

# A Method to Extract Dielectric Parameters from Transmission Lines with Conductor Surface Roughness at Microwave Frequencies

Binke Huang\* and Qi Jia

**Abstract**—This paper details an effective method to extract dielectric parameters including dielectric constant  $D_k$  and loss tangent  $D_f$  from transmission lines containing rough conductor surface. The concept of effective conductivity is firstly introduced to model conductor surface roughness in transmission lines. By using differential extrapolation method, propagation parameters of transmission lines can be extracted by removing the roughness effects. A curve-fitting method based on Genetic Algorithm (GA) is adopted to fit the propagation parameters in the smoothed case and to derive the dielectric parameters. The proposed method is especially accurate for parameter extraction at high frequency and is practical to all types of transmission lines.

## 1. INTRODUCTION

Knowledge about substrate's dielectric parameters in planar transmission lines is necessary for numerical design and analysis of microwave components. Material datasheets from vendor may provide constant parameters which are valid at low frequencies. However, these are not sufficient since dielectric parameters vary apparently when the frequency changes over a wide range. Another problem is the effect of conductor surface roughness, which introduces more transmission loss and reduces the accuracy of parameter extraction. When transmission frequency increases, skin depth may decrease to the level of surface roughness. Therefore, effects of roughness become more obvious. During the fabrication process of PCB, however, the conductor surface roughness is indispensable as it provides more adhesion to dielectric substrate. Therefore, separation of roughness effect in transmission parameter is necessary to extract dielectric parameters accurately.

Zhang et al. presented an effective method for extracting parameters of a dispersive medium over a wide-frequency range using GA with a transmission line model [1]. Toda and de Flaviis have provided the measured complex permittivity at 60 GHz of a variety of common FR-4 and FR-5 type packaging materials with the covered transmission-line method [2]. Narayanan presented a novel microstrip transmission line method for broadband relative permittivity measurement of planar dielectric substrate materials [3]. However, all above works did not consider the interfacial surface roughness effects while modeling the planar transmission line. Horn et al. experimentally showed that there is a substantial effect on both transmission line insertion loss and velocity of propagation due to surface roughness in conductors [4]. Guo et al. proposed an approach to evaluate the conductor surface roughness effects on signal propagation, including signal attenuation and phase-delay, where a periodic structure model has been proposed to calculate equivalent roughened conductor surface impedance [5].

Herein, an effective method of extracting dielectric parameters from transmission lines is presented. This method focuses on the removal of roughness effect, accurate estimation of dielectric constant and loss tangent. Section 2 provides introduction to three fundamental theories: effective conductivity, differential extrapolation method and dielectric parameter extraction method with GA. The complete

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\* Corresponding author: Binke Huang (bkhuang@mail.xjtu.edu.cn).

The authors are with the School of Electronics and Information Engineering, Xi'an Jiaotong University, Xi'an 710049, China.

extraction process is described in Section 3. Firstly, effective conductivity is adopted to facilitate the calculation of transmission line propagation parameters with rough conductor. With the help of differential extrapolation method, propagation parameters in the smoothened case are derived from those of rough case. Dielectric parameter extraction method with GA is finally used to obtain  $D_k$  and  $D_f$ . In Section 4, experimental and simulated results are presented to support the proposed method. Section 5 gives the accuracy analysis of the proposed method at lower and extreme high frequencies. Section 6 summarizes this paper.

## 2. FUNDAMENTAL THEORIES

### 2.1. Effective Conductivity for Modeling Conductor Surface Roughness

Conductivity gradient model [6] can be used to describe the variation of conductivity in a rough conductor. In such model, conductivity varies from zero to bulk conductivity when material transforms from dielectric to conductor. A Cumulative Distribution Function (CDF) is used to represent this variability. Conductivity in this model can be calculated by

$$\sigma(x) = \sigma_b CDF(x) = \sigma_b \frac{1}{\sqrt{2\pi}R_q} \int_{-\infty}^x e^{-\frac{u^2}{2R_q^2}} du \quad (1)$$

where  $\sigma_b$  is the bulk conductivity and  $R_q$  the RMS of peak-to-valley roughness amplitude.

