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Numerical Estimation of the Complex Refractive Indexes by the Altitude Depending on Wave Frequency in the Ionized Region of the Earth Atmosphere for Microwaves Information and Power Transmissions

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Abstract—The phase and group refractive indexes of microwaves in the ionosphere region of the earth atmosphere are very important for both the researching theoretical problems and practical problems in wireless information transmission (WIT) and wireless power transmission (WPT). So far, there have been many attempts devoted to discussing and determining characterizations of earth atmosphere's ionosphere region including the refractive indexes of microwaves concerning their velocities in ionized region, unfortunately due to the complicated features of the ionosphere region leading to research task facing many challenges. Up to recent, there is still a lack of systematic numerical data of complex refractive index by altitude depending on high frequencies of the electromagnetic waves in the ionosphere region. This paper outlines some theoretical analyses and discussions of some theoretical aspects of the complex refractive index in atmosphere's ionized region more in detail. Based on conductivities data and the complex relative permittivity by altitude determined previously, the numerically estimated data of complex refractive indexes by the altitude from 100 km up to 1000 km at the different frequencies are also shown and discussed.

1. INTRODUCTION

The refractive indexes concerning the signal transfer velocities contain all interactions between electromagnetic radiation (EMR) and atmospheric medium. The complex refractive index data of the ionosphere medium play a very important role in both the theoretical and practical research problems of the WIT as well as of the WPT by microwave and laser power beams from GEO to the earth surface [1–7]. Currently, the investigation and calculation of numerical solutions for WIT and WPT mathematical problems from the Earth orbits to the earth surface also attract much attention. In order to solve analytically or numerically the WIT as well as WPT mathematical problems, one must have numerical data of the transfer environment such as the refractive indexes (phase and group), relative permittivity (ε_r), relative magnetic permeability (μ_r), conductivity (σ), total free electron density (N_e), ions concentrations (O^+, N^+, H^+, He^+ ...), and plasma concentration in the ionosphere region of atmosphere [7–11]. Up to recently, there are a lot of works studying many aspects of ionosphere region such as distribution of electron density, analytical methods, application of the locality principle to radio occultation studies of ionospheric and atmospheric layers from radio occultation data [12, 13]. Some other works investigated the measurements from the low-orbit satellite GPS/MET in the atmosphere at 2–35 km altitudes, and these results have been used to obtain global distributions of mesoscale variances

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of the radio wave refraction index [15, 16] and investigate the refractive index effects at low frequency, 12 MHz [17]. Unfortunately, there are few results concerning the complex refractive indexes by altitude from the earth surface to the ionosphere that have been published [18–21].

In our previous works [7, 9, 10], some theoretical issues of WPT and numerically estimated data of the relative permittivity (ε_r) by the altitude both in the non-ionized region and in the ionized region have been outlined. Following these works, this paper briefly outlines some theoretical analyses of complex refractive indexes of electromagnetic waves. After that, the paper outlines numerical estimation data of the complex refractive index by the altitude depending on wave frequencies in the ionized region from 100 km to 1000 km with the aims for applications in the numerical solution methods to WIT and WPT mathematical problems.

2. BRIEFLY ON THE REFRACTIVE INDEXES AND THE RELATED VELOCITIES OF ELECTROMAGNETIC WAVES IN THE ENERGY TRANSFER MEDIUM

The common refractive index (n) of a light or electromagnetic wave in the transmission medium is defined as the ratio of the speed of wave in vacuum to the velocity of radiation traveling the medium. If we denote the wavelength of the wave in a medium λ, its wavelength in vacuum is λ_0, wave frequency f and wave's velocity v, then the common refractive index can be written as follows [1, 2, 5, 6, 8, 11, 14–16, 19–21]:

$$n = \frac{\lambda_o}{\lambda} = \frac{c/f}{v/f} = \frac{f \cdot c}{f \cdot v} = \frac{c}{v} \tag{1}$$

$$n = \frac{c}{v} = \frac{\frac{1}{\sqrt{\varepsilon_o \cdot \mu_o}}}{\frac{1}{\sqrt{\varepsilon_r \cdot \mu_r \varepsilon_o \cdot \mu_o}}} = \sqrt{\varepsilon_r \cdot \mu_r} \tag{2}$$

Here $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 299792458\,\text{m/s}$ is the speed of light in vacuum; ν is the velocity of wave in transfer medium; ε_r and μ_r are relative dielectric permittivity and relative magnetic permeability of the propagating medium, respectively. In order to determine the refractive index based on Eq. (2), either the related velocity (v) or relative dielectric permittivity (ε_r) and relative magnetic permeability (μ_r) must be determined. As known, there are many kinds of velocities related to aspects of the electromagnetic wave such as phase velocity, group velocity, particle velocity, wave velocity and energy velocity based on the classic approach, but here we investigate only two kinds, the phase and group velocities concerning phase and group refractive indexes [10, 20, 21]. The relationship between the phase refractive index and phase velocity as well as between the group refractive index and group phase velocity for all values of dispersion can be defined as follows [1, 2, 10, 20–23]:

