

Synthesis of Generalized Chebyshev Lossy Bandstop Filters with Non-Paraconjugate Transmission Zeros

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Abstract—A systematic procedure is presented for synthesis of generalized Chebyshev lossy bandstop filters with non-paraconjugate transmission zeros. From a lossy scattering parameters with the prescribed reflection zeros, the transformation formulas from the scattering matrix to the admittance matrix are obtained by reconstructing the non-paraconjugate transmission zeros as paraconjugate ones. The canonical transversal array is modeled by partial fraction expansion of the normalized admittance functions, resulting in an increased order of the final network provided there are nonparaconjugate transmission zeros. The methods are simpler and more general than the ones in the literature. So it shows great versatility, and can also accommodate lossless network or a transfer function with symmetrical transmission zeros. To illustrate the proposed synthesis procedure, three typical examples have been carried out to validate the synthesis method.

1. INTRODUCTION

Microwave filters are indispensable components in modern communication systems. With the continuous development of information technology over the last years, the demand for filter with high-performance has become more urgent. High-performance is defined as the most efficient use of the costly spectrum, that is, it preserves a very flat passband and a steep transition into the rejection band with a small size and mass [1]. It is usually a trade-off between in-band insertion loss variations, out of band isolation, size and mass to design this type of filter. High quality factor (Q) plays an important role in high-performance filters. In order to increase the Q , one often must increase the size of the resonator, or may necessitate expensive dielectric resonators. The former approach usually results in a larger volume, which can not be tolerated in some applications such as satellite communications. The latter approach increases the cost greatly.

Lossy filter has been developed using low Q resonators at the cost of a significant increase in the absolute insertion loss. In this type of filter, losses are intentionally added according to the design specifications. In other words, it is possible to enhance the performance or at least maintain similar performance by using low Q resonators. The prescribed insertion loss can be potentially compensated by an amplifier.

Recently, the synthesis of lossy filters has been widely discussed in literature and it has become one of the most important topics in microwave filter research. The commonly-used methods for the synthesis of lossy networks are: the predistortion, the lossy coupling matrix synthesis and the even/odd mode analysis. Predistortion can improve the insertion loss flatness by reflecting more power at the band center of the filter function. To compensate for the return-loss performance, nonreciprocal devices such as isolators and circulators are needed, resulting in a larger size. Comparing with predistortion, direct synthesis method of lossy filters [2, 3] which is based on the absorption using additional resistive elements rather than the reflection of power, leads to improved return loss. In 2008, $N \times N$ lossy coupling matrix

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applying an iterative algorithm by Mirafteb and Yu [3] was synthesized, where nonuniform dissipation and modified topologies with extra lossy coupling elements were used to realize a low Q filter with high flatness and good return loss. A systematic approach to the transversal $N+2$ coupling matrix were also proposed [4]. From the well-established mathematical theorems for complex matrices, a direct method to obtain the lossy coupling matrix has been developed in [5]. Oldoni et al. [6] presented a more general even- and odd-mode decomposition to synthesize a lossy network in 2010. It is not limited to symmetric networks.

The above mentioned coupling matrices are not realizable in their current form. Hence, further matrix operations are required to transform the coupling matrix to the matrix in a requested topology. For lossy case, both hyperbolic and trigonometric matrix rotation are applied to appropriately distribute the loss among the resonators. But it is quite cumbersome, time-consuming and even impossible for the lossy filter to find an appropriate sequence of rotations, especially for the filter with high order and complex topology since there are simultaneous resistive and reactive coupling. This method also lacks general rules for the rotation matrix. To overcome this limitation, optimizations are preferable to the derivation of the required coupling matrix for lossy filter by defining one or more appropriate cost functions [7, 8]. The aim is to minimize the difference between the synthesized filter response and a theoretical objective function at several frequency points. The coupling coefficients are used as the independent variables in minimizing a simple cost function. In [7], a fast synthesis technique for generalized Chebyshev lossy bandpass filters was proposed by solving a nonlinear least squares problem based on zeros and poles of filter's transfer functions. However, it may miss the best solution if an initial value is not sufficiently close to the global minimum. Evolutionary algorithms have the potential to find the global minimum by crossover and mutation operation. Zhao and Wang [8] applied it to a lossy coupling matrix synthesis by defining multi-objective functions.

The requirements for high-performance can be fulfilled by filters with transmission zeros. It is a common sense that zeros of the transfer function usually are purely imaginary or appear in a conjugate pair, in other words, they are restricted to be symmetrically distributed about the imaginary axis in the complex frequency plane, as shown in Fig. 1(a). This phenomenon is also called to be paraconjugate. To our knowledge, the filters in the previous literatures are all concerned about this form. But in a more general case, the transmission zeros are non-paraconjugate or asymmetrically distributed about the imaginary axis, as depicted in Fig. 1(b). The classical method of filter design is based on the assumption that the transmission zeros (TZs) are all paraconjugate. The existing methods are no longer valid for this non-paraconjugate network. Therefore, a more general theory for the synthesis of lossy filter with non-paraconjugate TZs is desired. Inspired by [9], we propose a novel synthesis technique belonging to this class. The synthesis starts from suitable characteristic polynomials by reconstructing the non-paraconjugate transmission zeros as paraconjugate ones. With this technique, zeros of the new transfer function are paraconjugate pair. It is shown that the presented approach is, however, more general than the existing ones because it does not have any limitation for the choice of the symmetrical transmission zeros.