The conductivity gradient model can account for the additional loss due to surface roughness. The concept of effective conductivity is introduced to facilitate the calculation of transmission loss when the conductor has a rough surface. This method assumes that the rough conductor has a virtual smooth surface whose bulk conductivity is equal to the effective conductivity. The virtual smooth surface is equivalent to the actual rough surface in the sense that transmission losses calculated from these two surfaces are same, which is illustrated in Eq. (2).

$$\int \frac{1}{2} \frac{|\bar{J}_{rough}(x)|^2}{\sigma_{rough}(x)} dx = \int \frac{1}{2} \frac{|\bar{J}_{smooth}|^2}{\sigma_{eff}} dx \quad (2)$$

where  $\sigma_{rough}$  is the actual conductivity acquired from Eq. (1) and  $\sigma_{eff}$  the effective conductivity to be calculated.  $\bar{J}_{rough}$  can be calculated from Maxwell's equations with inhomogeneous conductivity  $\sigma(x)$  as follows.

$$\nabla \times \bar{B} = \frac{1}{c^2} \frac{\partial \bar{E}}{\partial t} + \mu_0 \bar{J} \quad (3)$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad (4)$$

$$\bar{J} = \sigma(x) \bar{E} \quad (5)$$

$\bar{J}_{rough}$  can be expressed from the solutions of Maxwell's equations for virtual smooth surface model with the unknown effective conductivity  $\sigma_{eff}$ . It demonstrates that the transmission loss calculated from the effective conductivity (right hand side of Eq. (2)) for a virtual smooth conductor is equal to that of the actual conductor.

### 2.2. Differential Extrapolation Method for Removal of the Effect of Surface Roughness

Conductor surface roughness can change the attenuation constant  $\alpha$  and phase constant  $\beta$ . These changes are especially obvious with higher surface roughness. Differential extrapolation method [7] can effectively remove these additional modifications which are not necessary for dielectric parameter extraction procedures. This method assumes that both attenuation constant and phase constant are attributing to dielectric parts and conductor parts, where dielectric parts are proportional to a sum of  $\omega$  and  $\omega^2$  terms and the conductor parts are proportional to  $\sqrt{\omega}$  term. Therefore, both the attenuation

constant  $\alpha$  and phase constant  $\beta$  can be curve-fitted by equations of type shown in Eqs. (6) and (7).

$$\alpha = A_1\sqrt{\omega} + A_2\omega + A_3\omega^2 \quad (6)$$

$$\beta = B_1\sqrt{\omega} + B_2\omega + B_3\omega^2 \quad (7)$$

When the surface roughness is considered, the curve-fitted forms does not change, but the fitting coefficients  $A_i$ ,  $B_i$  ( $i = 1, 2, 3$ ) vary with the RMS of conductor surface roughness.

The fitting coefficients  $A_i$ ,  $B_i$  are then plotted as a function of  $R_q$ . By extrapolating these curves to the case of an ideally smooth surface, i.e.,  $R_q = 0$ , the coefficients corresponding to the smoothed conductor can be derived.

### 2.3. Dielectric Parameter Extraction with Genetic Algorithm Method

To extract the dielectric parameter in a wideband, it is necessary to determine the frequency dispersion relation. Debye dependence law is used to describe its dispersive behavior. Herein, as illustrated by Eqs. (8) and (9), an 8th order Debye model [8] is considered.

$$\epsilon'_r = \epsilon_\infty + (\epsilon_s - \epsilon_\infty) \sum_{n=1}^8 \frac{g_n}{1 + (\omega\tau_n)^2} \quad (8)$$

$$\epsilon''_r = (\epsilon_s - \epsilon_\infty) \sum_{n=1}^8 \frac{g_n\tau_n\omega}{1 + (\omega\tau_n)^2} + \frac{\sigma_b}{\omega\epsilon_0} \quad (9)$$

where  $\epsilon_s$ ,  $\epsilon_\infty$  are static dielectric constant and relative permittivity at high-frequency respectively, and  $\sigma_e$  is the effective conductivity to be solved. These three parameters along with  $g_n$  and  $\tau_n$  are coefficients that need to be determined.  $\epsilon_0$  is the permittivity of free-space.

An analytical solution is then calculated with Eq. (10) to predict the propagation constant [9].

$$\gamma = \alpha + \beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (10)$$

where  $R$ ,  $L$ ,  $G$  and  $C$  are per-unit-length (p.u.l.) resistance, inductance, conductance, and capacitance parameters, respectively. These values are functions of dielectric parameters, on which an 8th order behavior is supposed. Calculation of where  $R$ ,  $L$ ,  $G$  and  $C$  are detailed in [1].