$$n_{ph} = \frac{c}{v_{ph}} \tag{3}$$

$$n_{gr} = \frac{c}{v_{gr}} \tag{4}$$

If we multiply Eqs. (3) with (4), after arrangement, we have the general expression:

$$v_{ph} \cdot \nu_{gr} \cdot n_{ph} \cdot n_{gr} = c^2 \tag{5}$$

This is the essential general relation of relationship among phase velocity, group velocity, phase refractive index, group refractive index and speed of light. This relation is valid for the whole range of electromagnetic radiation. Unfortunately, in the cases of perfect lossless hollow waveguide with air inside and the matter waves exhibiting wave-particle duality used in quantum theory, several authors write a similar expression [23–25]

$$\nu_{ph} \cdot \nu_{gr} = c^2 \tag{6}$$

Eq. (6) is a special case of Eq. (5). In this case, the phase and group refractive indexes are equal to 1 because the role of transfer medium in these cases is considered as a vacuum, or the space medium is limited in the atomic space level... From Eq. (6), we can see that one kind of velocities could be larger than the light speed (c), and another kind of velocities is smaller than light speed (c) so that the result of their multiplication is equal to 1.

3. THE NUMERICAL ESTIMATION OF THE COMPLEX REFRACTIVE INDEX BY ALTITUDE IN THE IONOSPHERE REGION

3.1. General Expression of Complex Refractive Index in the Ionosphere Region

In the ionosphere region, there are many complex processes taking place simultaneously. The refractive index (n) contains in itself almost interactions among the propagating electromagnetic wave and the medium parameters being in the ionized region [1, 5, 6, 11, 14–16, 18–21]. Commonly, the phenomena of photo ionization, recombination, diffusion, photon absorption, and emission of ions as well as emission of electrons always take place in this region. The conductivity (σ) of the ionosphere region depends not only on wave frequency (f), electron collision frequency, free electron density (N_e), ion densities of particles (O^+ N^+, H^+, He^+ ...), but also on traveling direction of the electromagnetic wave. In addition, the effects of earth magnetic field (B_o) on free electrons and microwave signal also directly influence the refractive index (n) as well as the relative dielectric permittivity (ε_r), and relative magnetic permeability (μ_r) of the ionized atmosphere [1, 5, 6, 11, 18, 19, 23–27]. The ionosphere region is considered a dispersive medium with respect to microwave signals, which means that the propagation path of microwave signal through the ionosphere depends mainly on the frequency of the microwave signal. If the collision effects of the particles ions are ignored, the formula for the *phase ionosphere refractive index*, called Appleton-Hartree Equation, can be presented by as follows [1, 22, 23, 28].

$$n_{ph}^2 = 1 - \frac{A}{1 - jC - \left(\frac{B_T^2}{2(1-A-jC)}\right) \pm \left(\frac{B_T^4}{4(1-A-jC)^2} + B_L^2\right)^{1/2}} \tag{7a}$$

Here the dimensionless quantities A, B_T, B_L and C and related parameters are defined as follows:

$$A = f_N^2/f^4;\ B = f_B/f;\ B_L = \frac{f_B}{f}\cos\theta;\ B_T = \frac{f_B}{f}\sin\theta;\ C = \frac{f_c}{f};\ f_N = \left(\frac{N_e \cdot e^2}{\varepsilon_0 m_e}\right)^{1/2};\ f_B = \frac{B_o \cdot e}{m_e} \tag{7b}$$

where n_{ph} is the complex phase refractive index, ω the angular frequency of traveling wave, f the wave frequency, f_N the angular plasma frequency, f_B the electron gyro-frequency, θ the angle between the propagation direction and the Earth magnetic field, f_c the electron collision frequency, N_e the total electron density, ε_0 the permittivity of free space, B_0 the magnitude of the magnetic field vector, e the electron charge and m_e the electron mass.

We can see from Eq. (7a) that the phase refractive index of the ionosphere region is a complex number represented by the term of "jC" in the denominator.

Depending on the frequency of the electromagnetic wave and the angle (θ) between the propagation direction of the wave and the Earth magnetic field (B_0), the above Eq. (7a) will be modified by the discussions based on several conditions as follows:

a) When propagation direction of the traveling wave is roughly parallel to B_o, it is named the quasi-longitudinal (QL) approximation. The wave here is called the ordinary wave (O-wave); in this case, the E-field accelerates electrons parallel to the magnetic field. This means that the magnetic field has no influence because a magnetic field only imposes a force on charged particles moving perpendicular to the field. Corresponding to this case, the conductivity called field-aligned conductivity (σ_{FA}) is calculated [22–28].

b) When propagation direction of the wave is roughly perpendicular to B_o corresponding to $\theta \simeq 90°$ and $B_L \simeq 0$, the propagation direction is named quasi-transverse (QT) approximation. The wave here is called the extraordinary wave (X-wave). In this case, the E-field of the incident radiation accelerates the free electrons normal to the magnetic field. This means that it exerts a force on the free electrons and therefore modifies their motions. This causes the refractive index of the extraordinary wave to be different from the ordinary wave. According to this case, the conductivity so called Pedersen Conductivity (σ_P) is calculated [22–29].

c) When one takes the effects of the electron density within the ionosphere and the effect of the Earth's magnetic field and its interactions with the ionosphere, one will get the real refractive index expression of Appleton-Hertree formula with almost terms of the first order and the higher order [22, 23, 29, 30].
d) When one takes into account only the effects of the electron density within the ionosphere, one could have only the first order refractive index in Appleton-Hertree formulas, and then one will get very a simple form of the refractive index expression with the real value [22–24, 27–30]. Below, we will discuss this case and outline the real phase refractive index expression more in detail.)