This paper extends the work first presented in [9] to lossy resonator filter. Such a modification allows filters with more TZs to be realized, and in effect, higher selectivity or equalized group delay can be achieved. The paper is organized in the following way: Section 2 describes the details of polynomial synthesis for lossy resonator filter with non-paraconjugate transmission zeros. Section 3

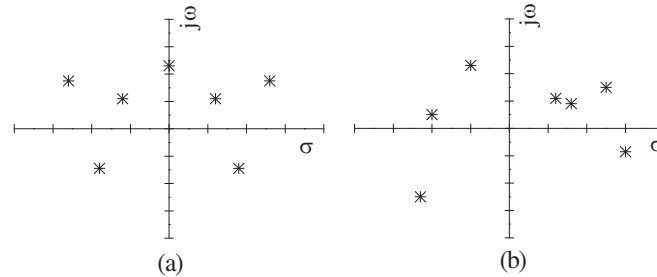


Figure 1. Distribution of the transmission zeros, (a) paraconjugate, (b) non-paraconjugate.

provides a method of the expressions for the complex Y parameter from the reconstructed scattering parameters. Three different examples are considered to cover bandstop filters with paraconjugate TZs, non-paraconjugate TZs and no reflection zero in Section 4 to validate the novel design method. Section 5 concludes this article.

2. BASIC THEORY

2.1. Characteristic Polynomials of Lossless Filter

Consider the scattering matrix of a lossless two-port bandstop filter network composed of a series of N cross-coupled resonators as

$$\begin{bmatrix} S'_{11} & S'_{12} \\ S'_{21} & S'_{22} \end{bmatrix} = \frac{1}{E(s)} \begin{bmatrix} P_{11}(s)/\varepsilon & F(s)/\varepsilon_R \\ F(s)/\varepsilon_R & P_{22}(s)/\varepsilon \end{bmatrix} \quad (1)$$

where $s = \sigma + j\omega$ is the complex frequency variable, $S'_{12} = S'_{21}$ since the network is reciprocal; $P_{11}(s)$, $P_{22}(s)$ and $F(s)$ are the reflection and transfer polynomials; the common denominator $E(s)$ is a strict Hurwitz polynomial whose roots must all lie in the left-hand plane of the s -plane. It is assumed that the polynomials $P_{11}(s)$, $P_{22}(s)$, $F(s)$ and $E(s)$ have been normalized to their respective highest degree coefficients such that their highest degree coefficients are all unity. For bandstop filter, both $F(s)$ and $E(s)$ are N th-degree polynomials, while the degree of $P_{11}(s)$ corresponds to the number of finite-position reflection zeros (RZs) that are originally prescribed. For a realizable network, the degree of $P_{11}(s)$ must not exceed N , i.e., $n_{rz} \leq N$. ε and ε_R are normalization factors to force $|S'_{11}|$ and $|S'_{21}| \leq 1$ at any frequency variable s . $\varepsilon_R = 1$ for all cases except for fully canonical filtering function, where all the prescribed RZs belong to finite frequencies, i.e., $n_{rz} = N$. In this case, ε_R is slightly greater than 1, which can be described as [10]

$$\varepsilon_R = \frac{\varepsilon}{\sqrt{\varepsilon^2 - 1}} \quad (2)$$

The value of ε can be found through the following equation:

$$\varepsilon = \frac{\varepsilon_R}{\sqrt{10^{RL/10} - 1}} \cdot \left| \frac{F(s)}{P_{11}(s)} \right|_{s=\pm j} \quad (3)$$

where RL is the prescribed stopband rejection loss in decibels. It is important to mention that Eq. (3) is slightly different that of [10]. In [10], incorrect results will appear when $n_{rz} = N$ because $\varepsilon_R \neq 1$ for the fully canonical case and it should not be neglected. Inserting Eq. (3) into Eq. (2) finally gives

$$\varepsilon_R = \sqrt{1 + (10^{RL/10} - 1) \left| \frac{P_{11}(s)}{F(s)} \right|_{s=\pm j}^2} \quad (4)$$

Polynomial $P_{11}(s)$ is determined from finite reflection zeros s_r . Once N , RL and s_r are given, a recursive procedure is used to obtain the polynomials $P_{11}(s)$, $F(s)$ and $E(s)$ by Cameron in [11].

