# A New Wideband Mutual Coupling Compensation Method for Adaptive Arrays Based on Cubic Hermite Interpolation

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**Abstract**—A new mutual coupling compensation method for wideband adaptive arrays is proposed. The new method is developed by combining the element pattern reconstruction method and the cubic Hermit interpolation method to achieve wideband mutual coupling compensation. For the employment of this method, mutual coupling matrices at some frequencies obtained by element pattern reconstruction method are needed and stored. By employing the cubic Hermit interpolation method, all entries of mutual coupling matrix for any frequency within the entire frequency band can be obtained accurately and efficiently. A uniform circular array with eight wideband dipole antennas is designed to verify the validity and effectiveness of the proposed wideband compensation method by the numerical examples.

## 1. INTRODUCTION

It is well known that adaptive antenna arrays are very useful in radar and communication systems. However, the performance of adaptive antenna arrays is deteriorated due to the mutual coupling effect of the array, especially for the wideband antenna arrays. Over the past years, many attempts have been made to compensate the mutual coupling effect and many methods have been proposed, such as the open circuit voltage method and receiving mutual impedance method (RMIM), the minimum norm method, the methods proposed by Yuan et al., Su et al., etc. [1–8]. The open circuit voltage method was proposed in 1983 [1]. In this method, the open circuit voltage was treated as the decoupled terminal voltage. The compensation effect of this method was reduced for the neglect of the open circuit scattering. Other methods have been proposed in the following research, such as RMIM and minimum norm method [2, 3]. RMIM has been verified with better calibration precision than the open circuit voltage method [2, 4]. Actually, the entries of the receiving mutual impedance matrix are functions against the space between the elements, and the mutual coupling impedance matrix is obtained by experimental method. In RMIM experiment, the mutual coupling impedance for each entry of the mutual coupling matrix is calculated by three measurement data. The three kinds of measurement data are the terminal voltage of the element in the isolated state and the terminal voltages of corresponding two elements are in array states. Due to the omnidirectional character of the simple wire antenna element, the terminal voltage of the antenna response to various direction incident signals in horizontal plane is the same. Therefore, this method has a good effect on wire elements. Although it is effective for the calibration of mutual coupling among wire antenna elements, this method is not effective generally for the microstrip antenna element due to the pattern is un-omnidirectional, and the responses to different incident signals directions are not consistent. Later, the transformation between the ideal point source patterns and the embedded element patterns were proposed in [5–7]. For the antenna array composed of dipoles or monopoles, $H$-plane pattern of the element is isotropic, the same as that of the ideal point source. In this case, this method can provide accurate mutual coupling compensation for the incident signals coming from

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the $H$-plane. However, the pattern of a microstrip antenna is dramatically different from that of the ideal point source. It is difficult to find a proper calibration matrix which is suitable for wide angle range of a microstrip antenna array. The universal steering vector was used in the DOA estimations so that the received voltages at the terminals of the array elements can be used directly to find the directions without calibration [8]. However, the universal steering vector is related to the incident angles and polarization of the signals, which limits the applications of this method in the adaptive antenna arrays. Recently, the element pattern reconstruction (EPR) method was proposed to compensate for mutual coupling effect of adaptive arrays [9–11]. This method obtains the compensation matrix by the transformation relation of the far-field pattern between embedded and isolated state elements. This method has been verified valid on the mutual coupling compensation of uniform line array and conformal antenna array.

For the wideband mutual coupling compensation, the research is relatively little. The system identification method was verified as an available one to obtain the wideband mutual coupling compensation matrix [12, 13]. And this method has been combined with RMIM and EPR methods to get the analytical expression for each entry of the wideband compensation matrix against frequency, respectively [14, 15]. However, when there is more than one extreme point in the entry curves of the wideband compensation matrix against frequency, the system identification method is almost invalid. Additionally, the calculation of the polynomial coefficients is complex, and orders of the entries are indeterminate, which are not suitable for real-time requirement.

In this paper, the shape preserving piecewise cubic Hermit interpolation method is employed in the compensation of the wideband mutual coupling effect. The compensation matrix at any frequent point within bandwidth can be calculated through the proposed interpolation method with high precision. Besides, this method has higher computational efficiency. A wideband monopole antenna array with eight elements in uniform circular array is designed to verify the validity and effectiveness of the wideband mutual coupling compensation method.