Besides, the actual propagation constant can also be obtained by either simulations or experiments. By appropriate curve-fitting method, the Debye coefficients and dielectric parameters can be extracted. Herein, a curve-fitting method based on GA is adopted. It is a heuristic search algorithm that mimics the process of natural selection [10]. This algorithm is especially effective for solving non-linear and higher dimensional optimization problems.

In this paper, objective function that needs to be minimized is defined by (11).

$$\Delta = \frac{1}{N} \sqrt{\sum_{i=1}^N \left\{ \left( \frac{|\alpha_m(f_i) - \alpha_a(f_i)|}{\max_i |\alpha_a(f_i)|} \right)^2 + \left( \frac{|\beta_m(f_i) - \beta_a(f_i)|}{\max_i |\beta_a(f_i)|} \right)^2 \right\}} \quad (11)$$

where  $\alpha_a(f_i)$  and  $\beta_a(f_i)$  are analytical propagation constants at a particular frequency  $f_i$ .  $\alpha_m(f_i)$  and  $\beta_m(f_i)$  are simulated or measured propagation constants at a particular frequency  $f_i$ .  $N$  is the number of frequencies considered in the extraction process.

## 3. THE PROCESS AND IMPLEMENTATION OF DIELECTRIC PARAMETERS

The flowchart of dielectric parameter extraction is described in Figure 1.

Firstly the effective conductivity is calculated for conductor with certain roughness, and this data is then imported into a commercial field solver (HFSS in this paper) to define a new material. Such material is equivalent to the original rough conductor if and only if the propagation constant is considered. Therefore, propagation constant of transmission line with rough conductor can be simulated easily in a field solver.

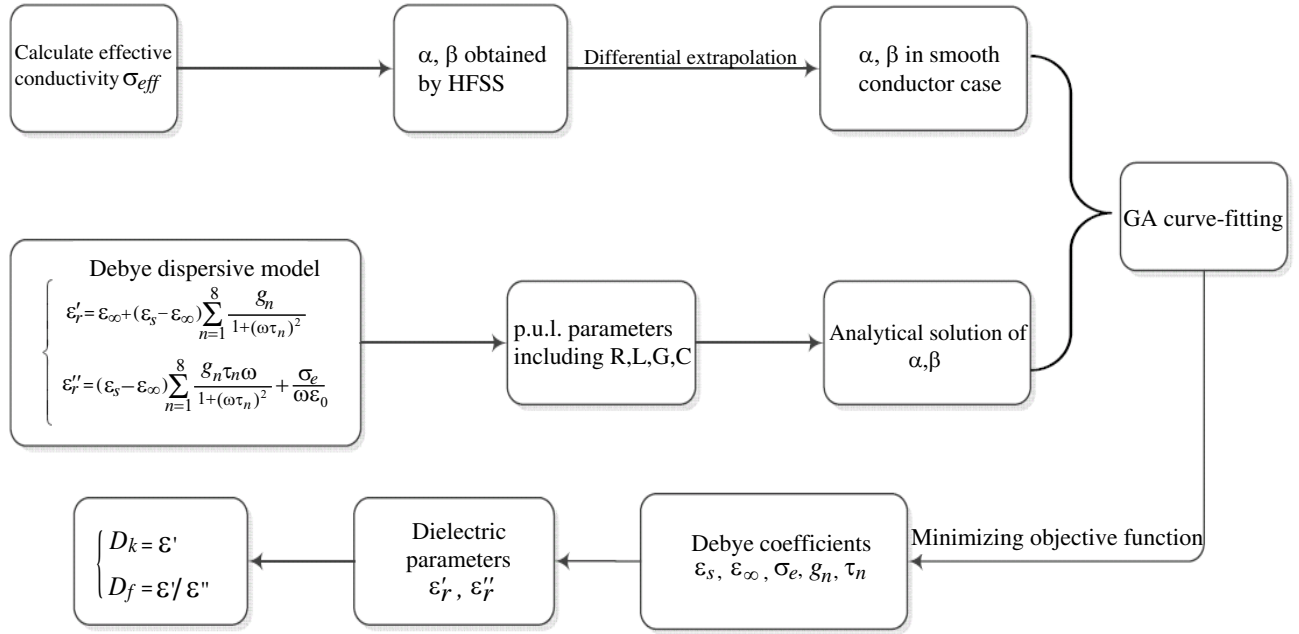
The following procedures require removal of surface roughness effects on the propagation constant. Thus, differential extrapolation method is taken to derive propagation constant of transmission lines containing smooth conductor from simulated or experimental values for rough conductor cases.