For a passive, lossless, and reciprocal system, the transfer and reflection vectors are orthogonal in order to satisfy the unitary conditions:

$$\begin{bmatrix} S'_{11} & S'_{21} \\ S'_{21} & S'_{22} \end{bmatrix}^t \begin{bmatrix} S'_{11} & S'_{21} \\ S'_{21} & S'_{22} \end{bmatrix}^* = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (5)$$

where superscript t indicates the transpose operation, $*$ is the conjugation symbol. Therefore, the following equations are obtained through multiplying row 1 by column 1 and row 1 by column 2, respectively

$$S'_{11}S'_{11}^* + S'_{21}S'_{21}^* = 1 \quad (6a)$$

$$S'_{22}S'_{21}^* + S'_{21}S'_{11}^* = 0 \quad (6b)$$

From Eq. (1), Eq. (6) may be found as follows

$$\frac{F(s)F(s)^*}{\varepsilon_R^2} + \frac{P_{11}(s)P_{11}(s)^*}{\varepsilon^2} = E(s)E(s)^* \quad (7a)$$

$$F(s)P_{11}(s)^* + F(s)^*P_{22}(s) = 0 \quad (7b)$$

The latter equation can be rewritten as

$$P_{22}(s) = -\frac{F(s)}{F(s)^*}P_{11}(s)^* \quad (8)$$

This equation is of the utmost importance, which will be discussed later. There is no obvious evidence in support of a theory that the TZs must be upon the imaginary axis. For a synthesizable network, however, the zeros from $S_{11}(s)$ and $S_{22}(s)$ must be located in the form of a mirror-image about the imaginary axis. In this case, the individual zeros of $S_{11}(s)$ may be chosen from the left-hand or right-hand side of each pair, and the remaining zeros from each pair to form the complementary function $S_{22}(s)$. For this reason the non-paraconjugate TZs, as shown in Fig. 1(b) are desirable. Thus, regardless of transmission zeros, the zeros of reflection function would no longer be located on the imaginary axis. Similar to [10], it is necessary to multiply the polynomial $F(s)$ by j whenever $N - n_{fz}$ is an even integer, where n_{fz} is the number of finite-position TZs.

2.2. Discussion of $\frac{F}{F^*}$

2.2.1. Case I: $F(s)$ with Paraconjugate Roots

Assumed that N -pole filter contains n_{fz} finite-position TZs (i.e., $s_1, s_2, \dots, s_{n_{fz}}$) and $n_{fz} \leq N$. All TZs are located symmetrically about the imaginary axis or upon the imaginary axis. This is the most general form. In this case, polynomial F can be written in terms of complex frequency:

$$F(s) = \begin{cases} j \prod_{i=1}^{n_{fz}} (s - s_i) & N - n_{fz} \text{ is an even integer} \\ \prod_{i=1}^{n_{fz}} (s - s_i) & \text{else} \end{cases} \quad (9)$$

As mentioned above, it is necessary to multiply the polynomial $F(s)$ by j whenever $N - n_{fz}$ is an even integer to satisfy the unitary conditions for the scattering matrix. Then the factor $rp = \frac{F(s)}{F(s)^*}$ can be formulated as

$$rp = \begin{cases} (-1)^{n_{fz}+1} & N - n_{fz} \text{ is an even integer} \\ (-1)^{n_{fz}} & \text{else} \end{cases} = (-1)^{N+1} \quad (10)$$

Noted that this equation is different from [9, Eq. (14)].

2.2.2. Case II: $F(s)$ with Non-Paraconjugate Roots

Generally, $F(s)$ may have non-paraconjugate roots, that is, one or more TZs is/are asymmetrically distributed about the imaginary axis. Without loss of generality, we assumed that filter have ns symmetrical TZs (i.e., s_1, s_2, \dots, s_{ns}) and na asymmetrical TZs (i.e., $s_{a1}, s_{a2}, \dots, s_{ana}$), where $n_{fz} = ns + na$. Polynomial $F(s)$ takes the form

$$F(s) = \begin{cases} j \prod_{i=1}^{ns} (s - s_i) \prod_{i=1}^{na} (s - s_{ai}) & N - n_{fz} \text{ is even} \\ \prod_{i=1}^{ns} (s - s_i) \prod_{i=1}^{na} (s - s_{ai}) & \text{else} \end{cases} \quad (11)$$

In this case, one has

$$rp = \begin{cases} (-1)^{ns+1} \frac{\prod_{i=1}^{na} (s - s_{ai})}{\prod_{i=1}^{na} (s + s_{ai}^*)} & N - n_{fz} \text{ is even} \\ (-1)^{ns} \frac{\prod_{i=1}^{na} (s - s_{ai})}{\prod_{i=1}^{na} (s + s_{ai}^*)} & \text{else} \end{cases} \quad (12)$$