3.2. The Phase Refractive Indexes Expression in the Ionized Region

For high frequency (HF) electromagnetic waves, the *phase refractive index* n_{ph} can be derived from the Appleton-Hartree formula (7a) because C dimensionless quantity is neglected at high frequency, yielding a simplified relation not containing the imaginary terms as the following [22, 27–30]:

$$n_{ph} = 1 - \frac{f_p^2}{2f^2} \pm \frac{f_p^2 \cdot f_g \cdot \cos\theta}{2f^3} - \frac{f_p^2}{4f^4}\left[\frac{f_p^2}{2} + f_g^2\left(1+\cos^2\theta\right)\right] \tag{8a}$$

where

$$f_p^2 = \frac{N_e \cdot e^2}{4\pi^2\varepsilon_o m_e}; \quad f_g = \frac{eB}{2\pi m_e} \tag{8b}$$

The meanings of parameters here are described in the above section. The wave with the upper (+) sign in Eq. (8a) is corresponding to the "O-wave" and is left-hand circularly polarized, whereas the wave related to the lower (−) sign is corresponding to the"X wave" and is right-hand circularly polarize [22, 26–28]. Furthermore, if we only take into account the effects of the electron density within the ionosphere, then we could have to consider only the first order refractive index in Appleton-Hertree simpler formula (8a). Since the third- and fourth-order terms are orders of magnitude smaller than the second-order term, they are usually neglected in the first approximation. The equation of phase reflective index in Eq. (8a) with the first two terms is denoted as the first order phase refractive index. After putting the constants values of e, m_e, π, ε_o, as mentioned in above, the approximated equation of the first order phase reflective index in ionosphere region can be reduced to the following form [28–31]:

$$n_{ph} \approx 1 - \frac{f_p^2}{2f^2} = 1 - \frac{N_e . e^2}{2.4\pi^2\varepsilon_o m_e f^2} = 1 - 40.31\frac{N_e}{f^2} \tag{9}$$

From Eq. (9) we can see that the value of phase reflective index could be negative if the value of frequency (f) is small, i.e., the value of the second term is larger than 1.

Briefly, the refractive indexes in the ionosphere region depend on "O-wave" and "X-wave". Each of these waves travels a completely independent path through the ionosphere. At higher frequencies, the ordinary and extraordinary waves often follow very similar paths. At lower frequencies, the ordinary and extraordinary waves will diverge more considerably [19, 20, 22, 28, 31–33].

3.3. The Expressions of Relative Permittivity and Complex Refractive Index for Numerical Estimation

In the dispersion plasma medium when the electromagnetic wave passes through the medium, some of their parts will be attenuated. Therefore, the refractive index will be described by a complex refractive index (here we denote it by n_c) as follows [19, 22, 23, 28]:

$$n_c = n \pm i\kappa \tag{10}$$

where "n" is the real part, concerning phase refractive index (equal n_{ph} in Eq. (9)), and is related to the phase velocity. The imaginary part (κ) is called the *extinction coefficient*, and it is used to quantify the gain or attenuation amount by absorption when the electromagnetic wave propagates through the medium. Eq. (10) can receive a positive or negative sign as denoted in Eq. (10). Here it is worth to note that κ term gives an exponential decay, since intensity is proportional to the square of the electric field and will have attenuation coefficient in the form of $\alpha = 4\pi\kappa/\lambda_0$ [19, 22, 27, 28]. Both n and κ are dependent on the frequency. The complex refractive index has an alternative form, and it could be $n_c = n - i\kappa$ instead of $n_c = n + i\kappa$ as in Eq. (10), but if $\kappa > 0$, it still corresponds to loss. So, these two forms of n_c are inconsistent, but these should not be confused as the difference is related to defining sinusoidal time dependence as the term of $\text{Re}[\exp(-i\omega t)]$ versus $\text{Re}[\exp(+i\omega t)]$ [1, 19, 22, 26–28]. In order to numerically estimate the complex refractive index here we have to use several conditions and constraints:

- The complex refractive index in ionosphere region described by the Eq. (7a) is a complicated form that includes many effects and interactions between microwave signal and ionosphere region. Eq. (7a) in principle can be described in the complex form as in the simpler form of Eq. (10) for the numerical estimation.
- From Eq. (2): $n = \sqrt{\varepsilon_r \cdot \mu_r}$ we see that the refractive index can be estimated numerically when the relative permittivity (ε_r) and relative magnetic permeability (μ_r) are given. Here we assume that the relative magnetic permeability $\mu_r \simeq 1$. This assumption is usually acceptable for non-magnetic medium, then we have $n = \sqrt{\varepsilon_r}$. *In this case*, the complex refractive index (n_c) can be written as follows:

$$n_c^2 = (n + i\kappa)^2 = \varepsilon_r = \varepsilon_r' + i\varepsilon_r'' \tag{11}$$

where ε_r' and ε_r'' are the real and imaginary parts of the relative permittivity of the ionosphere region.

