^2 - \varepsilon_{xx}) - 2\varepsilon_{xy}\varepsilon_{xz}\varepsilon_{yz} + \varepsilon_{xz}^2(\varepsilon_{yy} - \tau_1^2)$ .

If  $\theta = 0$  and  $\varphi = 0$  (EM wave propagates along  $Z$ -axis) we obtain biquadratic equation for the determinant  $\Delta_0$  having four poles:

$$\begin{aligned} q_1^2 &= \left[ (\eta' + \mu'') - \frac{1}{2\varepsilon'} (\varepsilon' + \eta' + \mu'') \sin^2 \theta_0 \right] - i \frac{s}{2\varepsilon'^2} \{ (\tilde{\eta}'' - \tilde{\mu}') \\ &\quad + [\tilde{\varepsilon}''(\eta' + \mu'') - 2\varepsilon'(\tilde{\eta}'' - \tilde{\mu}')] \sin^2 \theta_0 \} \equiv a - isb, \\ q_2^2 &= \left[ (\eta' - \mu'') - \frac{1}{2\varepsilon'} (\varepsilon' + \eta' - \mu'') \sin^2 \theta_0 \right] - i \frac{s}{2\varepsilon'^2} \{ (\tilde{\eta}'' + \tilde{\mu}') \\ &\quad + [\tilde{\varepsilon}''(\eta' - \mu'') - 2\varepsilon'(\tilde{\eta}'' + \tilde{\mu}')] \sin^2 \theta_0 \} \equiv c - isd, \end{aligned} \quad (9)$$

where  $\theta_0$  is an incidence angle on a plasma slab; components of dielectric permittivity tensor for the polar ionospheric collisional magnetized plasma ( $\alpha = 0^\circ$ ) at  $(1 \pm u_0) \gg s^2$  have the following form:

$$\begin{aligned} \varepsilon_{xx} = \varepsilon_{yy} = \eta' - is\tilde{\eta}'', \quad \varepsilon_{zz} = \varepsilon' - is\tilde{\varepsilon}'', \quad \varepsilon_{xy} = s\tilde{\mu}' - i\mu'', \quad \eta' = 1 - \frac{v_0}{1 - u_0}, \\ \tilde{\eta}'' = \frac{v_0(1 + u_0)}{(1 - u_0)^2}, \quad \tilde{\mu}' = \frac{2v_0\sqrt{u_0}}{(1 - u_0)^2}, \quad \mu'' = \frac{v_0\sqrt{u_0}}{1 - u_0}, \quad \varepsilon' = 1 - v_0, \quad \tilde{\varepsilon}'' = v_0. \end{aligned} \quad (10)$$

If EM wave propagates along the  $Z$ -axis and the external magnetic field is located in the  $YOZ$ -plane (principle plane), polarization coefficients for the collisional magnetized plasma are [7]:

$$\frac{\langle E_y \rangle_{1,2}}{\langle E_x \rangle_{1,2}} = K_{1,2} = -i \frac{2\sqrt{u_L}(g - v)}{u_T \mp \sqrt{u_T^2 + 4u_L}(g - v)^2}, \quad \frac{\langle E_z \rangle_{1,2}}{\langle E_x \rangle_{1,2}} = -i \frac{\sqrt{u_T}gv + K_{1,2}\sqrt{u_L}v \sin \alpha}{gu - g^2(g - v) - u_L v}. \quad (11)$$

Upper sign (index 1) corresponds to the ordinary wave; the lower sign (index 2) to the extraordinary wave;  $K_1 K_2 = 1$ ,  $|K_{1,2}|$  is the ratio of the ellipse semi-axis. In the polar ionosphere ( $\alpha = 0^\circ$ ) the waves are circularly polarized:  $(\langle E_y \rangle / \langle E_x \rangle) = \pm 1$ ,  $(\langle E_z \rangle / \langle E_x \rangle) = 0$ .

Fluctuating field of scattered EM waves satisfies set of stochastic differential equations taking into account both electron density and external magnetic field fluctuations

$$\left( \frac{\partial^2}{\partial x_i \partial x_j} - \Delta \delta_{ij} - k_0^2 \varepsilon_{ij} \right) e_j = j_i, \quad (12)$$

where current density is

$$\begin{aligned} \mathbf{j}(\mathbf{r}) = & -\frac{k_0^2 v_0 g}{g^2 - u_0} \left\{ -i \frac{\sqrt{u_0}}{g} [\langle \mathbf{E} \rangle \mathbf{h}'_0] - \frac{u_0}{g} (\langle \mathbf{E} \rangle \mathbf{m}) \mathbf{h}'_0 - \frac{u_0}{g^2} (\langle \mathbf{E} \rangle \mathbf{h}'_0) \mathbf{m} \right\} \\ & - \frac{k_0^2 v_0 g}{g^2 - u_0} \left\{ n' + \frac{2u_0}{g^2 - u_0} (\mathbf{m} \mathbf{h}'_0) \right\} \left\{ \langle \mathbf{E} \rangle - i \frac{\sqrt{u_0}}{g} [\langle \mathbf{E} \rangle \mathbf{m}] - \frac{u_0}{g^2} (\langle \mathbf{E} \rangle \mathbf{m}) \mathbf{m} \right\}, \end{aligned}$$

$\mathbf{h}'_0 = \mathbf{h}_0 / |\langle \mathbf{H}_0 \rangle|$  is the unit vector of fluctuating external magnetic field,  $n' = n / \langle N \rangle$  is the electron density fluctuation. Both are random functions of the spatial coordinates,  $v_0 = \omega_{p0}^2 / \omega^2$ ,  $\omega_{p0}^2 = 4\pi e^2 \langle N \rangle / m \omega^2$ ,  $u_0 = (e \langle H_0 \rangle / mc \omega)^2$ . Index “zero” indicates the mean values of non-dimensional magneto-ionic plasma parameters.