3. LOSSY ADMITTANCE POLYNOMIAL SYNTHESIS

Following [3], the lossy S -parameter polynomials with different loss levels at input and output ports scattering parameters are put in the form

$$S_{11} = K\alpha S'_{11} \quad S_{22} = \frac{K}{\alpha} S'_{22} \quad S_{12} = S_{21} = KS'_{21} \quad (13)$$

where K is the attenuation factor of filter. The attenuation level of the return loss can be decided by the control parameter α ($0 < K \leq 1$, $K \leq \alpha \leq 1/K$). With the evaluated scattering polynomials, the admittance parameters can be found. Using classic two-port S matrix to Y matrix transformation formulas with normalized characteristic impedances, one obtains the admittance matrix (variable s is omitted for readability)

$$y_{21n} = -\frac{2K}{\varepsilon_R} \frac{F}{E} \quad (14a)$$

$$y_d = 1 + \frac{K\alpha P_{11}}{\varepsilon E} + \frac{KP_{22}}{\alpha \varepsilon E} + \frac{K^2 P_{11} P_{22}}{\varepsilon^2 E^2} - \frac{K^2 F^2}{\varepsilon_R^2 E^2} \quad (14b)$$

$$y_{11n} = 1 - \frac{K\alpha P_{11}}{\varepsilon E} + \frac{KP_{22}}{\alpha \varepsilon E} - \frac{K^2 P_{11} P_{22}}{\varepsilon^2 E^2} + \frac{K^2 F^2}{\varepsilon_R^2 E^2} \quad (14c)$$

$$y_{22n} = 1 + \frac{K\alpha P_{11}}{\varepsilon E} - \frac{KP_{22}}{\alpha \varepsilon E} - \frac{K^2 P_{11} P_{22}}{\varepsilon^2 E^2} + \frac{K^2 F^2}{\varepsilon_R^2 E^2} \quad (14d)$$

where y_{21n} , y_{11n} , y_{22n} are the numerators for polynomials y_{21} , y_{11} , y_{22} , and y_d is the common denominator, $y_{21} = y_{21n}/y_d$, $y_{11} = y_{11n}/y_d$, $y_{22} = y_{22n}/y_d$. Inserting Eq. (8) into Eq. (14) and applying Eq. (7a), admittance matrix becomes in the following form after some algebraic manipulations:

$$y_{11} = \frac{E - \frac{K\alpha}{\varepsilon} P_{11} + \frac{F}{F^*} \left(K^2 E - \frac{K}{\alpha \varepsilon} P_{11} \right)^*}{E + \frac{K\alpha}{\varepsilon} P_{11} - \frac{F}{F^*} \left(K^2 E + \frac{K}{\alpha \varepsilon} P_{11} \right)^*} \quad (15a)$$

$$y_{21} = \frac{-\frac{2K}{\varepsilon_R} F}{E + \frac{K\alpha}{\varepsilon} P_{11} - \frac{F}{F^*} \left(K^2 E + \frac{K}{\alpha \varepsilon} P_{11} \right)^*} \quad (15b)$$

$$y_{22} = \frac{E + \frac{K\alpha}{\varepsilon} P_{11} + \frac{F}{F^*} \left(K^2 E + \frac{K}{\alpha \varepsilon} P_{11} \right)^*}{E + \frac{K\alpha}{\varepsilon} P_{11} - \frac{F}{F^*} \left(K^2 E + \frac{K}{\alpha \varepsilon} P_{11} \right)^*} \quad (15c)$$

3.1. Particular Case: $F(s)$ with Paraconjugate Roots

By substituting Eq. (10) into Eq. (15), the Y -parameter functions can be derived as

$$y_{11} = \frac{E - \frac{K\alpha}{\varepsilon} P_{11} - (-1)^N \left(K^2 E - \frac{K}{\alpha \varepsilon} P_{11} \right)^*}{E + \frac{K\alpha}{\varepsilon} P_{11} + (-1)^N \left(K^2 E + \frac{K}{\alpha \varepsilon} P_{11} \right)^*} \quad (16a)$$

$$y_{21} = \frac{-\frac{2K}{\varepsilon_R} F}{E + \frac{K\alpha}{\varepsilon} P_{11} + (-1)^N \left(K^2 E + \frac{K}{\alpha \varepsilon} P_{11} \right)^*} \quad (16b)$$

$$y_{22} = \frac{E + \frac{K\alpha}{\varepsilon}P_{11} - (-1)^N \left(K^2E + \frac{K}{\alpha\varepsilon}P_{11} \right)^*}{E + \frac{K\alpha}{\varepsilon}P_{11} + (-1)^N \left(K^2E + \frac{K}{\alpha\varepsilon}P_{11} \right)^*} \quad (16c)$$