As known, the complex relative dielectric permittivity in the ionized atmospheric region can be written by the following form [9, 10, 22, 23, 26–28]:

$$\varepsilon_r = 1 - 4\pi \frac{N_e \cdot e^2}{\varepsilon_o m_e \omega^2} - i\frac{4\pi\sigma}{\omega} = \varepsilon_r' + i\varepsilon_r'' \tag{12}$$

where the real and imaginary parts of the relative permittivity can be separately written:

$$\varepsilon_r' = 1 - 4\pi \frac{N_e \cdot e^2}{m_e \varepsilon_o \omega^2} \tag{13}$$

$$\varepsilon_r'' = \frac{4\pi\sigma}{\omega} \tag{14}$$

where σ is denoted for common conductivity in the ionosphere region. The meanings of other parameters have been described in the above. Here it is worth to notice that Eqs. (11) and (12) are related to the phase complex refractive index corresponding to phase velocity of signal. Based on these assumptions, now the real and imaginary parts of the phase complex refractive index can be written as follows:

$$n = \frac{1}{\sqrt{2}}\left(\left(\varepsilon_r'^2 + \varepsilon_r''^2\right)^{1/2} + \varepsilon_r'\right)^{1/2} \tag{15}$$

$$\kappa = \frac{1}{\sqrt{2}}\left(\left(\varepsilon_r'^2 + \varepsilon_r''^2\right)^{1/2} - \varepsilon_r'\right)^{1/2} \tag{16}$$

3.4. Numerical Estimation Results of Complex Refractive Index by Altitude in the Ionized Region

As known, the literature shows a few results on estimating relative permittivity and refractive index by altitude from 100 km to 1000 km depending on the frequency of the electromagnetic wave in the ionized region. This could be due to a lack of publically reliable models and systematic experiment data on total free electron density and conductivities by altitude in whole range from 100 km — about 1000 km as previously mentioned [7, 9, 10, 12–17, 22–28, 32].

Depending on the density and moving directions of electrons as well as of particles ions under the electric and magnetic fields in the ionized region, different kinds of conductivities for electromagnetic waves are introduced; they are parallel field-aligned conductivity (σ_{FA}), transverse conductivities Pedersen conductivity (σ_p) and Hall conductivity (σ_h). The parallel field-aligned conductivity strongly increases with altitude due to decreasing collision frequency of electrons with the neutral gas or ions.

The parallel conductivity value is always much higher than that of the conductivity perpendicular to the magnetic field [29, 31–35]. As Hall conductivity (σ_h) data are only up to 400 km, their values are small, so in this paper, we only use two kinds of conductivities published [9, 25, 34]: the parallel field-aligned conductivity (σ_{FA}) and transverse conductivities-Pedersen conductivity σ_p) for numerical estimations.

In order to numerically estimate the values of the phase complex refractive indexes with altitude in the ionized atmosphere region based on Eqs. (12) to (16), we get the values of constants as follows: $\varepsilon_0 = 8.854187817 \times 10^{-12}\,\mathrm{F{\cdot}m^{-1}}$, $e = 1.602176487 \times 10^{-19}\,\mathrm{C}$, $m_e = 9.10938215 \times 10^{-31}\,\mathrm{kg}$. ω is angular frequency of the electromagnetic wave and is chosen at several different values for numerical estimations. The free electron density values (N_e) with altitude, values of field-aligned conductivity (σ_{FA}) and Pedersen conductivity (σ_p) by altitude have been determined in our previous work [9] from the published graphs and data [25, 34] by the fitting procedure (Get data Graph Digitizer) on computer [9]. The determined data of the ionized electron density (N_e) and two kinds of conductivities (field-aligned, and Pedersen) are also shown in work [9]. The numerical estimation of complex relative permittivity at different frequencies (20 MHz, 30 MHz, 40 MHz, 50 MHz, 100 MHz, 1 GHz and 2.45 GHz) is also calculated again, and their results are shown in Figs. 1–12.