We will use Fourier integral over  $x$  and  $y$  coordinates:

$$\mathbf{e}(\mathbf{r}) = \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \tilde{\mathbf{e}}(k_x, k_y, z) \exp[i(k_x x + k_y y)], \quad \mathbf{j}(\mathbf{r}) = \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \tilde{\mathbf{g}}(k_x, k_y, z) \exp[i(k_x x + k_y y)]$$

The boundary conditions should be added to these equations. Fluctuations are negligible below and above plasma layer. If the plane  $XOY$  coincides with the lower boundary of slab, at  $z \geq L$  the EM waves propagating in the negative direction must be absent, and at  $z \leq 0$  — in the positive direction. Since all functions are finite in a turbulent slab,  $0 \leq z \leq L$ , we solve the set of Equation (12) via the spectral method [13]:

$$\begin{aligned} \tilde{e}_x(z) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} dq A(q) \exp[-i(L - z)k_0 q], \quad \tilde{e}_y(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dq B(q) \exp[-i(L - z)k_0 q], \\ \tilde{e}_z(z) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} dq C(q) \exp[-i(L - z)k_0 q], \quad \tilde{g}_x(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dq F_1(q) \exp[-i(L - z)k_0 q], \\ \tilde{g}_y(z) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} dq F_2(q) \exp[-i(L - z)k_0 q], \quad \tilde{g}_z(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dq F_3(q) \exp[-i(L - z)k_0 q]. \end{aligned} \quad (13)$$

Boundary conditions also include Snell's law:  $\tau_1^2 + \tau_2^2 = N^2 \sin^2 \theta_0$  [7]; at the beginning of a slab ( $z = 0$ )  $\theta = \theta_0$ ,  $N = 1$ , and hence  $\tau_1^2 + \tau_2^2 \approx \sin^2 \theta_0$ .

Fourier components of the fluctuating field for arbitrary angle  $\alpha$  (vector  $\mathbf{m}$  lies in the  $YOZ$  plane) satisfy set of differential equations:

$$\begin{aligned} \frac{\partial^2 \tilde{e}_x}{\partial z^2} - ik_0 \tau_1 \frac{\partial \tilde{e}_z}{\partial z} + k_0^2 (\varepsilon_{xx} - \tau_2^2) \tilde{e}_x + k_0^2 (\varepsilon_{xy} + \tau_1 \tau_2) \tilde{e}_y + k_0^2 \varepsilon_{xz} \tilde{e}_z &= -\tilde{g}_x \\ \frac{\partial^2 \tilde{e}_y}{\partial z^2} - ik_0 \tau_2 \frac{\partial \tilde{e}_z}{\partial z} + k_0^2 (-\varepsilon_{xy} + \tau_1 \tau_2) \tilde{e}_x + k_0^2 (\varepsilon_{xx} - \tau_1^2) \tilde{e}_y + k_0^2 \varepsilon_{yz} \tilde{e}_z &= -\tilde{g}_y \\ ik_0 \tau_1 \frac{\partial \tilde{e}_x}{\partial z} + ik_0 \tau_2 \frac{\partial \tilde{e}_y}{\partial z} + k_0^2 \varepsilon_{xz} \tilde{e}_x - k_0^2 \varepsilon_{yz} \tilde{e}_y + k_0^2 (\tau_1^2 + \tau_2^2 - \varepsilon_{zz}) \tilde{e}_z &= \tilde{g}_z, \end{aligned} \quad (14)$$

Particularly, for the  $Y$ -component and  $\alpha \neq 0$  in the collisional magnetized ionospheric plasma we have:

$$\begin{aligned} \tilde{g}_y(\mathbf{a}, z) = \frac{v_0}{1 - u_0} k_0^2 \langle E_x \rangle \left\{ (-iB_1 + sB_3)n' + (B_5 + isB_8)h'_{0x} + [-i(B_6 + B_2 \sin \alpha) + s(B_9 + B_4 \sin \alpha)]h'_{0y} \right. \\ \left. + [-i(B_7 + B_2 \cos \alpha) + s(B_{10} + B_4 \cos \alpha)]h'_{0z} \right\}, \end{aligned} \quad (15)$$

where:  $B_1 = -R_y + \sqrt{u_0} \cos \alpha + u_0 R_2 \sin \alpha$ ,  $B_2 = 2u_0(-R_y + \sqrt{u_0} \cos \alpha + u_0 R_2 \sin \alpha)/(1 - u_0)$ ,  $B_3 = [-(1 + u_0)R_y + 2\sqrt{u_0} \cos \alpha + u_0(3 - u_0)R_2 \sin \alpha]/(1 - u_0)$ ,  $B_5 = \sqrt{u_0}R_z + u_0 \sin \alpha$ ,  $B_4 = 2u_0[-(3 + u_0)R_y + 4\sqrt{u_0} \cos \alpha + u_0(5 - u_0)R_2 \sin \alpha]/(1 - u_0)^2$ ,  $B_6 = u_0(R_2 + R_y \sin \alpha)$ ,  $B_7 = \sqrt{u_0} + u_0 R_z \sin \alpha$ ,  $B_8 = [2\sqrt{u_0}R_z + u_0(3 - u_0) \sin \alpha]/(1 - u_0)$ ,  $B_9 = u_0(3 - u_0)(R_2 + R_y \sin \alpha)/(1 - u_0)$ ,  $B_{10} = [2\sqrt{u_0} + u_0(3 - u_0)R_z \sin \alpha]/(1 - u_0)$ ,  $R_1 = R_y \cos \alpha - R_z \sin \alpha$ ,  $R_2 = R_y \sin \alpha + R_z \cos \alpha$ ,  $\langle E_y \rangle / \langle E_x \rangle = -iR_y \langle E_z \rangle / \langle E_x \rangle = -iR_z$  are the polarization coefficients determined by (11),  $n' \equiv n'(\mathbf{a}, z)$ ,  $h'_{0i} \equiv h'_{0i}(\mathbf{a}, z)$ ,  $\mathbf{a} = \{k_x, k_y\}$ .

In the polar ionosphere, at  $\alpha = 0^\circ$ , formula (15) simplifies:

$$\tilde{g}_y = \mp ik_0 \langle E_x \rangle [(D_1 + isD_3)n' - (D_2 + isD_4)h'_{0z}], \quad (16)$$

$$\tilde{g}_z(\mathbf{a}, z) = k_0^2 \langle E_x \rangle (F_1 + isF_2) [h'_{0x}(\mathbf{a}, z) \pm ih'_{0y}(\mathbf{a}, z)], \quad (17)$$

$\tilde{g}_y(\mathbf{a}, z) = \pm i\tilde{g}_x(\mathbf{a}, z)$ ; for the collisionless magnetized plasma it coincides with [13], where:

$$D_1 = \frac{v_0}{1 - u_0} (1 \pm \sqrt{u_0}), \quad D_2 = -\frac{v_0}{1 - u_0} \left[ \pm \sqrt{u_0} + \frac{2u_0(1 \pm \sqrt{u_0})}{1 - u_0} \right], \quad D_3 = \frac{v_0}{(1 - u_0)^2} (1 \pm \sqrt{u_0})^2,$$

$$D_4 = -\frac{2v_0}{(1 - u_0)^2} \left[ \pm \sqrt{u_0} + \frac{u_0(3 + u_0 \pm 4\sqrt{u_0})}{1 - u_0} \right], \quad F_1 = \frac{v_0}{1 - u_0} (u_0 \pm \sqrt{u_0}),$$

$$F_2 = \frac{v_0}{(1 - u_0)^2} [u_0(3 - u_0) \pm 2\sqrt{u_0}].$$

Hence, current density fluctuations in the  $XOY$  plane are caused by both electron density and external magnetic field fluctuations, while the  $Z$  component contains only magnetic field fluctuations.