3.2. Generalized Case: $F(s)$ with Non-Paraconjugate Roots

Substituting the expressions of Eq. (12) into Eq. (15) yields the following equation for the admittance matrix

$$y_{11} = \frac{E' - \frac{K\alpha}{\varepsilon}P'_{11} \mp (-1)^{ns} \left(K^2E' - \frac{K}{\alpha\varepsilon}P'_{11} \right)^*}{E' + \frac{K\alpha}{\varepsilon}P'_{11} \pm (-1)^{ns} \left(K^2E' + \frac{K}{\alpha\varepsilon}P'_{11} \right)^*} \quad (17a)$$

$$y_{21} = \frac{-\frac{2K}{\varepsilon_R}F'}{E' + \frac{K\alpha}{\varepsilon}P'_{11} \pm (-1)^{ns} \left(K^2E' + \frac{K}{\alpha\varepsilon}P'_{11} \right)^*} \quad (17b)$$

$$y_{22} = \frac{E' + \frac{K\alpha}{\varepsilon}P'_{11} \mp (-1)^{ns} \left(K^2E' + \frac{K}{\alpha\varepsilon}P'_{11} \right)^*}{E' + \frac{K\alpha}{\varepsilon}P'_{11} \pm (-1)^{ns} \left(K^2E' + \frac{K}{\alpha\varepsilon}P'_{11} \right)^*} \quad (17c)$$

where $E' = E \prod_{i=1}^{na} (s + s_{ai}^*)$, $P'_{11} = P_{11} \prod_{i=1}^{na} (s + s_{ai}^*)$, and $F' = F \prod_{i=1}^{na} (s + s_{ai}^*)$. When $N - n_{fz}$ is an even integer, Eq. (17) takes the upper sign. For example, when $N - n_{fz}$ is an even integer, operator ' \pm ' is replaced by operator '+' only, vice versa.

From the general expression of the proposed admittance function, the main properties can be summarized as follows:

- 1) When there exists non-paraconjugate TZs, the order of y_d is larger than that of $E(s)$. The order of the denominator of Eq. (15) increases by na after multiplication by $\prod_{i=1}^{na} (s + s_{ai}^*)$. It is found that the corresponding admittance polynomials have an order of $N+na$ provided that $F(s)$ has na asymmetrical roots and $E(s)$ is of order N .
- 2) It is clear from the above formulas that E' , F' and P'_{11} are used as substitutions by multiplying a common factor $\prod_{i=1}^{na} (s + s_{ai}^*)$, as compared with the conventional admittance matrix. The roots of P multiplying by a common factor are paraconjugate.
- 3) Eq. (17) is more universal whether the TZs are symmetrical or not. So [3, Eq. (25)] can be regarded as one of the particular case of Eq. (17).

The lossy coupling matrix can be synthesized from the transversal array circuit model. Once the admittances have been assigned, the lossy $N+2$ complex coupling matrix can be synthesized by partial fraction expansion which was presented in [3]. To further obtain a practical coupling matrix, it is necessary to transform the transversal array into the requested topology by using hyperbolic rotations [3, 4] or optimization [7, 8].

4. SYNTHESIS EXAMPLES

In this section, three typical examples are presented to illustrate the procedures described above and transversal coupling matrix M is obtained that yields the correct filter response. The examples are selected to cover the theory presented in this paper. These include: 1) fully symmetrical Tzs with the

same return and insertion loss levels; 2) fully asymmetrical TZs; and 3) filter without reflection zero. The first example in detail proves that this method is an extension of [3]. The second example shows the effectiveness of the approach for filter networks with only asymmetrical TZs. It represents a case where the synthesis method in [3] cannot be used. The third example is simplified to the case where Chebyshev bandstop filter is presented compared to the conventional filter function. The simulations demonstrate that the proposed synthesis method is indeed more universal whether TZs are symmetrical or not even if there is no reflection zero.

4.1. Symmetrical TZs

The first example to synthesize is a fourth-order bandstop filter with two reflection zeros at $j1.3$ and $-j1.8$. The lossless stopband rejection is 20 dB shifted down by 6 dB ($K = 0.5, \alpha = 1$). The four transmission zeros occur at $[-j0.93, j0.9543, j0.501, -j0.359]$, which are upon the imaginary axis. The coefficients of the admittance polynomials are calculated using the previous synthesis method and shown in Table 1. The transversal coupling matrix element values can be obtained in Table 2 by applying a partial fraction expansion of the normalized admittance functions [3]. For some variables see [3].