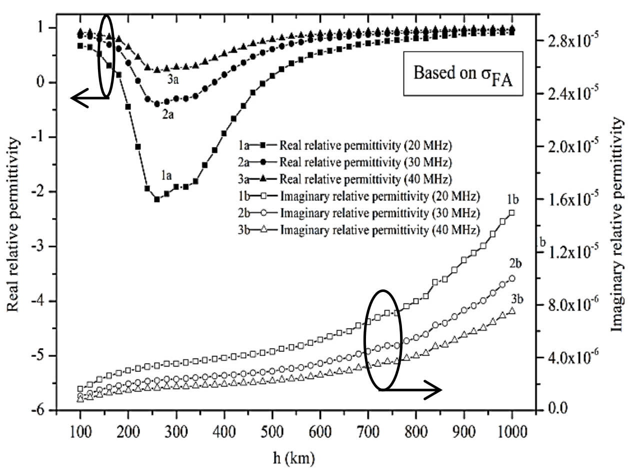

Figure 1. Numerical estimated data of complex relative permittivity (ε_r) by altitude based on the parallel Field-Aligned conductivity (σ_{FA}) at frequency 20 MHz (1a, 1b), at frequency 30 MHz (2a, 2b) and at frequency 40 MHz (3a, 3b).

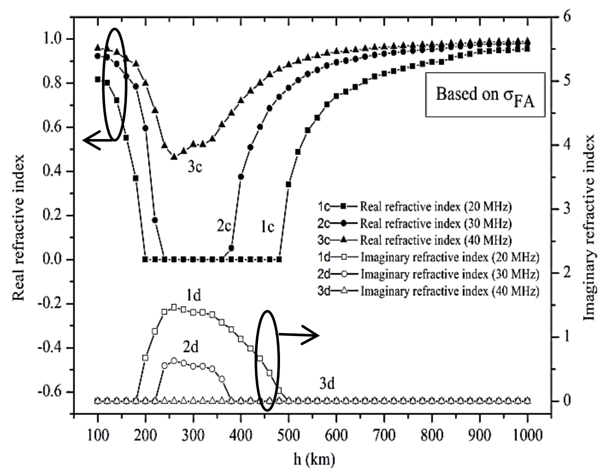

Figure 2. Numerical estimated data of complex refractive index (n_c) by altitude based on the parallel Field-Aligned conductivity (σ_{FA}) at frequency 20 MHz (1c, 1d), at frequency 30 MHz (2c, 2d), and at frequency 40 GHz (3c, 3d).

4. DISCUSSIONS AND CONCLUSIONS

Figures 1 to 4 show the calculated results of the complex relative permittivity and refractive index based on the parallel field-aligned conductivity (σ_{FA}). The real parts of relative permittivity (ε'_r) varied from the negative value of "-2" at frequency 20 MHz to about nearly "1" at 1 GHz. The magnitudes of the complex parts (ε''_r) of relative permittivity estimated based on the parallel field-aligned conductivity are values large enough, and their magnitudes varied in the range of 10^{-6} to 10^{-5}. The real parts of refractive indexes varied in the range of from 0.7 to 1 value; meanwhile, the values of the imaginary parts varied in the range of 1 to 10^{-6} when frequency varied from 20 MHz to 1 GHz.

Figures 5 to 8 show the estimated results of the complex relative permittivity and refractive index based on the transverse Pedersen conductivity (σ_P) at different frequencies. The real parts of relative permittivity (ε'_r) varied from "-2" value at frequency 20 MHz to nearly "1" at 1 GHz. The magnitudes of the complex parts (ε''_r) of relative permittivity are very small, and their magnitudes varied in the range of from 0–1.6×10^{-19}. The real parts of refractive indexes based on the transverse Pedersen conductivities varied in the range of 0.7 to 1 value; meanwhile, their imaginary parts were also very high in the altitude from 200 km to 500 km at frequencies of 20 MHz and -30 MHz.

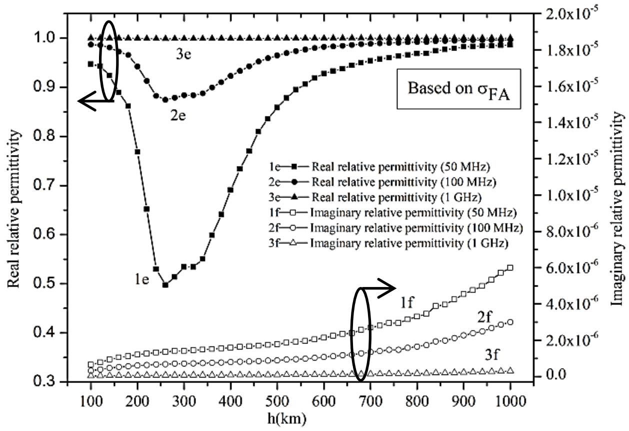

Figure 3. Numerical estimated data of complex relative permittivity (ε_r) by altitude based on the parallel Field-Aligned conductivity (σ_{FA}) at frequency 50 MHz (1e, 1f), at frequency 100 MHz (2e, 2f), and at frequency 1 GHz (3e, 3f).