In the case of  $\theta \neq 0$ ,  $\varphi \neq 0$  and  $\alpha = 0^\circ$ , at  $(1 \pm u_0) \gg s^2$  coefficients of Equation (13) are determined as:

$$\begin{aligned} A(q) = -\frac{1}{k_0^2 \Delta_0} \left\{ [(P_1 q^2 + P_2) + is(Q_0 q^2 + Q_1)] F_1(q) + [(P_3 q^2 + P'_4 + iP''_4) + s(Q'_2 + iQ''_2)] F_2(q) \right. \\ \left. + [P_5 q^3 + (P'_6 + iP''_6)q + s(Q'_3 + iQ''_3)] F_3(x) \right\}, \end{aligned}$$

$$\begin{aligned} B(q) = -\frac{1}{k_0^2 \Delta_0} \left\{ [(P_3 q^2 + T'_2 + iT''_2) + s(Q'_4 + iQ''_4)] F_1(q) + [(T_3 q^2 + T_4) + is(Q_5 q^2 + Q_6)] F_2(q) \right. \\ \left. + [T_5 q^3 + (T'_6 + iT''_6)q + s(Q'_7 + iQ''_7)] F_3(x) \right\}, \end{aligned}$$

$$C(q) = -\frac{1}{k_0^2 \Delta_0} \left\{ [(P_5 q^3 + (P'_6 - iP''_6)q + s(-Q'_3 + iQ''_3)q] F_1(q) + [T_5 q^3 + (T'_6 - iT''_6)q + s(-Q'_7 + iQ''_7)q] F_2(q) + [(q^4 + B_5 q^2 + B_6) + is(D'_3 q^2 - D'_4)] F_3(q) \right\}, \quad (18)$$

here:  $P_1 = \tau_1^2 - \varepsilon'$ ,  $P_2 = (\varepsilon' - \sin^2 \theta_0)(\eta' - \tau_1^2)$ ,  $Q_0 = \tilde{\varepsilon}''$ ,  $Q_1 = \tilde{\varepsilon}''(\tau_1^2 - \eta') - \tilde{\eta}''(\varepsilon' - \sin^2 \theta_0)$ ,  $P_3 = \tau_1 \tau_2$ ,  $P'_4 = -\tau_1 \tau_2(\varepsilon' - \sin^2 \theta_0)$ ,  $P''_4 = \mu''(\varepsilon' - \sin^2 \theta_0)$ ,  $Q'_2 = \mu'' \tilde{\varepsilon}'' - \tilde{\mu}'(\varepsilon' - \sin^2 \theta_0)$ ,  $Q''_2 = \tau_1 \tau_2 \tilde{\varepsilon}''$ ,  $P_5 = \tau_1$ ,  $P'_6 = -\tau_1(\eta' - \sin^2 \theta_0)$ ,  $P''_6 = -\tau_2 \mu''$ ,  $Q'_3 = \tau_2 \tilde{\mu}'$ ,  $Q''_3 = \tau_1 \tilde{\eta}''$ ,  $T'_2 = \tau_1 \tau_2$ ,  $T''_2 = \mu''(-\varepsilon' + \sin^2 \theta_0)$ ,  $Q'_4 = \tilde{\mu}'(\varepsilon' - \sin^2 \theta_0) - \mu'' \tilde{\varepsilon}''$ ,  $Q''_4 = \tau_1 \tau_2 \tilde{\varepsilon}''$ ,  $T_3 = \tau_2^2 - \varepsilon'$ ,  $T_4 = \eta' \varepsilon' - \eta' \tau_1^2 - (\varepsilon' + \eta') \tau_2^2$ ,  $T_5 = \tilde{\varepsilon}''$ ,  $Q_6 = -(\eta' \tilde{\varepsilon}'' + \varepsilon' \tilde{\eta}'') + \tilde{\eta}'' \tau_1^2 + (\tilde{\eta}'' + \tilde{\varepsilon}'') \tau_2^2$ ,  $T_5 = \tau_2$ ,  $T'_6 = -\eta' \tau_2$ ,  $T''_6 = \tau_1 \mu''$ ,  $Q'_7 = -\tau_1 \tilde{\mu}'$ ,  $Q''_7 = \tau_2 \tilde{\eta}''$ ,  $B_5 = \sin^2 \theta_0 - 2\eta'$ ,  $B_6 = \eta'(\eta' + \sin^2 \theta_0) - \mu''^2$ ,  $D'_3 = 2\tilde{\eta}''$ ,  $D'_4 = B_5 \tilde{\eta}'' - 2\tilde{\mu}' \mu''$ .

Substituting (18) into (13) and taking into account (9), Fourier integrals should be calculated using the residue theory.

### 3. SECOND ORDER STATISTICAL MOMENTS OF SCATTERED RADIATION

In this section we will investigate second order statistical moments of the ordinary EM waves scattered in the turbulent collision magnetized plasma caused by electron density fluctuations if waves propagates along the external magnetic field.

Substituting (14)–(17) into (13), taking into account that phase fluctuation  $\varphi_1$  is determined by the expression  $\varphi_1 = \text{Im}(e/\langle E_x \rangle)$ , applying the residue theory, for the  $Y$  component of scattered radiation and arbitrary correlation function of the phased fluctuations  $W_\varphi(\mathbf{r}_1, \mathbf{r}_2) = \langle \varphi_1(\mathbf{r}_1) \varphi_1^*(\mathbf{r}_2) \rangle$  at the observation points  $\mathbf{r}_1$  and  $\mathbf{r}_2$  caused by electron density fluctuations has the following form:

$$\begin{aligned} \langle \varphi_1(x + \rho_x, y + \rho_y, L) \varphi_1^*(x, y, L) \rangle_{YD} = & -4D_1 k_0^2 L \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \int_{-\infty}^{\infty} d\rho_z W_D(k_x, k_y, \rho_z) \\ & \exp(ik_x \rho_x + ik_y \rho_y) \left\{ \frac{2t_0}{\delta_1^2} (G'_1 + s \cdot G''_1) \left[ \frac{\sin(2k_{01}L)}{2k_{01}L} \text{ch}(s \cdot r_1 k_{01} \rho_z) - \cos(k_{01} \rho_z) \right] \right. \\ & + \frac{2t'_0}{\delta_1^2} (G'_2 + s \cdot G''_2) \left[ \frac{\sin(2k_{02}L)}{2k_{02}L} \text{ch}(s \cdot r_2 k_{02} \rho_z) - \cos(k_{02} \rho_z) \right] \\ & - \frac{4}{\delta_1 \delta_2} (G'_3 + s \cdot G''_3) \left[ \frac{\sin(p_2 k_0 L)}{p_2 k_0 L} \cos\left(\frac{p_1}{2} k_0 \rho_z\right) \text{ch}\left(s \cdot \frac{q_1}{2} k_0 \rho_z\right) - (J_1 + s \cdot J_2) \cos\left(\frac{p_2}{2} k_0 \rho_z\right) \right. \\ & \left. \left. \cdot \text{ch}\left(s \cdot \frac{q_2}{2} k_0 \rho_z\right) + (J_3 - s \cdot J_4) \sin\left(\frac{p_2}{2} k_0 \rho_z\right) \text{sh}\left(s \cdot \frac{q_2}{2} k_0 \rho_z\right) \right] \right\}, \quad (19) \end{aligned}$$

where observation points are spaced apart at a very small distances  $\rho_x$  and  $\rho_y$  perpendicular to the principle plane,  $W_n(\mathbf{k})$  is the arbitrary correlation function of the electron density fluctuations,  $G'_1 = D_1(t_0 + 2t_1 \tau_1^2 + 2t_2 \tau_2^2)$ ,  $G''_1 = 2D_3 h \tau_1 \tau_2$ ,  $G'_2 = D_1(t'_0 + 2t'_1 \tau_1^2 + 2t'_2 \tau_2^2)$ ,  $G''_2 = 2D_3 h' \tau_1 \tau_2$ ,  $G'_3 = D_1[t_0(t'_0 + t'_1 \tau_1^2 + t'_2 \tau_2^2) + t'_0(t_1 \tau_1^2 + t_2 \tau_2^2)]$ ,  $G''_3 = D_3(ht'_0 + h't_0) \tau_1 \tau_2$ ,  $t_0 = -2\mu'' \varepsilon'$ ,  $t_1 = (\varepsilon' - \eta' + 3\mu'')/2$ ,  $t_2 = (-\varepsilon' + \eta' + 5\mu'')/2$ ,  $h = \eta' + \mu'' - \varepsilon'$ ; stroke terms  $t'_{0,1,2}$  and  $h'$  are obtained using replacements:  $\mu'' \rightarrow -\mu''$ ,  $\tilde{\mu}' \rightarrow -\tilde{\mu}'$ ;  $r_1 = b_1/2a_1$ ,  $r_2 = d_1/2c_1$ ,  $k_{01} = k_0 \sqrt{a_1}$ ,  $k_{02} = k_0 \sqrt{c_1}$ ,  $p_{1,2} = p'_{1,2} - p''_{1,2}$ ,  $q_{1,2} = q'_{1,2} + q''_{1,2} \sin^2 \theta_0$ ,  $p'_{1,2} = \sqrt{a_1} \pm \sqrt{c_1}$ ,  $q'_{1,2} = (d_1/2 \sqrt{c_1}) \mp (b_1/2 \sqrt{a_1})$ ,  $p''_{1,2} = (a_2/2 \sqrt{a_1}) \pm (c_2/2 \sqrt{c_1})$ ,  $q''_{1,2} = (d_2/2 \sqrt{c_1}) \mp (b_2/2 \sqrt{a_1})$ ,  $a_1 = \eta' + \mu''$ ,  $a_2 = (\varepsilon' + \eta' + \mu''/2\varepsilon')$ ,  $b_1 = (\tilde{\eta}'' - \tilde{\mu}'/2\varepsilon'^2)$ ,  $b_2 = [\tilde{\varepsilon}''(\eta' + \mu'') - 2\varepsilon'(\tilde{\eta}'' - \tilde{\mu}')] / 2\varepsilon'^2$ ,  $c_1 = \eta' - \mu''$ ,  $c_2 = (\varepsilon' + \eta' - \mu''/2\varepsilon')$ ,  $d_1 = (\tilde{\eta}'' + \tilde{\mu}')/2\varepsilon'^2$ ,  $d_2 = [\tilde{\varepsilon}''(\eta' - \mu'') - 2\varepsilon'(\tilde{\eta}'' + \tilde{\mu}')] / 2\varepsilon'^2$ ,  $J_1 = \frac{\sin(p_1 k_0 L)}{p_1 k_0 L} \text{ch}(s \cdot q_1 k_0 L)$ ,  $J_2 = \frac{q_1}{p_1} \frac{\cos(p_1 k_0 L)}{p_1 k_0 L} \text{sh}(s \cdot q_1 k_0 L)$ ,  $J_3 = \frac{\cos(p_1 k_0 L)}{p_1 k_0 L} \text{sh}(s \cdot q_1 k_0 L)$ ,  $J_4 = \frac{q_1}{p_1} \frac{\sin(p_1 k_0 L)}{p_1 k_0 L} \text{ch}(s \cdot q_1 k_0 L)$ ;  $\delta_1 = 4\mu'' \sqrt{\eta' + \mu''}$ ,  $\delta_2 = -4\mu'' \sqrt{\eta' - \mu''}$ .

This expression is applicable for the near ( $R \ll 1$ ) and far ( $R \gg 1$ ) zones from a plasma boundary,  $R = L/k_0 l_{\parallel}^2$  is the wavy parameter. Double integral in the wave number space depends only on the shape of the fluctuation spectrum but not on the strength of the fluctuations.

For statistically isotropic and homogeneous random magnetic field we have [16]:

$$V_{ij}(\rho) \equiv \langle h'_{0i}(\mathbf{r}_1) h'_{0j}(\mathbf{r}_2) \rangle = \frac{\sigma_{h0}^2}{12} \left( \frac{\partial^2}{\partial \rho_i \partial \rho_j} - \delta_{ij} \frac{\partial^2}{\partial \rho_m \partial \rho_m} \right) \Phi(\rho), \quad (20)$$

where:  $\rho = |\mathbf{r}_1 - \mathbf{r}_2|$ ,  $\sigma_{h0}^2$  is the variance of the fluctuating field,  $\Phi(\rho)$  is the arbitrary scalar function having characteristic spatial scale  $L_0$ . Random magnetic field within one irregularity (linear scale  $L_0$ ) displaces plasma particles on small angle. At the same time the displacement of particles by regular field  $\mathbf{H}_0$  on the same distance maybe not small.