## 2. THEORY

### 2.1. Element Pattern Reconstruction Method

The EPR method is based on the fact that if the reconstructed pattern of the embedded element is consistent with that in the isolated state, then the corresponding calibrated received signal will be consistent with the received signal without mutual coupling effect. In EPR method, there is an antenna array composed of $N$ elements, and each element is terminated with the same match load $Z_L$. In Eq. (1), $E_n(\theta, \varphi)$ and $E_n^i(\theta, \varphi)$ with $n = 1, 2, \ldots, N$ represent the embedded element pattern and the isolated element pattern for each element, respectively. The transformation relation of the electric field main polarization component between two kinds of element pattern can be written as

$$\begin{bmatrix} E_1^i\,(\theta,\varphi) \\ E_2^i\,(\theta,\varphi) \\ \vdots \\ E_N^i\,(\theta,\varphi) \end{bmatrix} = \mathbf{C} \begin{bmatrix} E_1\,(\theta,\varphi) \\ E_2\,(\theta,\varphi) \\ \vdots \\ E_N(\theta,\varphi) \end{bmatrix} \tag{1}$$

where $\mathbf{C}$ is the calibration matrix. In order to calculate the calibration matrix, $M$ directions are sampled, and one obtains

$$\begin{bmatrix} E_1^i\,(\theta_1,\varphi_1) & \ldots & E_1^i\,(\theta_M,\varphi_M) \\ E_2^i\,(\theta_1,\varphi_1) & \ldots & E_2^i\,(\theta_M,\varphi_M) \\ \vdots & & \\ E_N^i\,(\theta_1,\varphi_1) & \ldots & E_N^i\,(\theta_M,\varphi_M) \end{bmatrix} = \mathbf{C} \begin{bmatrix} E_1(\theta_1,\varphi_1) & \ldots & E_1(\theta_M,\varphi_M) \\ E_2(\theta_1,\varphi_1) & \ldots & E_2(\theta_M,\varphi_M) \\ \vdots & & \\ E_N(\theta_1,\varphi_1) & \ldots & E_N(\theta_M,\varphi_M) \end{bmatrix} \tag{2}$$

A least-square solution that satisfies $\min ||\mathbf{CE} - \mathbf{E}^i||$ can be given by

$$\mathbf{C} = \mathbf{E}^i \mathbf{E}^H \left( \mathbf{EE}^H \right)^{-1} \tag{3}$$

where $\mathbf{E}$ and $\mathbf{E}^i$ represent the pattern matrices for all embedded elements and for the isolated elements, respectively. The operator $||\cdot||$ denotes the $F$-norm of a matrix and superscript $H$ the complex conjugate

transpose. The statement above is the basic theory of the element pattern reconstructed method for single frequency or narrowband width signal, and the wideband coupling calibration methods are described in the below.

## 2.2. Wideband Mutual Coupling Compensation

For wideband systems, each entry of the wideband compensation matrix will vary with the frequency. In the system identification method, the form of the algebraic expression is usually expressed as a ratio of two frequency-dependent polynomials. The polynomial coefficients and the polynomial orders for each entry are to be determined. However, when the bandwidth is very broad, the curves of each entry against frequency are un-monotonic functions, which means that there may be multi-local maximum or minimum in the entry curves of the wideband compensation matrix against frequency. In this case, the system identification method is almost invalid. Besides, there are many polynomial coefficients, and the polynomial orders for all entries are high.

To adapt to wider bandwidth, the interpolation method is an alternative method for numerical-approaching. Compared with the complex-curve fitting method, the interpolation method is simpler and more accurate, and there are many common interpolation methods, such as linear interpolation, shape-preserving piecewise cubic Hermite interpolation, cubic spline interpolation, etc. However, each interpolation method has its own advantages and the applications of curve type. The interpolation function is not smooth at the endpoints of the subspace for linear interpolation method, which means that the first-order derivative is not continuous at the endpoints. Although the cubic spline method can obtain a smooth interpolation function with continuous first-order derivative and continuous curvature (that is a continuous second-order derivative), there may be overshoots and oscillation for unsmooth function sample data. In view of the fact that the curves for each entry of the wideband compensation matrix against frequency are not smooth and with multiple extreme value points, shape-preserving piecewise cubic Hermite interpolation will be a good choice, which is an improvement of the piecewise cubic Hermite interpolation. One of the advantages of this method is its simplicity, and the interpolation functions are continuous at the endpoint of the subspace. Moreover, it makes the interpolation functions save the shape of the curve and avoids too many overshoots and oscillation from sample data, where the first-order derivative is continuous, and there is no requirement on the second-order derivative of the interpolation functions.

According to the principle of the shape-preserving piecewise cubic Hermite interpolation method, the process to obtain entries of the wideband mutual coupling calibration matrix is presented in the following.