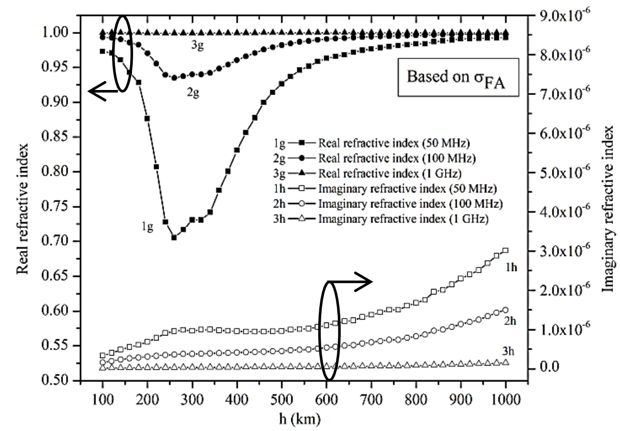

Figure 4. Numerical estimated data of complex refractive index (n_c) by altitude based on the Field-Aligned conductivity (σ_{FA}) at frequency 50 MHz (1g, 1h), at frequency 100 MHz (2g, 2h), and at frequency 1 GHz (3g, 3h).

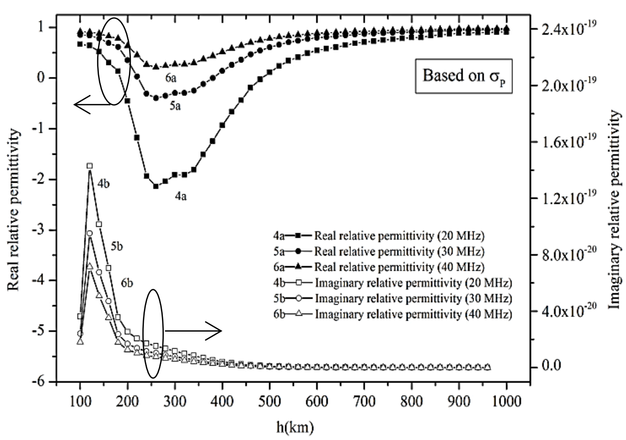

Figure 5. Numerical estimated data of complex relative permittivity (ε_r) by altitude based on the transverse Pedersen conductivity (σ_p) at frequency 20 MHz (4a, 4b), at frequency 30 MHz (5a, 5b) and at frequency 40 MHz (6a, 6b).

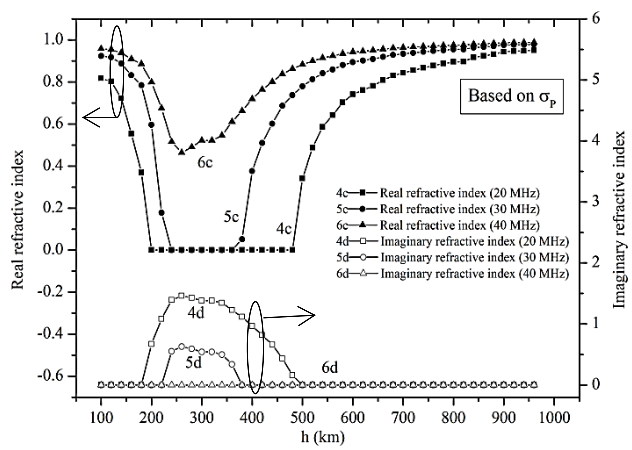

Figure 6. Numerical estimated data of complex refractive index (n_c) by altitude based on Pedersen conductivity (σ_p) at frequency 20 MHz (4c, 4d), at frequency 30 MHz (5c, 5d) and at frequency 40 MHz (6c, 6d).

From Fig. 1 and Fig. 5, we also see that at the altitude range of ionosphere region from 200 km to 500 km where the total free electron density has abnormal behavior [25], the numerical estimated results of relative permittivity have negative values at 20 MHz and 30 MHz. This phenomenon is explained based on Eq. (13) where the value of the second term in Eq. (13) depending on the frequency, which could be larger than 1, leading to the fact that real part (ε'_r) will get a negative number.

The calculated results also show that at the low or medium microwave frequencies, the refractive index is a complex number. When frequency (f) of an electromagnetic wave comes to very high frequency, for example, to 1 GHz, the imaginary parts of refractive index decreases to nearly zero value, leading to the refractive index becoming the real value. This fact is agreed with the theoretical estimation based on Eq. (7a) where the "C" terms in Eq. (7a) come to zero ($C = f_c/f \rightarrow 0$) when the wave frequency is increased. The refractive index becomes a real value.

Concerning the negative relative permittivity and negative refractive index, as known, the recent

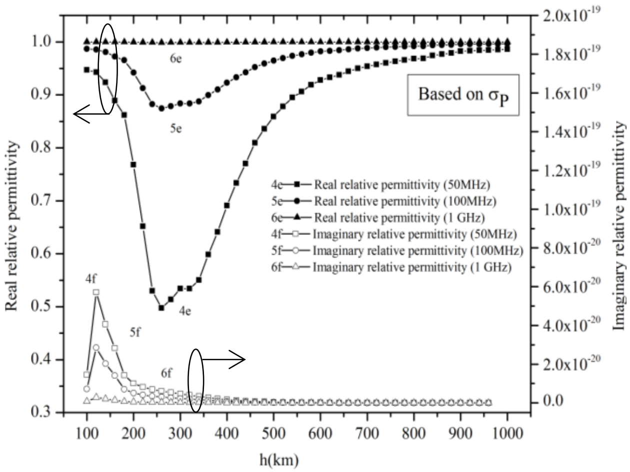

Figure 7. Numerical estimated data of complex relative permittivity (ε_r) by altitude based on the transverse Pedersen conductivity (σ_p) at frequency 50 MHz (4e, 4f), at frequency 100 MHz (5e, 5f), and at frequency 1 GHz (6e, 6f).