Correlation function of the phase fluctuation of scattered ordinary EM wave in the collisional magnetized plasma caused by the external magnetic field fluctuations in a the zero-order approximation is

$$\begin{aligned} & V(\rho_x, \rho_y, L) \\ &= 2k_0^2 D_2^2 M_0 \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \int_{-\infty}^{\infty} d\rho_z V_{zz}(k_x, k_y, \rho_z) \left( \frac{t_0^2}{\delta_1^2} \left[ \frac{\sin(2k_{01}L)}{2k_{01}L} \text{ch}(s \cdot \alpha_1 k_{01} \rho_z) - \cos(k_{01} \rho_z) \right] \right. \\ &+ \frac{t_0'^2}{\delta_1^2} \left[ \frac{\sin(2k_{02}L)}{2k_{02}L} \text{ch}(s \cdot \alpha_2 k_{02} \rho_z) - \cos(k_{02} \rho_z) \right] + 4 \frac{t_0 t_0'}{\delta_1 \delta_2} \left\{ \frac{\sin(p_1 k_0 L)}{p_1 k_0 L} \cos\left(\frac{p_2}{2} k_0 L\right) \text{ch}\left(s \cdot \frac{q_2}{2} k_0 \rho_z\right) \right. \\ &+ \left[ \frac{\cos(p_2 k_0 L)}{p_2 k_0 L} \sin\left(\frac{p_1}{2} k_0 L\right) - \frac{\sin(p_2 k_0 L)}{p_2 k_0 L} \cos\left(\frac{p_1}{2} k_0 L\right) \right] \cdot \text{ch}\left(s \cdot \frac{q_2}{2} k_0 \rho_z\right) + s \left[ -\frac{q_1}{p_1} k_0 L \cos(p_1 k_0 L) \right. \\ &\cdot \sin\left(\frac{p_2}{2} k_0 \rho_z\right) \text{sh}\left(s \cdot \frac{q_1}{2} k_0 \rho_z\right) + \frac{q_1}{p_1} \sin(p_1 k_0 L) \sin\left(\frac{p_2}{2} k_0 \rho_z\right) \text{sh}\left(s \cdot \frac{q_2}{2} k_0 \rho_z\right) + \frac{q_1}{p_1} \cos n\left(\frac{p_1}{2} k_0 \rho_z\right) \\ &\cdot \left. \left. \left. [\cos(p_2 k_0 L) - 1] \text{sh}\left(s \cdot \frac{q_1}{2} k_0 \rho_z\right) \right] \right\} \right). \end{aligned} \quad (21)$$

It contains only ZZ component, while in the first and second approximations cross-correlation functions should be taken into account. We will not write theirs out having quite bulky forms.

#### 4. NUMERICAL CALCULATIONS

Currently polar ionosphere is of great interest [8, 17]. This is connected with creation of global model of ionospheric storms and providing stable navigation in polar region. In some cases a strong positive correlation between the phase acceleration variations and TEC variations in the variation frequency band of 0.08–1 Hz. It is well known that this frequency band relates to the 1-st Fresnel zone sized ionospheric irregularities. A clear positive correlation between the increase in phase acceleration and the growth of TEC variation spectral components of 0.08–0.6 Hz exists [18]. This frequency range corresponds to the scope of small-scale ionospheric irregularities with sizes from hundred meters to several kilometers.

Small-scale ionospheric irregularities are primarily responsible for the occurrence of rapid variations of amplitude and phase confirming a possible link between non-stationary phase changes and small-amplitude irregularities in the ionosphere.

Let us consider some results of simulation, where the obtained formulas have been used. Analytical and numerical calculations are out for the anisotropic Gaussian correlation function of electron density fluctuation [11, 19]:

$$W_D(k_x, k_y, \rho_z) = \frac{\sigma_D^2}{4\pi} \frac{l_{\parallel}^2}{\chi \Gamma_0} \exp\left(-\frac{m^2}{l_{\parallel}^2} \rho_z^2 + i n k_x \rho_z\right) \exp\left(-\frac{k_x^2 l_{\parallel}^2}{4\Gamma_0^2} - \frac{k_y^2 l_{\parallel}^2}{4\chi^2}\right), \quad (22)$$

where:  $m^2 = \chi^2/\Gamma_0^2$ ,  $\Gamma_0^2 = \sin^2 \gamma_0 + \chi^2 \cos^2 \gamma_0$ ,  $n = (\chi^2 - 1) \sin \gamma_0 \cos \gamma_0 / \Gamma_0^2$ ,  $\sigma_D^2$  is the variance of the density fluctuations. The average shape of electron density irregularities has the form of elongate ellipsoid of rotation. The rotation axis is located in the plane of geomagnetic meridian. The ellipsoid is characterized with two parameters: the anisotropy factor for the irregularities equaling to the ratio of ellipsoid axes  $\chi = l_{\parallel}/l_{\perp}$  (ratio of longitudinal and transverse sizes of plasma irregularities with respect to the external magnetic field) and orientation characterizes by the inclination angle  $\gamma_0$  of the prolate

irregularities with respect to the lines of forces of an external magnetic field (sometimes with respect to horizon). Anisotropy of the shape of irregularities is connected with the difference of diffusion coefficients in the field align and field perpendicular directions.

For statistically isotropic and homogeneous random magnetic field  $\mathbf{H}_0$  Gaussian correlation function [18]  $\Phi(\rho) = L_0^2 \exp(-\rho^2/L_0^2)$  having characteristic linear scale  $L_0$  formula (20) can be rewritten as:

$$V_{ij}(\rho) = \frac{\sigma_{h0}^2}{3} \left[ \left(1 - \frac{\rho^2}{L_0^2}\right) \delta_{ij} + \frac{\rho_i \rho_j}{L_0^2} \right] \exp(-\rho^2/L_0^2) \quad (23)$$

and its spectral representation is

$$V_{ij}(\mathbf{k}) = M_0 (k^2 \delta_{ij} - k_i k_j) \exp\left(-\frac{k^2 L_0^2}{4}\right), \quad (24)$$

where  $M_0 \equiv \sigma_{h0}^2 \pi^{3/2} L_0^5 / 12$ ,  $\sigma_{h0}^2$  is the variance of an external magnetic field fluctuations.