Consider an unknown compensation matrix function $C(f)$ against frequency whose values are sampled at different frequency points $f_0, f_1, \ldots, f_n$, where $f_0 < f_1 < \ldots < f_n$. And the compensation matrix $C(f)$ at sample points are $C(f_0)$, $C(f_1)$, ..., $C(f_n)$, respectively. Interpolation involves the estimation of values of $C(f)$ at point $f$ in the interval $(f_0, f_n)$. Assume that the first-order derivatives at the sampled point are $d_0, d_1, \ldots, d_n$, respectively, then the piecewise cubic Hermite interpolation function $C(f_i)$ should meet the following two conditions:

$$\text{a) } C(f_i) = y_i, \ C'(f_i) = d_i, \quad i = 0, 1, 2, \ldots, n \tag{4}$$

$$\text{b) } C_k(f) = a_{i0} + a_{i1}f + a_i f^2 + a_{i3}f^3, \quad k = 1, 2, \ldots, n-1 \tag{5}$$

In each subspace $[f_{i-1}, f_i]$, the maximum exponent of the interpolation function is cubic. For calculation convenience, functions (4) and (5) can be written by the following matrix:

$$
\begin{bmatrix} C(f_{i-1}) \\ C(f_i) \\ d_{i-1} \\ d_i \end{bmatrix}
=
\begin{bmatrix}
1 & f_{i-1} & f_{i-1}^2 & f_{i-1}^3 \\
1 & f_i & f_i^2 & f_i^3 \\
0 & 1 & 2f_{i-1} & 3f_{i-1}^2 \\
0 & 1 & 2f_i & 3f_i^2
\end{bmatrix}
\begin{bmatrix} a_{i0} \\ a_{i1} \\ a_{i2} \\ a_{i3} \end{bmatrix}
\tag{6}
$$

The coefficients $(a_{i0}, a_{i1}, a_{i2}, a_{i3})$ can be obtained through the calculation of Eq. (7)

$$
\begin{bmatrix} a_{i0} \\ a_{i1} \\ a_{i2} \\ a_{i3} \end{bmatrix}
=
\begin{bmatrix}
1 & f_{i-1} & f_{i-1}^2 & f_{i-1}^3 \\
1 & f_i & f_i^2 & f_i^3 \\
0 & 1 & 2f_{i-1} & 3f_{i-1}^2 \\
0 & 1 & 2f_i & 3f_i^2
\end{bmatrix}^{-1}
\begin{bmatrix} C(f_{i-1}) \\ C(f_i) \\ d_{i-1} \\ d_i \end{bmatrix}
\tag{7}
$$

Substituting Eq. (7) into Eq. (5), we have the interpolation function $C(f_i)$ for the subspace $[f_{i-1},$ $f_i]$. According to the calculation principle of derivative in the shape-preserving piecewise cubic Hermite interpolation method [16, 17], it is simple to get the derivative according to the different locations of sampled points, which are one for the un-end sample point and another for the starting and the end point. Therefore, the compensation matrix $C(f)$ at any frequency can be got.

The root mean squared errors (RMSE) are used to test the accuracy of this method. The definition of the RMSE is

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(C(f_i) - C(f_i)^t\right)^2}, \tag{8}$$

where $C(f_i)$ is the value through interpolation method and $C(f_i)^t$ the true value of the compensation matrix function.

On the basis of the above principle of the shape-preserving piecewise cubic Hermite interpolation method, one can obtain the wideband mutual coupling compensation matrix. This method is advantageous in many aspects compared with the system identification method. Firstly, the calculation is simpler and real-time. The order of the polynomial in this method is just cubic, while the order in the system identification method reaches seven. Secondly, the bias of this method is less. The system identification method is implemented on the entire bandwidth and it is only effective when the mutual coupling matrix function against frequency is the monotonic function, which is invalid for the un-monotonic functions. In general case, the curves for each entry of the wideband compensation matrix against frequency are un-monotonic function with multiple extremum points when the bandwidth is broad. As comparison, the shape-preserving piecewise cubic Hermite interpolation method is more suitable for the un-monotonic function. Only extreme point and one or some points among the extreme value points are chosen as feature points for the application of this method, and one can obtain accurate wideband compensation matrix by the interpolation method. Once the compensation matrix is obtained, it can be employed on many super-resolution techniques such as direction of arrival (DOA) estimation algorithms and anti-jamming algorithms for wideband signals.

## 3. NUMERICAL EXAMPLES

In this section, a wideband dipole antenna and array with eight elements in uniform circular array are designed to verify the validity of the wideband compensation method proposed in this paper. Firstly, the antenna design and array structure are presented. Secondly, the effect of the proposed method for wideband mutual coupling is demonstrated by numerical examples.