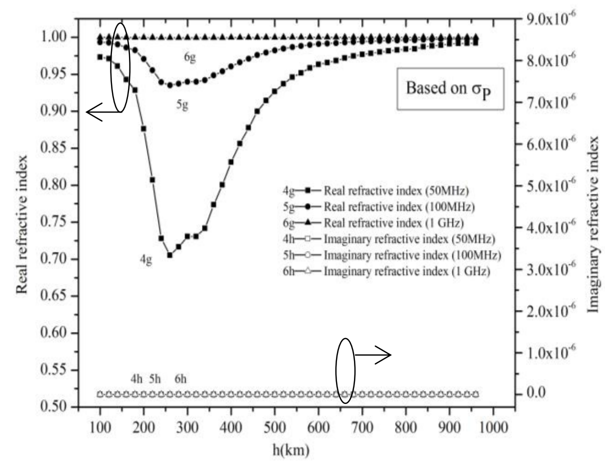

Figure 8. Numerical estimated data of complex refractive index (n_c) by altitude based on the transverse Pedersen conductivity (σ_p) at frequency 50 MHz (4g, 4h), at frequency 100 MHz (5g, 5h) and at frequency 1 GHz (6g, 6h).

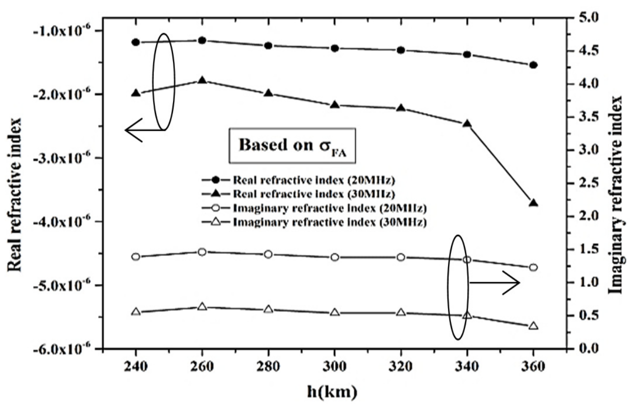

Figure 9. Numerical estimated data of negative complex refractive index (n_c) corresponding to the negative relative permittivity being in Fig. 1 by altitude from 240 km to 360 km based on the parallel Field-Aligned conductivity (σ_{FA}) at frequency 20 MHz, and at frequency 30 MHz.

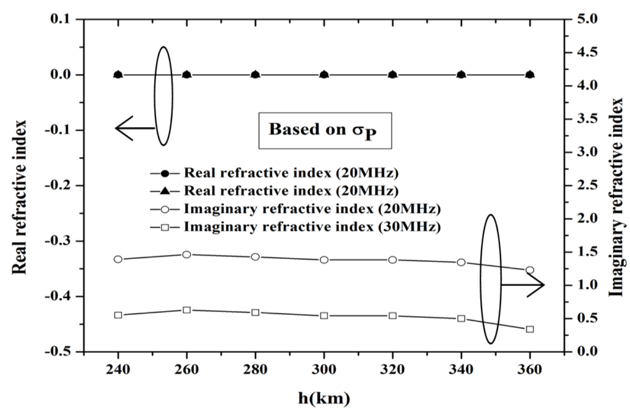

Figure 10. Numerical estimated data of the negative complex reflective index (n_c) corresponding to the negative relative permittivity being in Fig. 5 by altitude from 240 km to 360 km based on the transverse Pedersen conductivity (σ_p) at frequency 20 MHz, at frequency 30 MHz.

researches have also shown the existence of medium (material) with a negative refractive index. In principle, the phase refractive index expressed in Eq. (2) can receive the results in two cases: $n = \pm\sqrt{\varepsilon_r \cdot \mu_r}$. This means that the value of phase refractive index could get both the positive and negative values. If the refractive index has negative value ($n < 0$) according to Eq. (1), this phenomenon occurs only when the phase velocity is negative, leading to that the relative dielectric permittivity and magnetic permeability are also negative ($\varepsilon_r < 0$; $\mu_r < 0$). In the causal theory of waves, relative permittivity (ε_r) is a complex quantity, where the imaginary part corresponds to a phase shift of the polarization $\mathbf{P}$ relative to $\mathbf{E}$ and leads to the attenuation of electromagnetic waves passing through the medium. For plane waves propagating in a negative phase velocity medium, the electric field, magnetic field and wave vector follow a left-hand rule. This kind of medium is, so called, Left-Handed Material (medium). The negative permittivity makes propagation loss as well. Since the wave number is related to the square root of the product of the permittivity and permeability, a negative permittivity and