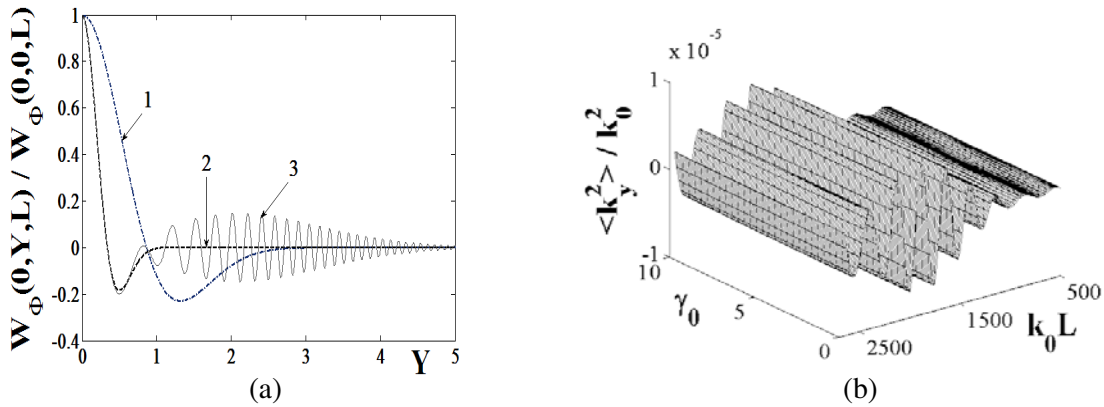
Numerical calculations are carried out for an incident wave 3 MHz ( $k_0 = 6.28 \cdot 10^{-2}$  m) using the following dimensionless parameters:  $v_0 = 0.28$ ,  $u_0 = 0.22$ . The effective collision frequency of electron with other plasma particles in the ionospheric  $F$ -region (at altitudes 160–340 km) is of the order of  $\sim 10^{-2}$  and hence  $s \sim 10^{-4}$  for 3 MHz operating frequency. Variance of electron density fluctuations is in the interval  $\sigma_D^2 \sim 10^{-4} - 10^{-6}$  [20].

It has been established that small irregularities ( $\leq 10$  km) in  $F$ -region are highly elongated along the direction of the magnetic field and are strongly anisotropic. On the other hand, large irregularities ( $\geq 10$  km) are weakly anisotropic [21].

Spaced receiver measurements made at Kingston, Jamaica, show [22] that the irregularities causing the scintillation of signals from earth satellites are closely aligned along the magnetic field in the  $F$ -region, between heights of 153 and 617 km with an average height of 362 km. The signals observed were the 41-MHz transmissions from the moving satellites BE-B and BE-C. The orientation of the irregularities in the ionosphere was measured with respect to the geographic north. The dip angle of the irregularities in the ionosphere was in the interval  $0^\circ \div 18^\circ$  mainly within  $16^\circ$  of dip.

Statistical characteristics of scattered radiation are calculated at  $k_0 L \gg 1$  using the stationary phase method [23]. Substituting (22) into (19) in nonmagnetic ( $u_0 = 0$ ) isotropic ( $\chi = 1$ ) collisionless plasma ( $s = 0$ ) at  $\rho_x = \rho_y = 0$  we obtain the well-known formula for the variance of the phase fluctuations  $W_{\varphi D} = \sqrt{\pi} \sigma_D^2 v_0^2 k_0^2 L l / 4$  [8].

Figure 1 depicts the dependence of the normalized correlation function of the phase fluctuations of scattered ordinary wave versus nondimensional space parameter  $\rho_y / l_{\parallel}$  in the turbulent collisional magnetized plasma at  $s = 10^{-4}$ ,  $k_0 L = 2 \cdot 10^3$  (corresponding thickness of plasma slab 32 km). Angle of

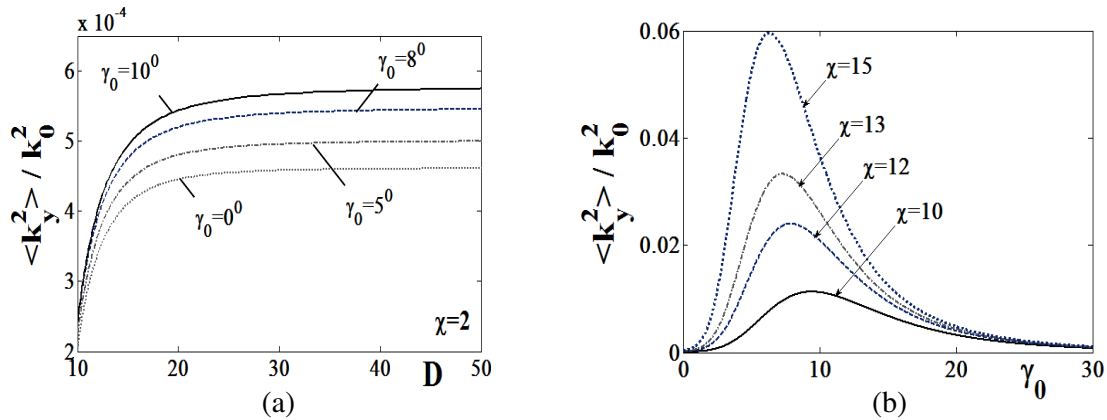


**Figure 1.** (a) Depicts correlation function of the phase fluctuations of scattered ordinary wave in the collisional magnetized plasma with electron density fluctuations as a function of distance between observation points. 3D picture of the broadening of the SPS versus angle of inclination and thickness of plasma slab is presented on (b).



inclination of prolate irregularities is equal  $\gamma_0 = 10^\circ$ ,  $l_{\parallel} = 3$  km. If  $l_{\parallel}/L = 0.01$ , in the case of isotropic irregularities ( $\chi = 1$ ) increasing anisotropic factor up to  $\chi = 5$ , correlation function fast dumping. Decreasing thickness of a slab  $l_{\parallel}/L = 0.1$  correlation function of the phase fluctuation oscillates from  $Y = 1$  up to  $Y = 5$  solely due to the collision of electrons with other plasma particles; stationary interval of oscillations exists in the interval  $Y = 1.5$ – $2.5$ . Simulations show that oscillations increase in proportion to the parameter  $l_{\parallel}/L$ . Varying thickness of plasma slab and inclination angle  $\gamma_0$ , broadening of the SPS in the collision magnetized plasma oscillates (Figure 1(b)).