### 3.1. Wideband Monopole Antenna and Array

To achieve wider bandwidth of dipole antenna, the method of inductive loading is applied. The profile of the element and the structure of the array are shown in Figure 1. As shown, there is an inductance in the middle of the two parts of one element. EM simulation tool Ansoft HFSS (High Frequency Structure Simulator) 15.0 is utilized to calculate the electric fields of the elements and the array. The VSWR (Voltage Standing Wave Ratio) curves of the element in isolate and array states are shown in Figure 2. The frequency bandwidth of the isolated element covers about 2.25 octaves with VSWR $\leq 2.2$. Besides, the center frequency is 368.5 MHz, and the relative bandwidth reaches 78%, which is much more than the relative bandwidth in previous methods [14, 15]. The VSWR of the array state is terrible compared with that of the isolated state, which shows that the performance of the array is deteriorated by the mutual coupling of the antenna elements. The radiation $R$ of the array is 450 mm, and the space of two adjacent elements is about $0.423\lambda_0$, where $\lambda_0$ is the wavelength of the center frequency.

### 3.2. Wideband Mutual Coupling Compensation

To realize the wideband mutual coupling compensation, the bandwidth is decomposed into 20 uniform narrowband components, and the sampling frequency interval $f_d$ is 14.35 MHz where there are 21 sampled points within the entire bandwidth. The element pattern reconstruction method is employed to
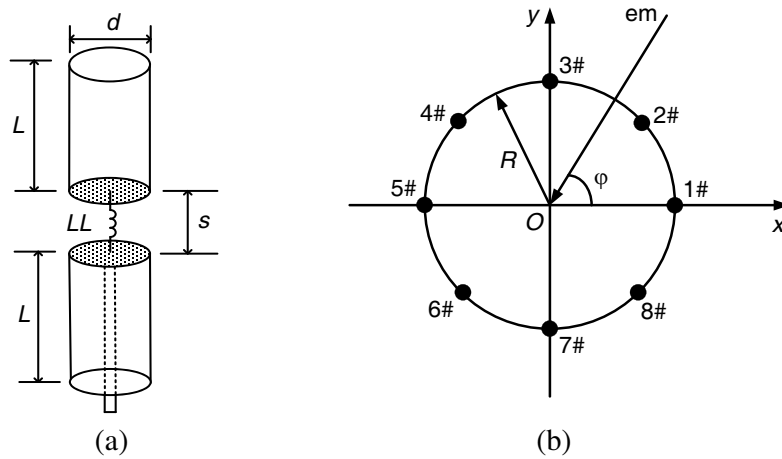
**Figure 1.** (a) Wideband dipole element profile and (b) array structures. $L = 235\,\text{mm}$, $d = 50\,\text{mm}$, $s = 8\,\text{mm}$, $LL = 0.02e - 6H$, $R = 450\,\text{mm}$.
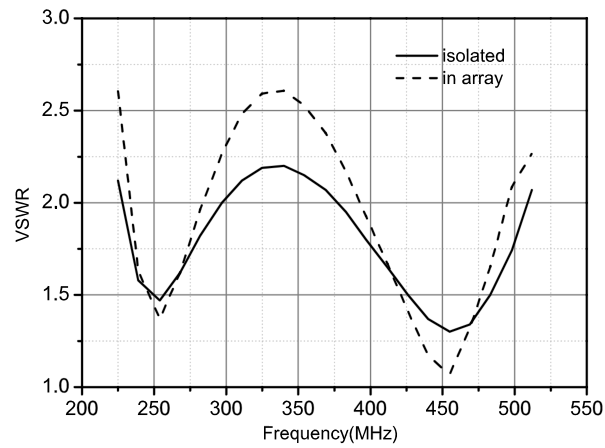


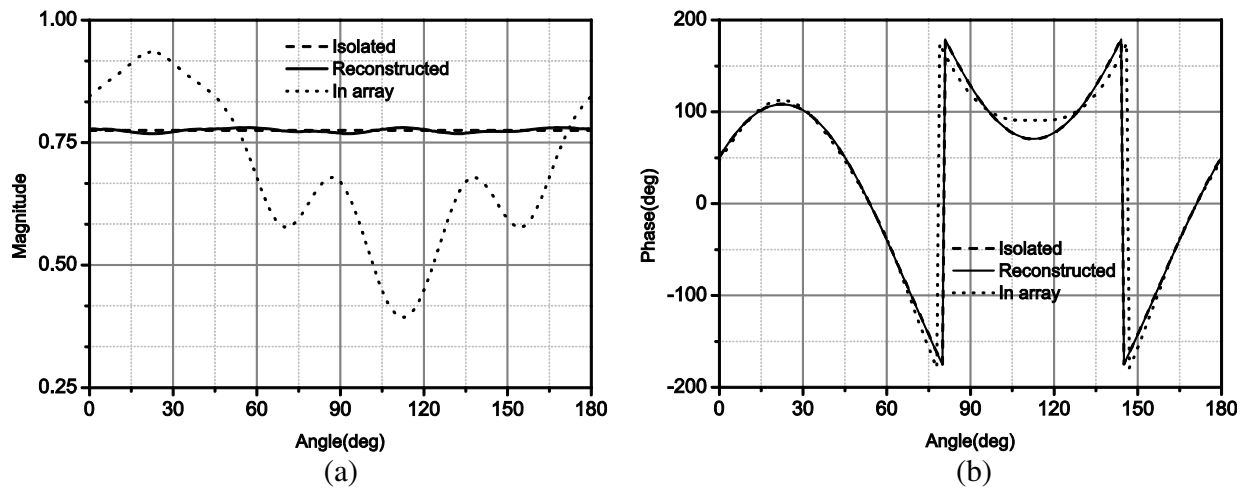**Figure 2.** VSWR curves of the element in various states.