On the other hand, an 8th order Debye model containing unknown coefficients is adopted. With such model, analytical solution to the propagation constants, which contains the same undetermined coefficients, can be obtained from (10).

Since the analytical and experimental/simulated solution describes the same behavior, two curves need to coincide theoretically. Gap between the two curves is defined as the objective function in (11) and GA curve-fitting method is adopted to minimize it.

To accelerate the GA curve-fitting procedure, coefficients to be determined are recommended to fall in reasonable intervals.  $\tau_n$  is positive and smaller than 1e-9 since it denotes the relaxation time.  $g_n$  is usually greater than 0 and smaller than 1.  $\varepsilon_s$  and  $\varepsilon_\infty$  are in the order of static relative permittivity.  $\sigma_e$  is approximately equal to the bulk conductivity.

Once unknown coefficients including  $g_n$ ,  $\tau_n$ ,  $\varepsilon_s$ ,  $\varepsilon_\infty$  and  $\sigma_e$  are determined, one can have full knowledge of the proposed Debye dispersive model. Therefore, dielectric parameters including dielectric constant and loss tangent are finally extracted.

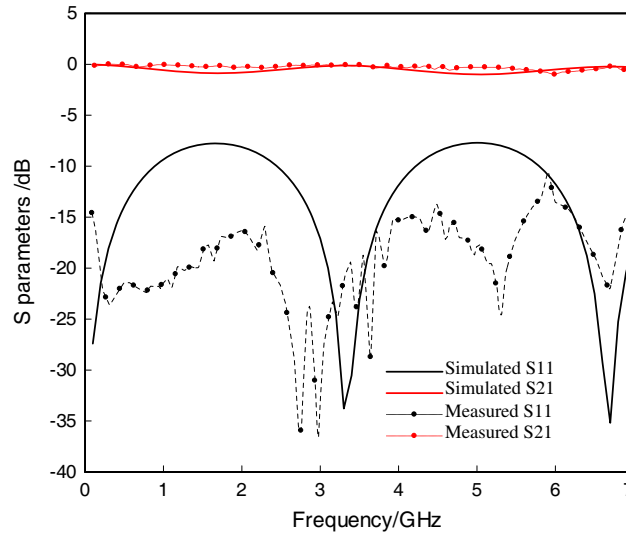


**Figure 1.** Flowing chart of the proposed method.

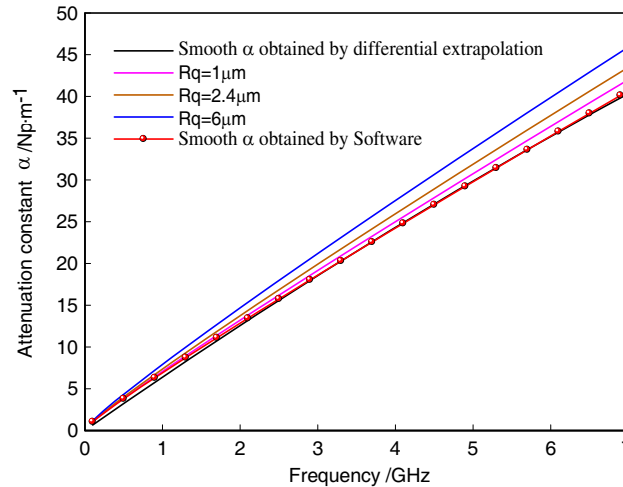
#### 4. RESULTS

Three microstrips test boards with different surface roughness ( $R_q$ ) are made to verify the proposed method. Characteristic impedance of these microstrips is 50 Ohm. All microstrips under test have the same geometrical specifications with width of the trace  $w = 1.6$  mm and thickness of the trace  $t = 0.035$  mm. The thickness of dielectric layer is 0.39 mm and width is 20 mm. Conductors of all the three groups are made of standard copper with a conductivity  $\sigma_{bulk}$  of 5.818e7 S/m, and dielectric material for all the microstrips is FR4. Surface roughness ( $R_q$ ) values of conductors in three microstrips are different, with values of 1  $\mu$ m, 2.4  $\mu$ m and 6  $\mu$ m respectively.

The  $S$ -parameters of one test board with the surface roughness  $R_q = 2.4$   $\mu$ m is measured with a Vector Network Analyzer (Agilent E8363B). Measured  $S_{21}$  and  $S_{11}$  are shown in Figure 2. Besides, a microstrip model with the same dimension is built in HFSS, and conductor in the model has an effective conductivity equivalent to the same degree of surface roughness. Figure 2 also gives the simulated  $S_{21}$  and  $S_{11}$  from HFSS.