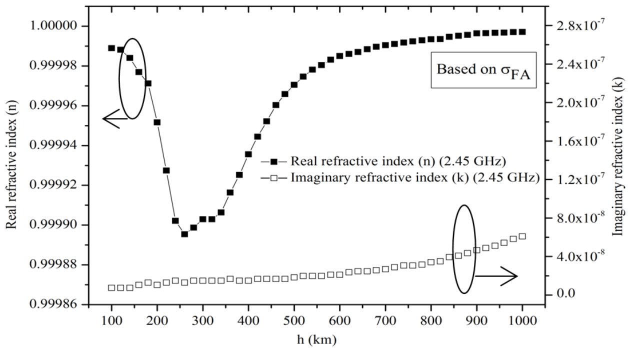

Figure 11. Numerical estimated data of complex refractive index (n_c) by altitude based on the Field-Aligned conductivity (σ_{FA}) at frequency 2.45 GHz.

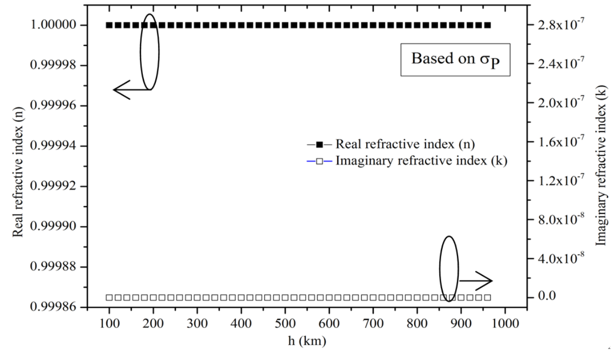

Figure 12. Numerical estimated data of complex refractive index (n_c) by altitude based on the Pedersen conductivity (σ_p) at frequency at frequency 2.45 GHz. Almost data of real parts are equal 1 and almost data of imaginary parts are equal or nearly zero (0).

positive permeability result in a loss wave number [36–39].

For demonstration of negative refractive index, Figs. 10 and 11 show the numerical estimation results of the negative refractive index corresponding to the calculated negative relative permittivity for both cases in Fig. 1 and Fig. 5 at the altitude range from 200 km to 500 km based on the parallel field-aligned conductivity (σ_o) and the transverse Pedersen conductivity (σ_P), respectively. The values of the imaginary parts of refractive indexes for both cases are considerably high at 20 MHz and 30 MHz. Their values are decreased nearly to zero at high frequencies. Fig. 11 and Fig. 12 also show the results of the refractive indexes at very high frequency based on the parallel field-aligned conductivity (σ_{FA}) and transverse Pedersen conductivity (σ_P), respectively for applications in the theoretical and practical problems of WPT [7, 9]. The numerically estimated results show that the refractive indexes are very small in the altitude range from 100 km to 1000 km.

In conclusion, we have discussed and analyzed, more in detail, some theoretical aspects of the complex refractive index expressions concerning the velocities of electromagnetic radiation (EMR) in ionosphere region of earth atmosphere. The initial equation of the complex reflective index-Appleton-Hertree formula — in the ionosphere region is discussed and approximated with the first order for using numerical estimation. The complex refractive index by altitude from 100 km to 1000 km in the ionized region was calculated in the first time based on the data of complex relative permittivity, the published data of the free electron density and the conductivities for two cases of parallel field-aligned conductivity (σ_{FA}) and transverse Pedersen conductivity (σ_P). The numerical estimation data of complex refractive indexes and complex relative permittivity in the ionized region agree with the theoretical considerations. The obtained data show that at 20 MHz to 50 MHz frequencies the real parts of the refractive indexes in the altitude range of 200–500 km are changed greatly due to the large variation of free electron densities in this range. In the high-frequency range, the variations of the real parts of the refractive index are very small. Their values are approximated to "1" value as in vacuum medium. At the frequency lower than 40 MHz, both the real parts of the complex relative permittivity and complex refractive index for ionized atmosphere region become negative values. These results could hint at modification of some theory considerations and conditions used for the former discussions.

Here we would like also to note that the numerically estimated data here are only draft data and have local characterization, which means that at different locations of ionosphere regions in the earth atmosphere, there will be different total free electron contents by altitude, leading to different relative permittivities and refractive indexes. Our numerical estimated data would be continuously studied and discussed more in detailed. The result presented in this paper is encouraging and implying various potential applications. Firstly, the estimated result can be used as input data for researches on various problems concerning the WIT as well as WPT, GPS signal transfer in the ionosphere region, overcoming the limitations of lacking data. Secondly, using the estimated relative permittivities and refractive indexes by altitude (or using their average values for a whole region), we are able to investigate and find a numerical solution of the energy transfer flux problem to determine transfer efficiencies for WIT, GPS and WPT.

The numerical data of phase refractive index are outlined here, but the numerical data of the group refractive index by altitude from the earth surface to over 1000 km has not yet been discussed and estimated and could be discussed in the next forthcoming paper.

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