**Figure 3.** Pattern reconstruction of electric field main polarization component for Element $8^{\#}$ at 368.5 MHz, (a) magnitude, (b) phase.

get the compensation matrix at each frequency point. As an illustration, the mutual coupling calibration of the array by EPR method at 368.5 MHz is shown in Figure 3 and Figure 4. In the *xoy* plane, the main polarization component is the $\theta$ component of the electric field. In order to calculate the calibration matrix via Eq. (3), 181 directions of the $\theta$ component of electric field are sampled from the angle range of $[0°, 180°]$ in the *xoy* plane. The magnitude and phase of the reconstructed pattern for element $8^{\#}$ at 368.5 MHz are shown in Figure 3. It shows that the isolated and reconstructed curves are almost consistent. The obtained calibration matrix is employed in the DOA estimation by the multiple signal classification (MUSIC) algorithm, as shown in Figure 4. It can be seen that the incident signals from $\varphi = 35°$ and $\varphi = 65°$ can be estimated by the MUSIC algorithm with calibration matrix obtained by element pattern reconstruct method. It demonstrates that the EPR method is effective for mutual coupling calibration.

The calibration matrix at 368.5 MHz is obtained through the element pattern reconstructed method by Equation (3), and the mutual coupling calibration effect can be seen in Figure 4. Similar to this example at 368.5 MHz, the other 20 sample frequency points can be obtained. Based on these compensation matrices, we can map the curves for each entry of the wideband compensation matrix against frequency. In the wideband compensation, we only need to select the extreme value points and one or some other sample points among the extreme value points for the accuracy. The mutual coupling compensation matrix at any frequency within the bandwidth can be obtained through this interpolation
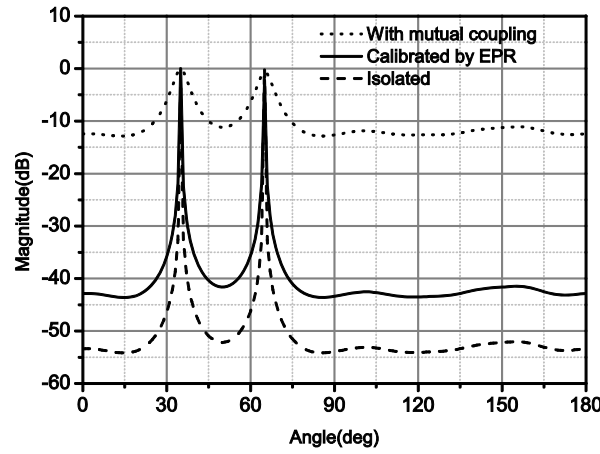


**Figure 4.** Spatial spectrum of the MUSIC algorithm for two incident signals at 368.5 MHz.
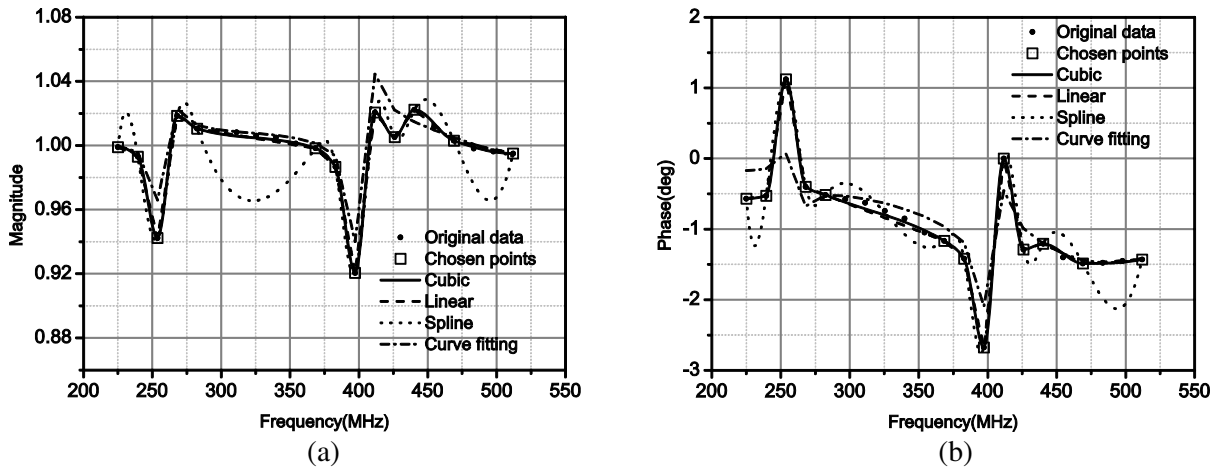