**Figure 2.** Comparison between simulated and measured  $S$ -parameters.

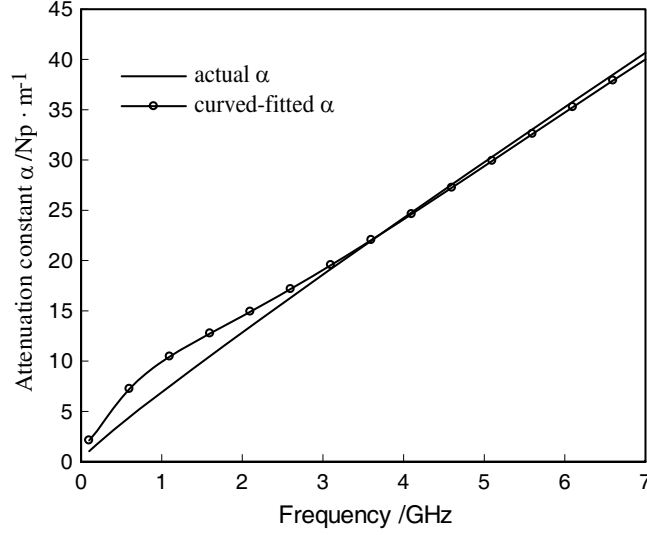


**Figure 3.** Comparison of attenuation constant obtained from different methods.

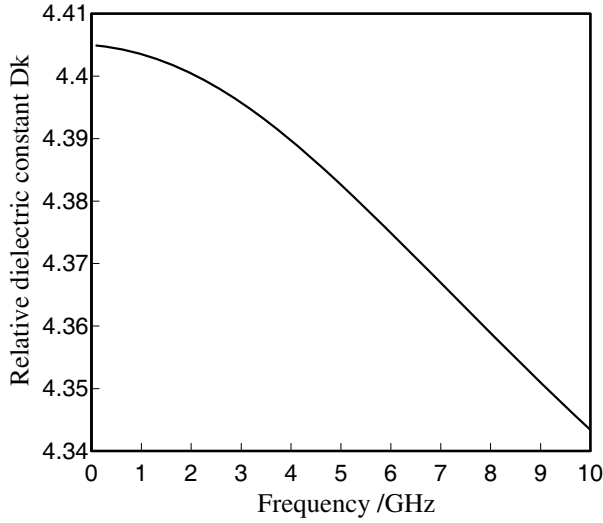
Figure 2 shows that the simulated  $s$  and measured  $S$ -parameters are in close agreement, which implies that the effective conductivity allows for an accurate prediction of microstrip propagation constant due to conductor surface roughness. Due to the material limitations and for the purpose of simplicity, simulated data are used hereinafter to extract the dielectric parameters.

Transmission losses of three microstrips containing rough conductor are plotted in Figure 3, where  $R_q$  of  $1\text{ }\mu\text{m}$ ,  $2.4\text{ }\mu\text{m}$  and  $6\text{ }\mu\text{m}$  are considered. With the differential extrapolation method, transmission loss curve corresponding to microstrip with smooth conductor is derived. Besides, the curve obtained directly from simulation for the same smoothened case is also given. The comparison shows that the two curves of same smoothness are very close to each other, which demonstrates that differential extrapolation method is effective and one can remove the effects of conductor surface roughness accurately.

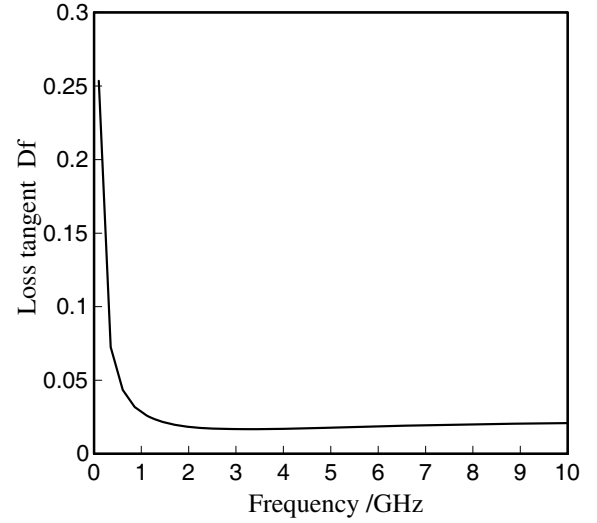
The analytical solution of attenuation constant with Debye parameters derived from curve-fitting method and simulated solution from a commercial field solver are shown in Figure 4. Discrepancy between these two curves is very small, especially at high frequency. Since the analytical solution depends on Debye parameters, a small discrepancy between analytical and simulated solution indicates an accuracy estimation of these parameters. Efficiency of GA curve-fitting method can therefore be corroborated.