Table 1. Coefficients of the lossy admittance functions.

s^i, i	y_d	y_{11n} and y_{22n}	y_{21n}
0	$3.6394 - j0.3911$	$1.184 - j0.6534$	$-j0.1601$
1	$2.06922 - j0.3391$	$3.4575 - j0.4154$	-0.1307
2	$4.7046 - j0.2822$	$2.3907 - j0.4716$	$-j1.0668$
3	$1.5449 - j0.2076$	$2.5814 - j0.1243$	-0.1664
4	1.2512	0.7488	$-j1.0024$

Table 2. Transversal array element values. $G_S = G_L = 0.5985$, $J_{SL} = -0.8011$.

k	B_k	G_k	J_{SK}	J_{LK}
1	-1.5345	0.5282	$0.7941 + j0.2928$	$0.7941 + j0.2928$
2	1.3627	0.5732	$-0.8297 + j0.2915$	$0.8297 - j0.2915$
3	1.1050	0.0892	$0.3452 + j0.0429$	$0.3452 + j0.0429$
4	-1.0992	0.0442	$-0.2432 + j0.0225$	$0.2432 - j0.0225$

The $N + 2$ lossy coupling matrix is shown in Eq. (18)

$$M = \begin{bmatrix} -j0.5985 & 0.7941 + j0.2928 & -0.8297 + j0.2915 & 0.3452 + j0.0429 & -0.2432 + j0.0225 & -0.8011 \\ 0.7941 + j0.2928 & -1.5345 - j0.5282 & 0 & 0 & 0 & 0.7941 + j0.2928 \\ -0.8297 + j0.2915 & 0 & 1.3627 - j0.5732 & 0 & 0 & 0.8297 - j0.2915 \\ 0.3452 + j0.0429 & 0 & 0 & 1.1050 - j0.0892 & 0 & 0.3452 + j0.0429 \\ -0.2432 + j0.0225 & 0 & 0 & 0 & -1.0992 - j0.0442 & 0.2432 - j0.0225 \\ -0.8011 & 0.7941 + j0.2928 & 0.8297 - j0.2915 & 0.3452 + j0.0429 & 0.2432 - j0.0225 & -j0.5985 \end{bmatrix} \quad (18)$$

The synthesized filter has a response shown in Fig. 2, from which it is found that the response obtained from the coupling matrix is indistinguishable from that of the prototype one.

4.2. Asymmetrical TZs

Consider the lossy 3-pole filter with the stopband rejection of 22 dB. The different loss levels for S_{11} and S_{22} are 3 and 9 dB compared to the lossless response, respectively. The prescribed RZs are $[0.5 - j1.8, j1.5, 1.2 + j2]$. Three asymmetrical transmission zeros located at $[-0.0481 - j0.85458, -0.01596 + j0.9298, -0.1441 + j0.2203]$ are computed from a simple recursion relation given by Cameron [11]. The coefficients for the new polynomials after multiplying by a common factor are summarized in Table 3. In this case, since there is three asymmetrical Tzs, i.e., $ns = 3$, the order of y_d increases to $N + ns = 6$, and their coefficients are shown in Table 4. Comparing Table 4 with Table 1, it

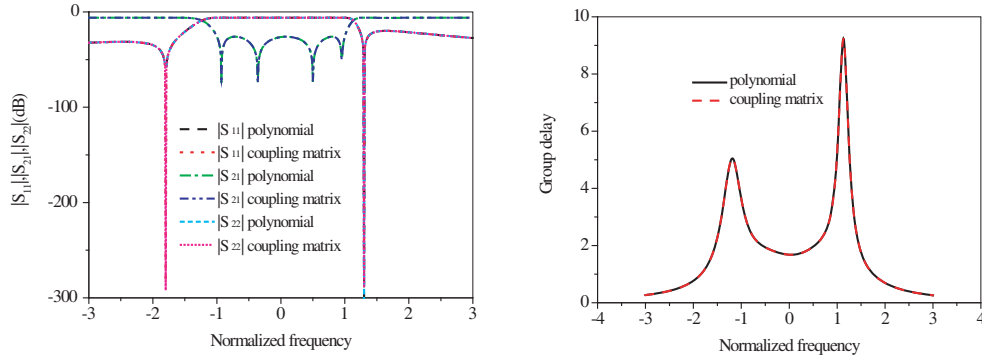


Figure 2. Frequency response and group delay for symmetrical TZs.

is obviously found that $y_{11n} \neq y_{22n}$ due to asymmetrical transmission zeros. The transversal coupling matrix element values with shunt conductances are: $G_S = 0.3159$ and $G_L = 0.5177$ are calculated in Table 5 with only four digits. The obtained transversal coupling matrix can be further transformed to the targeted topology through a series of complex rotations. It is not shown here and will be our future work. For comparison purposes, Fig. 3 shows the results obtained by coupling matrix (Table 5) and polynomials (13). As can be seen, it is almost identical to that of prototype synthesized with analytical polynomials.