**Figure 5.** The curves for $C_{11}$ entry of the wideband compensation matrix against frequency, (a) magnitude, (b) phase.

method on the storage of necessary sampled points.

Based on the above analysis, we got the magnitude and phase curves for $C_{11}$ to $C_{15}$ entries of the wideband compensation matrix against frequency. These curves are listed as examples from Figure 5 to Figure 9, respectively. In the figures, the original data are original sample data, which are the data of the 21 sample frequency points. The chosen points are the feature points which we need to storage and in which the interpolation method will be employed. It is shown that various feature points are chosen corresponding to different curves. The curve fitting represents the complex-curve fitting method which was employed in [14, 15]. Three interpolation methods are compared, of which the Cubic is the method employed in this paper; the Linear is the linear interpolation; the Spline is the cubic spline interpolation. It can be seen that the linear interpolation function curve is not smooth at the feature points of the frequency band though the error is little. Although the curve obtained by the cubic spline interpolation method is smooth, there are overshoots and oscillation for the entire bandwidth, which are serious deviation from the original sample data. In comparison, the shape-preserving piecewise cubic Hermite interpolation method gets more accurate data, and the curves are smooth at the feature points, which avoids too many overshoots and oscillation from the original sample data.
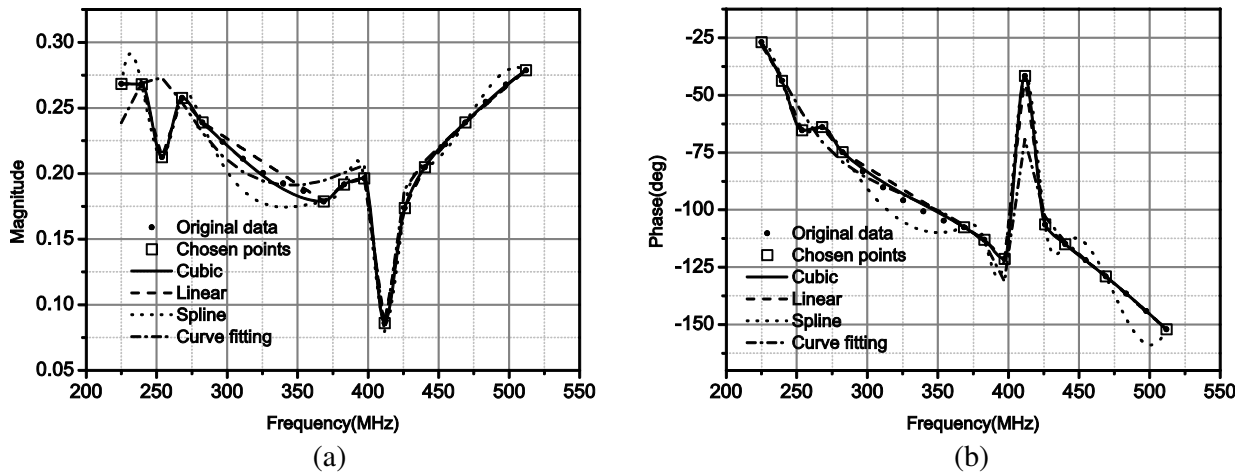