**Figure 4.** GA curve-fitting result of attenuation constant.



**Figure 5.** Extracted  $D_k$  with proposed method.



**Figure 6.** Extracted  $D_f$  with proposed method.

When Debye parameters are determined by the proposed curve-fitting method, Eqs. (8) and (9) can be directly used to calculate the dielectric parameters. Figure 5 and Figure 6 show final results of extracted  $D_k$  and  $D_f$ . These parameters are extracted from HFSS microstrip model where  $D_k$  and  $D_f$  are known a priori. The actual  $D_k$  is 4.4 and  $D_f$  is equal to 0.02. At the frequency higher than 1.5 GHz, both  $D_k$  and  $D_f$  are accurate, with errors less than 1.3% and 4.6% separately. At a lower frequency, discrepancy between the actual  $D_f$  and the extracted  $D_f$  is large. Such result can be interpreted by the fact that Debye model describes the dispersive behavior of material in high frequency, which is not accurate in low frequency.

## 5. DISCUSSION OF THE PROPOSED METHOD

The proposed method mainly focuses on the GHz frequency region. As indicated in Section 4, the error of parameters extraction is less than 5% at this frequency range. However, the accuracy decreases at lower and extreme high frequencies. At lower frequencies, dipolar polarization of dielectric material can

still follow the oscillations of the external electric field, and the material doesn't exhibit phenomenon of dielectric relaxation and dielectric dispersion [11]. However, Debye model adopted in our proposed method is a dispersive dielectric model [12], and mainly matches properties of dielectric material in GHz range. At lower frequency region dielectric parameters are approximately constant, it is not necessary to use the Debye model to describe dielectric's behavior. In addition, if a dispersive Debye model is adopted to fit behaviors of non-dispersive dielectric, i.e., material with constant dielectric parameters, more errors may be introduced during the fitting process. For example, in [1] the reconstructed Debye dispersive material (FR4) parameters are accurate at GHz frequency, while the loss tangent predicted by the Debye coefficients is more than 0.14, which disagrees with the actual value of 0.014 provided by [13].

At extreme high frequencies (such as the sub-millimeter and terahertz waves), several problems may bring inaccuracies of the proposed method. Firstly, conductivity of conductors may follow Drude dispersion model and is no longer constant [14]. As a consequence, conductor loss will not be proportional to  $\sqrt{\omega}$ , which makes it meaningless to use the  $A_1\sqrt{\omega} + A_2\omega + A_3\omega^2$  form to fit the total loss and to remove surface the roughness of conductors. Secondly, the rule of conductor loss with frequency will also modify the derivation formulas of effective conductivity, because the calculation of effective conductivity is based on the conductor loss equivalent between the effective conductors with smooth surface and the actual rough conductors. Finally, at higher frequencies, radiation (or leakage) loss may not be negligible and non-TEM waves may appear in actual measurement. The multiple modes problem [9] makes finding analytical solution of total loss in optimization much more difficult. At present, we study the multiple modes separation method with parallel-plate transmission line model at terahertz band.

## 6. CONCLUSION

A method to extract dispersive dielectric properties for substrate in transmission line with rough conductor is presented in this paper. With the help of effective conductivity, transmission line containing rough surface was modeled in a field solver. Using differential extrapolation, experimental or simulated solution of smoothened case is then derived from the prior results. A curve-fitting method based on GA is used for determining the Debye coefficients in analytical expression.  $D_k$  and  $D_f$  can be finally extracted from curve fitting.

Due to a limitation of experimental materials, experimental data is only used to prove effectiveness of the modeling of the conductor surface roughness with effective conductivity and the method proposed in this paper mainly adopts the simulated data. However, this method is practical to the real material and can be applied to various types of transmission line. Results obtained with the proposed method are in good agreement with the actual dielectric parameters, especially at microwave frequencies. To describe dielectric parameters accurately in lower and extreme high frequencies, further research and experiments need to be conducted. The method proposed in this paper can throw a light on such further research.

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