Table 3. Coefficients of $E'(s)$, $F'_{11}(s)$ and $P'(s)$ polynomials.

s^i, i	Coefficients of polynomial $E'(s)$	Coefficients of polynomial $F'_{11}(s)$	Coefficients of polynomial $P'(s)$
0	$-0.6791 - j0.1851$	$-j0.0439$	$-0.9443 + j0.9916$
1	$1.5714 - j2.2082$	0.2711	$-1.1888 - j5.9126$
2	$1.0583 - j0.8836$	$j0.4731i$	$1.3739 + j3.6281$
3	$3.1451 - j3.7321$	0.8017	$-3.4933 - j9.3563$
4	$3.4004 - j1.0961$	$j1.4454$	$4.5396 + j2.3022$
5	$1.9031 - j0.9610i$	0.5910	$-1.9082 - j1.9955$
6	1.0	j	1.0

Table 4. Coefficients of the lossy admittance functions.

s^i, i	y_d	y_{11n}	y_{22n}	y_{21n}
0	$-1.3648 + j0.0412$	$-0.3374 - j0.7725$	$-0.6797 + j0.3094$	$j0.0378$
1	$0.9612 - j5.9883$	$2.6147 - j0.5818$	$1.3176 - j2.7253$	-0.2332
2	$2.0736 - j0.0040$	$0.5435 - j3.0847$	$1.0415 + j0.8736$	$-j0.4070$
3	$1.7219 - j9.7737$	$5.8407 - j1.0986$	$2.0294 - j4.4906$	-0.6897
4	$6.7310 - j0.4034$	$1.7234 - j2.6273$	$3.3691 - j0.1155$	$-j1.2434$
5	$1.0792 - j2.2910$	$3.4220 - j0.3579$	$1.3401 - j1.0813$	-0.5084
6	1.7967	0.56755	0.9301	$-j0.8602$

4.3. Chebyshev Lossy Bandstop Filter

In this section, a 4-pole Chebyshev lossy bandstop filter with different loss levels is considered. Loss levels for S_{11} and S_{22} are 3 and 9 dB, respectively, and a lossless stopband rejection of 23 dB. The calculated transmission zeros are $[\pm j0.9239, \pm j0.3827]$. Y-matrix polynomial coefficients and transversal coupling matrix element values are calculated in Table 6 and Table 7.

Table 5. Transversal array element values $G_S = 0.3159$, $G_L = 0.5177$, $J_{SL} = -0.4788$.

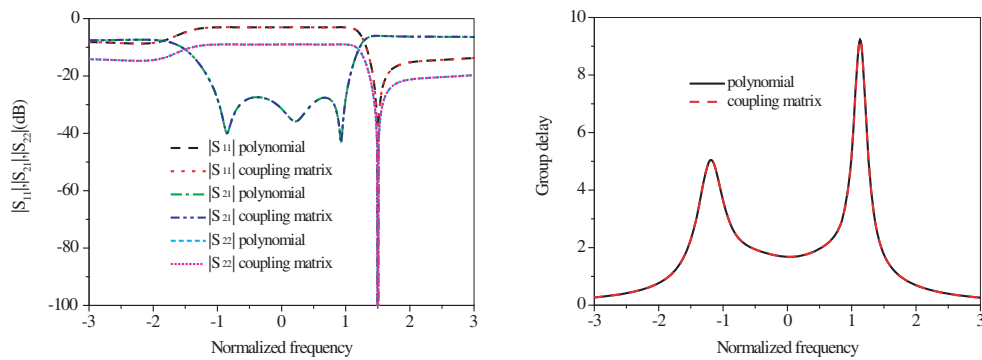
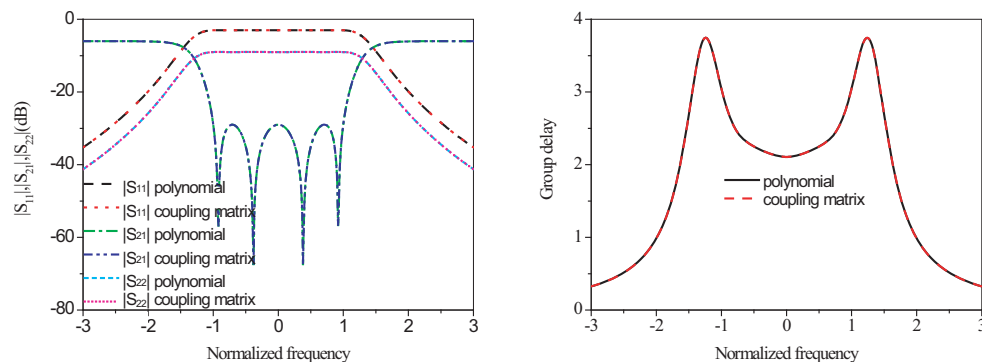
k	B_k	G_k	J_{SK}	J_{LK}
1	1.8296	$j0.1712$	$-0.7272 + j0.1183$	$0.3452 - j0.0503$
2	-1.6758	$j0.4702$	$0.9520 + j0.2489$	$0.6844 + j0.0244$
3	-1.0854	$j0.0738$	$-0.6019 + j0.0817$	$0.1194 - j0.0094$
4	-0.9408	$-j0.0140$	$-0.0038 + j0.0006$	$0.0751 + j0.1291$
5	0.8568	$-j0.0231$	$-0.0002 + j0.0017$	$0.0055 - j0.1932$
6	-0.2595	$-j0.0775$	$0.0035 + j0.0056$	$0.0644 + j0.3456$