**Figure 6.** The curves for $C_{12}$ entry of the wideband compensation matrix against frequency, (a) magnitude, (b) phase.
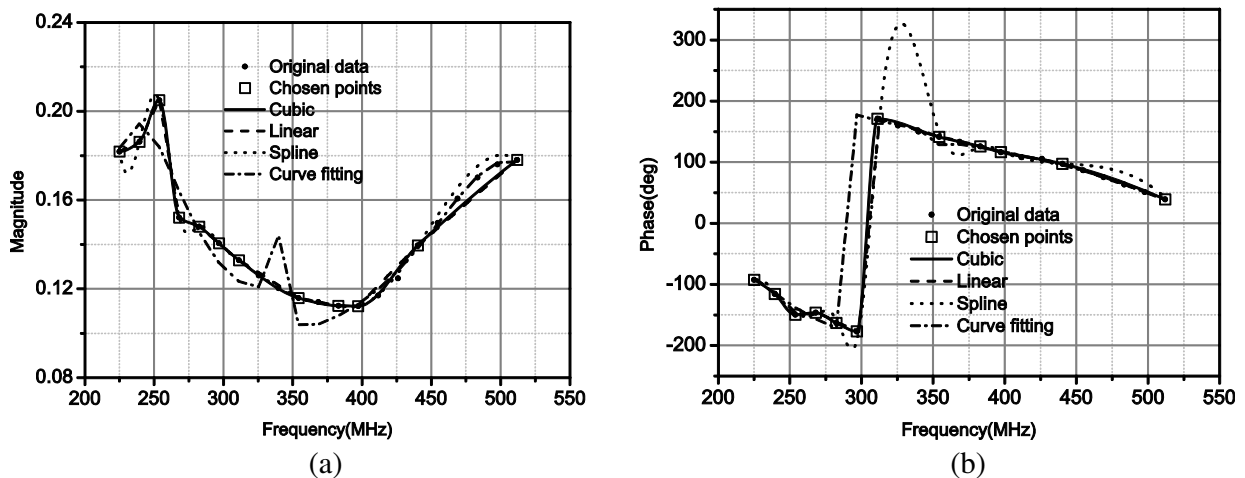


**Figure 7.** The curves for $C_{13}$ entry of the wideband compensation matrix against frequency, (a) magnitude, (b) phase.
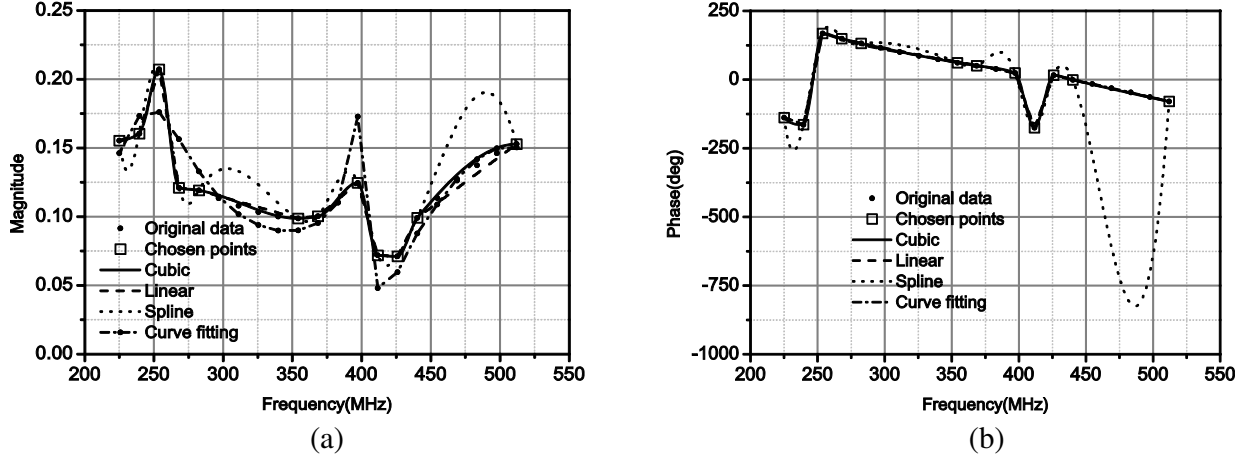
**Figure 8.** The curves for $C_{14}$ entry of the wideband compensation matrix against frequency, (a) magnitude, (b) phase.
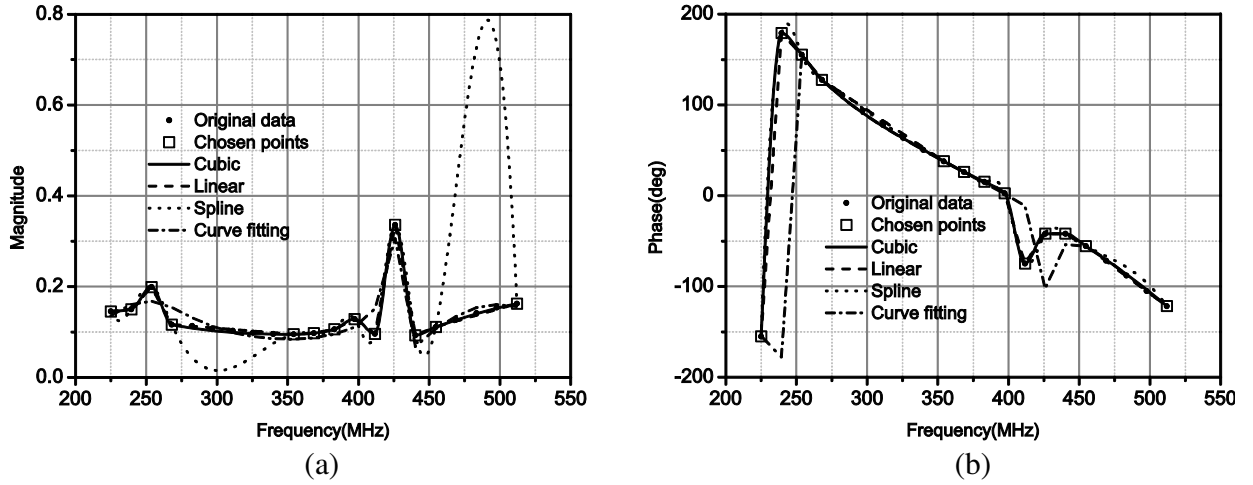