In Figure 2 the dependences of the SPS broadening are shown as a function of nondimensional spatial parameter  $D = L/l_{\parallel}$  in the principle plane for different inclination angle  $\gamma_0 = 0^\circ$ – $10^\circ$  of prolate electron density irregularities with respect to the external magnetic field, at fixed anisotropy factor  $\chi = 2$  (Figure 2(a)) and as a function of the inclination angle for different anisotropy factors (Figure 2(b)). If prolate irregularities are strongly stretched along the external magnetic field,  $\gamma_0 = 0^\circ$ , the broadening of the spatial spectrum tends to the saturation at  $D = 22$ . Increasing inclination angle  $\gamma_0 = 5^\circ$  saturation is started from  $D = 25$ , while if  $\gamma_0 = 10^\circ$ , at  $D = 40$ . Increasing angle in the interval  $\gamma_0 = 0^\circ$ – $10^\circ$  the broadening of the SPS of scattered ordinary EM waves tends to the saturation in proportion to the thickness of plasma slab and broadens in 45 percent. Figure 2(b) illustrates APS versus inclination angle  $\gamma_0$  for different anisotropy factor. If  $\chi = 10$  spectrum has maximum at  $\gamma_0 = 10^\circ$ , if  $\chi = 13$  maximum is displaced to the left,  $\gamma_0 = 7^\circ$ , while at  $\chi = 15$  we obtain  $\gamma_0 = 6^\circ$ . Maximum of the SPS is displaced to the left, to the small angles  $\gamma_0$ , increasing anisotropy factor.



**Figure 2.** Broadening of the SPS of scattered ordinary wave in the collisional plasma as a function of non-dimensional parameter (a)  $D = L/l_{\parallel}$  and (b) inclination angle  $\gamma_0$ .

Hence, thickness of a slab, characteristic spatial scale of electron density fluctuations along the external magnetic field, anisotropy factor and inclination factor have a substantial influence on the behavior of the second order statistical characteristics of scattered ordinary waves in the magnetized collision plasma.

Spatial correlation function of the phase fluctuations  $W_D(\rho)$  may be simply converted into temporal correlation function if rigid irregularities moving with the velocity  $V_0$  in perpendicular direction to the radio path,  $W_D(t) = W_D(V_0 t)$  and hence, for the broadening of the spectrum we obtain:  $\langle \Omega^2 \rangle = -V_0^2 \partial^2 W_D / \partial \rho_y^2 = V_0^2 \langle k_y^2 \rangle$ . Spaced receive measurement shows [24] that small-scale irregularities in the nightly ionospheric  $F$ -region having elliptic form and anisotropy axis are oriented along the Earth magnetic field. Irregularities are drift to the south-west direction with the velocity of the order 40–100 m/sec. Numerical calculations carrying out for the velocity  $V_0 \sim 100$  m/sec show that  $\langle \Omega^2 \rangle^{1/2} \sim 0.1$  Hz, which is in agreement with experimental data [25].

As we consider statistical characteristics of scattered radiation in the polar ionospheric collision magnetized plasma with vertical line of force of the geomagnetic field, the normalized phase portraits  $\Phi_{YM}(X, Y, L) / \Phi_{YM}(0, 0, L)$  are calculated analytically and numerically for three regions: principle plane — location of both external magnetic field and wave vector, perpendicular plane and transition area connecting these regions.

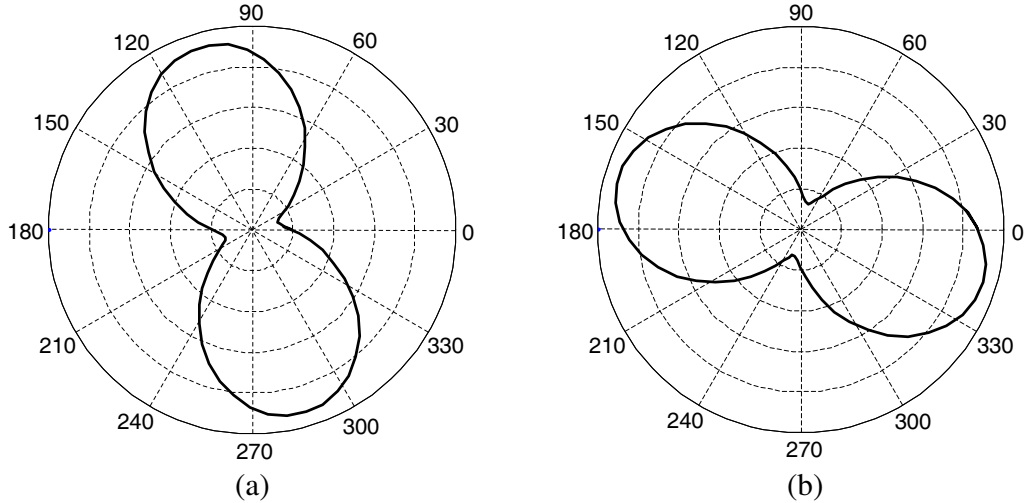
Substituting (24) into (21) using the polar coordinate system in the principle plane we obtain:

$$\langle \varphi_1 \varphi_1^* \rangle_{YM} = -\frac{2}{3} \pi^3 \sigma_{h0}^2 D_2^2 k_0^2 L_0 L \tau \Xi \int_0^{2\pi} d\varphi \left\{ -\frac{i e_1}{2\tau^2} \left( 3 - \frac{e_1^2}{2\tau^2} \right) \exp \left( -\frac{e_1^2}{4\tau^2} \right) \left[ 1 - \frac{i}{\pi} \left( \frac{e_1}{2\tau} + 2F_0 \right) \right] + \frac{1}{4\tau^4} \left( 1 - \frac{e_1^2}{2\tau^2} \right) \right\}, \quad (25)$$

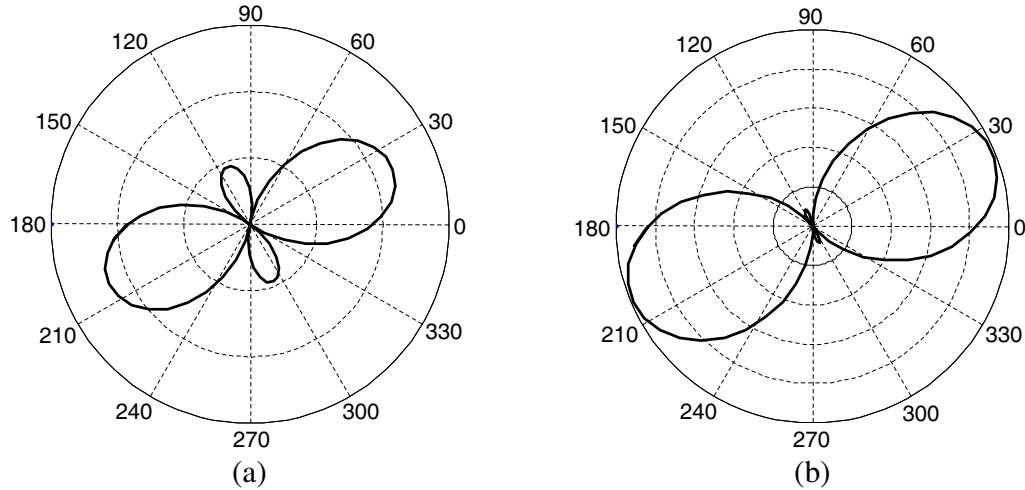
where:

$$\begin{aligned} \Xi &= \frac{1}{\eta' + \mu''} \left[ \frac{\sin(2\sqrt{a_1} k_0 L)}{2\sqrt{a_1} k_0 L} - \exp(-a_1 \Omega_0^2) \right] + \frac{1}{\eta' - \mu''} \left[ \frac{\sin(2\sqrt{c_1} k_0 L)}{2\sqrt{c_1} k_0 L} - \exp(-c_1 \Omega_0^2) \right] \\ &\quad - \frac{16}{\eta'^2 - \mu''^2} \left\{ \frac{\sin(p_1 k_0 L)}{p_1 k_0 L} \cos \theta_2 \exp \left( -\frac{p_2^2}{4} \Omega_0^2 \right) - \frac{\sin(p_2 k_0 L)}{p_2 k_0 L} \cos \theta_1 \exp \left( -\frac{p_1^2}{4} \Omega_0^2 \right) \right. \\ &\quad \left. + s \frac{q_1}{p_1} \left[ \frac{\sin(p_1 k_0 L)}{p_1 k_0 L} - \cos(p_1 k_0 L) \right] \sin \theta_2 \exp \left( -\frac{p_2^2}{4} \Omega_0^2 \right) \right\}, \\ \theta_{1,2} &= s \frac{p_{1,2} q_{1,2}}{2} \Omega_0^2, \quad \Omega_0 = \frac{1}{2} k_0 L_0, \quad e_1 = X \cos \varphi + Y \sin \varphi, \quad X = \frac{\rho_x}{l_{\parallel}}, \quad Y = \frac{\rho_y}{l_{\parallel}}, \\ F_0 &= \sum_{n=1}^{\infty} \frac{1}{n} sh \left( n \frac{e_1}{2\tau} \right) \exp \left( -\frac{n^2}{2} \right), \quad \tau = \frac{L_0}{2l_{\parallel}}. \end{aligned} \quad (26)$$

Analytical calculations show that in the transition region only  $V_{xz}$  and  $V_{yz}$  components, while in the  $XOY$  plane (perpendicular to the location of the external magnetic field)  $X$  and  $Y$  components of the second rank tensor  $V_{ij}$  give the contribution correlation function of an external magnetic field fluctuation. Phase portraits of the phase fluctuations for the ordinary wave has been constructed at:  $B = k_0 L = 100$ ,  $s = 10^{-3}$ . Figure 3 represents evaluation of the phasor lobes at  $\tau < 1$  ( $L_0 = 10$ ,  $l_{\parallel} = 900$ ). Figure 3(a) corresponds  $X = 7 \cdot 10^{-3}$  and  $Y = 2 \cdot 10^{-3}$ ; replacing  $X$  and  $Y$  phase portrait turns on the counterclockwise direction on  $60^\circ$ . Analysis show that the orientation of the phase portrait depends on the location of the observation points leading to the narrowing of the lobes if  $L_0 \gg l_{\parallel}$ . Figure 4 depicts phase portraits of the phase fluctuations at  $\tau > 1$  for  $X = 0.5$ ,  $Y = 0.2$ . Figure 4(a) corresponds:  $L_0 = 100$ ,  $l_{\parallel} = 10$ ; Figure 4(b)  $L_0 = 1000$ ,  $l_{\parallel} = 30$ . Increasing parameter  $\tau$  three times small lobes are decreased, while the main one are increased due to energy transfer from the electron



**Figure 3.** Phasor diagrams of the phase fluctuations caused by the external magnetic field fluctuations in the collision magnetized plasma at  $\tau = 6 \cdot 10^{-3}$ .



**Figure 4.** Phasor plots of the phase fluctuations caused by magnetic field fluctuations in the collision magnetized plasma for (a)  $\tau = 5$ ; (b)  $\tau = 16$ .

density fluctuations to the external magnetic field fluctuations not varying orientation of the phase portrait.

Geomagnetic field fluctuations are more intensive at the geomagnetic storms belong to the most striking phenomena impacting the space weather. A variety of complicated ionospheric patterns resulting from the interplay between different space weather factors were revealed by low-orbiting satellite radio tomography (LORT) during the geomagnetic storms [26]. Some features are common of the most storms, and the others are event-specific traits. The LORT images of the disturbed ionosphere demonstrate plasma distributions with numerous enhancements and depletions on various scaled, rapid rearrangement of the ionospheric structures, uplifting of the ionospheric  $F$ -layer, variations in the position and shape of the main ionization trough, waves and wavelike disturbances, etc.

## 5. CONCLUSION

Correlation function of the phase fluctuation oscillates in the collision magnetized ionospheric plasma. Oscillations are increased decreasing thickness of a slab.

The broadening of the SPS of scattered ordinary waves in magnetized plasma with anisotropic irregularities tends to the saturation inversely to the thickness of a slab increasing characteristic spatial scale of electron density fluctuations  $l_{\parallel}$  along the external magnetic field. Increasing anisotropy factor maximum of the SPS is displaced to the small inclination angles.

Converting spatial correlation function of the phase fluctuations into temporal correlation function and if rigid small-scale irregularities in the nightly ionospheric  $F$ -region irregularities moving with the velocity  $V_0 \sim 100$  m/sec in perpendicular direction to the radio path, the broadening of the spectrum is of the order 0.1 Hz which is in agreement with the experimental data.

Phasor diagrams of the phase fluctuations caused by the external magnetic field fluctuations are constructed for the collisional magnetized plasma at different spatial parameter  $\tau = (L_0/2l_{\parallel})$  is the ratio of the spatial scale of an external magnetic field fluctuations  $L_0$  and  $l_{\parallel}$  varying location of the observation points. Increasing parameter  $\tau$  three times small lobes decrease, while the main lobes increase due to energy transfer from the electron density fluctuations to the external magnetic field fluctuations not varying orientation of the phase portrait. More conventional way of investigating the anisotropy of ionospheric fluctuations is studying the statistical properties of the diffraction pattern implying at least three closely-spaced receivers. The multi-receiver method provides the anisotropy of the diffraction pattern in both field-aligned and perpendicular anisotropies of the irregularities, which cannot be done using a single receiver.

The above theory can be applied in determining the anisotropy parameters of the ionospheric irregularities observing statistical characteristics caused by the phase fluctuations.

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