Table 6. Coefficients of the lossy admittance functions.

s^i, i	y_d	y_{11n} and y_{22n}	y_{21n}
0	4.0810	0.7002	$-j0.1253$
1	2.7849	4.6532	0
2	4.9891	2.9859	$-j1.0024$
3	1.8304	3.0584	0
4	1.2512	0.7488	$-j1.0024$

Table 7. Transversal array element values $G_S = G_L = 0.5985$, $J_{SL} = -0.8011$.

k	B_k	G_k	J_{SK}	J_{LK}
1	1.3946	0.6317	$-0.7810 + j0.3231$	$0.9101 - j0.3013$
2	-1.3946	0.6317	$0.7810 + j0.3231$	$0.9101 + j0.3013$
3	-1.1754	0.0998	$-0.5337 - j0.0771$	$0.2599 - j0.1437$
4	1.1754	0.0998	$0.5337 - j0.0771$	$0.2599 + j0.1437$

**Figure 3.** Frequency response and group delay for asymmetrical TZs.**Figure 4.** Frequency response and group delay for Chebyshev lossy bandstop filter.

The synthesized filter response is shown in Fig. 4, where the results from analytical polynomials are also depicted for comparison. From Fig. 4, it can be found that the curves agree quite well.

5. CONCLUSIONS

This paper presents an analytical synthesis method for generalized Chebyshev bandstop filters whose transmission zeros are asymmetrically distributed about the imaginary axis in the complex frequency plane. From a rational polynomial of the lossless scattering parameters with the prescribed reflection zeros, the new polynomials are constructed by multiplying a common factor. This transformation forces the non-paraconjugate roots of transmission function to be paraconjugate by increasing the orders of polynomials. Reconstructing the non-paraconjugate transmission zeros as paraconjugate ones leads to the expressions for the complex Y parameters. The lossy transversal coupling matrix can be synthesized by classic partial fraction expansion of the normalized admittance functions. Finally the proposed methodology was verified through three examples. Comparing with the existing techniques used for lossy filter synthesis, the method does not have any limitation for the choice of the symmetrical transmission zeros. So it may be considered as a complement to previously known solutions, showing great versatility. To make the coupling matrix realizable, further matrix operations are required to find realizable coupling matrices. This work is left for future research.

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REFERENCES

1. Yu, M. and V. Mirmoftab, "Shrinking microwave filters," *IEEE Microwave Magazine*, Vol. 9, No. 5, 40–54, 2008.
2. Guyette, A., I. Hunter, and R. Pollard, "The design of microwave bandpass filters using resonators with nonuniform," *IEEE Trans. Microw. Theory Tech.*, Vol. 54, No. 11, 3914–3922, 2006.
3. Mirmoftab, V. and M. Yu, "Advanced coupling matrix and admittance function synthesis techniques for dissipative microwave filters," *IEEE Trans. Microw. Theory Tech.*, Vol. 57, No. 10, 2429–2438, 2009.
4. Mirmoftab, V. and M. Yu, "Generalized lossy microwave filter coupling matrix synthesis and design using mixed technologies," *IEEE Trans. Microw. Theory Tech.*, Vol. 56, No. 12, 3016–3027, 2008.
5. Navarro-Tapia, D., E. Sorolla, P. Otero, and M. Mattes, "Coupling matrix synthesis of dissipative microwave filters," *IET Microwaves, Antennas and Propagation*, Vol. 5, No. 8, 895–900, 2011.
6. Oldoni M., G. Macchiarella, G. Gentili, and C. Ernst, "A new approach to the synthesis of microwave lossy filters," *IEEE Trans. Microw. Theory Tech.*, Vol. 58, No. 5, 1222–1229, 2010.
7. Szydlowski, L., A. Lamecki, and M. Mrozowski, "Synthesis of coupled-lossy resonator filters," *IEEE Microwave and Wireless Components Letters*, Vol. 20, No. 7, 366–368, 2010.
8. Zhao, Y. and G. Wang, "Multi-objective optimisation technique for coupling matrix synthesis of lossy filters," *IET Microw. Antennas Propag.*, No. 7, No. 11, 926–933, 2013.
9. Zhang, Q. and C. Caloz, "Alternative construction of the coupling matrix of filters with non-paraconjugate transmission zeros," *IEEE Microwave and Wireless Components Letters*, Vol. 23, No. 10, 509–511 2013.
10. Cameron, R., "Advanced coupling matrix synthesis techniques for microwave filters," *IEEE Trans. Microw. Theory Tech.*, Vol. 51, No. 1, 1–10, 2003.
11. Cameron, R., "General coupling matrix synthesis methods for Chebyshev filtering functions," *IEEE Trans. Microw. Theory Tech.*, Vol. 47, No. 4, 433–442, 1999.