**Figure 9.** The curves for $C_{15}$ entry of the wideband compensation matrix against frequency, (a) magnitude, (b) phase.

**Table 1.** RMSE of the magnitude and phase for various methods.

| Method | Cubic | Linear | Spline | Piecewise Curve fitting |
|---|---|---|---|---|
| RMSE_magnitude | 0.0018 | 0.0053 | 0.1287 | 0.0305 |
| RMSE_phase (deg) | 1.1749 | 5.2021 | 120.8893 | 34.5095 |

In order to further illustrate the effectiveness of this method, the RMSE corresponding to the magnitude and phase for $C_{11}$ to $C_{15}$ are listed in Table 1. According to the character of the complex curve fitting method, the frequency band of the array is divided into two sections, and the curve fitting is employed on the divided two sections of the bandwidth. The data of piecewise curve fitting is obtained by the complex curve fitting method in the two sections.

It is shown that the RMSE of this method is much smaller than that of the other methods, and it has obvious advantage especially in the phase fitting. The entire calibration matrix can be obtained by the proposed method with less bias and more accuracy than other methods.

According to the structure of the circle array, the mutual coupling matrix is a circulant Toeplitz structure matrix. The wideband mutual coupling can be achieved by the following form:

$$C(f) = \begin{bmatrix} C_{11}(f) & C_{12}(f) & \ldots & C_{15}(f) & \ldots & C_{12}(f) \\ C_{12}(f) & C_{11}(f) & \ldots & C_{14}(f) & \ldots & C_{13}(f) \\ \vdots & \ddots & \ddots & \vdots & \ddots & \vdots \\ C_{15}(f) & C_{14}(f) & \ldots & C_{11}(f) & \ldots & C_{14}(f) \\ \vdots & \ddots & \ddots & \vdots & \ddots & \vdots \\ C_{12}(f) & C_{13}(f) & \ldots & C_{14}(f) & \ldots & C_{11}(f) \end{bmatrix}$$

The calibration matrix then can be employed on adaptive signal process algorithms, such as DOA estimation algorithms, beamforming algorithms and interference suppression algorithms, etc. Although the absolute bias on the RMSE of the magnitude between this method and the linear interpolation method is little, the relative bias to the amplitude range is large because the absolute amplitude range is small. Additionally, it takes only 0.4 ms to get the wideband mutual coupling compensation matrix which well satisfies the requirement of the real-time. The computation is operated by the MATLAB (MATrixLABoratory) 2012.b on the compute with 16 GBytes RAM (random access memory), Gigabyte P61-S3-B3 (Intel H61 (Cougar Point)) motherboard, 64-bit Windows7 operating system and Intel Core i5-2300 CPU. Compared with the complex curve fitting method, this method needs to store more data. However, it is very easy to implement and does not require more cost on the hardware for the digital circuit technology at present.

The results illustrate that this method has a good effect on wideband mutual coupling compensation. Although it also has some faults, its advantages for wideband application are more obvious than other methods.

## 4. CONCLUSION

This paper describes a new mutual coupling compensation method for wideband adaptive arrays. The shape-preserving piecewise cubic Hermit interpolation method is combined with the element pattern reconstruction method for wideband mutual coupling compensation matrices. Through the application of the element pattern reconstruction method, the curves for all entries of frequency-dependent mutual coupling matrix are obtained. The proposed method uses the feature points of the curves to provide accurate mutual coupling compensation matrix real-timely with less operation complexity. The validity of the wideband compensation method has been demonstrated by a wideband dipole antenna array with eight elements in uniform circular array.

## ACKNOWLEDGMENT

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