Synthesis of Dual Beam Pattern of Planar Array Antenna in a Range of Azimuth Plane Using Evolutionary Algorithm

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Abstract—In this paper a pattern synthesis method based on Differential Evolution Algorithm (DE) is presented to generate dual beam patterns from a planar array of isotropic antennas. These are $cosec^2$ pattern and pencil beam pattern. These patterns are obtained by finding out an optimum set of common elements amplitude (for $cosec^2$ pattern as well as a pencil beam pattern), and a set of phases, for $cosec^2$ pattern only. 4-bit discrete amplitudes and 5-bit discrete phases are used to reduce the design complexity of feed network. The beam patterns have been generated in two different azimuth planes instead of one particular plane. The evaluated excitations are also verified by considering a range of arbitrarily chosen azimuth planes, where the patterns are generated with some minor variations of the desired parameters. Obtained results clearly established the effectiveness of the proposed method.

1. INTRODUCTION

A reconfigurable planar array antenna is often required in satellite communication and radar related applications. However, generation of $cosec^2$ beam and pencil beam using a common set of discrete elements amplitudes often faces high side lobe with large ripple problem. Several approaches reported in the literature for generating beam patterns are as follows [1–10].

Diaz et al. proposed a method of generation of phase differentiated multiple beam patterns using simulated annealing algorithm [3]. Durr et al. generates shaped beams (flattop and cosecant) from linear antenna array where both uniform and Gaussian distributions of common amplitudes are used for the generation of patterns, and different sets of phases for both beams are computed using Woodward-Lawson technique [4]. Azaro et al. find the desired values of VSWR by minimizing the linear dimension of a monopole antenna using PSO, and the simulated results are also compared with experimental values [5]. An integrated multifunction/multiband [6] antenna was designed using the stochastic multiphases optimization technique by Azaro et al. Morabito et al. proposed a technique for optimal synthesis of linear phase-only reconfigurable arrays which are able to commute their pattern among different radiation models. Numerical results of actual interest and its implementation of realistic element patterns are also assessed [7]. Chatterjee et al. developed a method based on GSA for finding out optimum sets of 4-bit radial amplitudes and 5-bit phases to generate dual beams such as pencil-pencil, pencil-flattop and flattop-flattop of the concentric ring array antenna [8]. A noble method to eliminate interfering signals adaptively for planar array antenna using customized genetic algorithm was proposed by Massa et al. [9]. Mandal et al. proposed a method of synthesizing dual radiation patterns, flattop and pencil from a rectangular planar array antenna using evolutionary algorithm [10].

In this paper, a $cosec^2$ beam and a pencil beam patterns from a planar array [1, 2] of isotropic elements are obtained by finding out the optimum set of common elements amplitudes for both the patterns and a set of phases for $cosec^2$ shaped beam using DE. The patterns have been generated in two

predefined φ planes using these excitations, where the excitations for both amplitudes and phases are discrete in nature to provide lower Dynamic Range Ratio (DRR). As the DRR is low, lower number of attenuators and phase shifters are required, thus reducing the design complexity of the feed networks. These patterns are not restricted to a single predefined φ plane rather a range of azimuth planes with some minor variations in design parameters.

2. PROBLEM FORMULATIONS

A planar array of isotropic elements is considered. The far-field pattern of the array shown in Figure 1 can be written as [1, 2]:

$$AF(\theta, \varphi) = \sum_{m=1}^{M} \sum_{n=1}^{N} I_{mn} e^{j[kmd_x \sin\theta \cos\varphi + knd_y \sin\theta \sin\varphi + \alpha_{mn}]} \tag{1}$$

where, I_{mn} is the excitation amplitude of the mn-th element; M and N denote number of isotropic elements in x and y directions, respectively; $k = \frac{2\pi}{\lambda}$ represents wave number; inter element spacing along x and y directions, represented by d_x and d_y respectively, are considered as 0.5λ; θ, φ are polar and azimuth angles; phase excitation of the mn-th element is denoted by α_{mn}.

The fitness function for the dual beam patterns is defined as:

$$F(\rho) = k_1 \{peakSLL^{d_1} - \max_{\theta \in A_1} \{AF_{dB}^{\rho}(\theta, \varphi)\}\}^2 + k_2 \times \triangle + k_3 \{peakSLL^{d_2} - \max_{\theta \in A_2} (AF_{dB}^{\rho}(\theta, \varphi))\}^2 \tag{2}$$

where, $\triangle$ is defined as:

$$\triangle = \sum_{\theta_i \in \{0° - 30°\}} |AF_{dB}^{\rho}(\theta_i, \varphi) - D(\theta_i, \varphi)| \tag{3}$$

In Equations (2) and (3), $\varphi \in (10° – 20°)$ plane.

ρ is the unknown parameter set responsible for the desired beam patterns for this approach. ρ is defined as follows:

$$\rho = \{I_{mn}, \alpha_{mn}\}; \quad 1 \le m \le M \And 1 \le n \le N \tag{4}$$

$peakSLL^{d_1}$ and $peakSLL^{d_2}$ are the desired values of peak SLL for $cosec^2$ and pencil beam pattern. A_1 and A_2 are the sidelobe region for both patterns. $D_{dB}(\theta, \phi)$ is the desired pattern shown in Figure 2 at $\varphi = 10°, 20°$ plane. The range of θ_i for this approach is $0°$ to $30°$. k_1, k_2 and k_3 are the weighting factors. The fitness function has to be minimized by finding out optimum set of 4-bit amplitudes and 5-bit phases using Differential Evolution (DE) Algorithm.
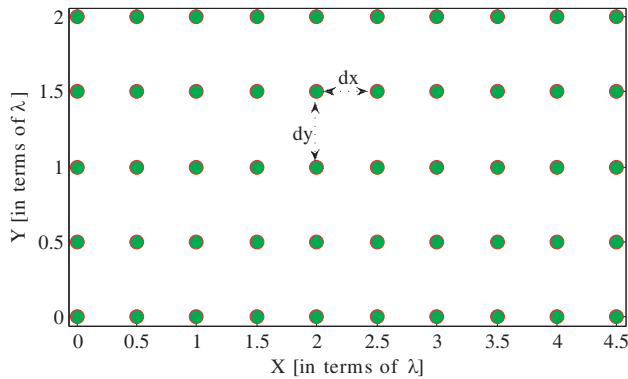

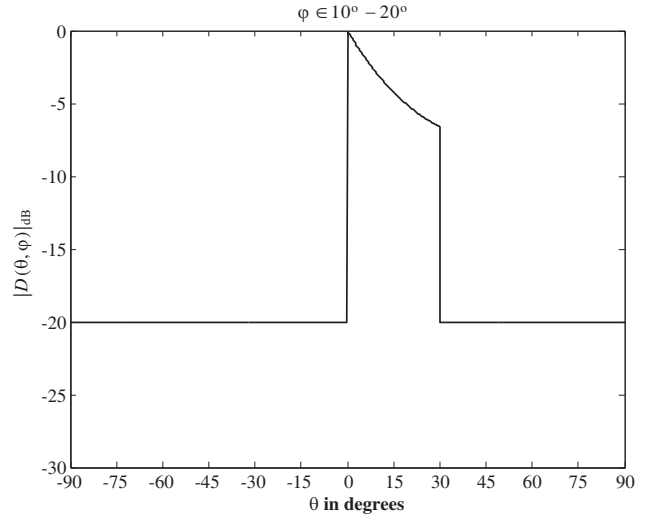**Figure 1.** Geometry of a planar array of 50 isotropic elements.

Figure 2. Desired $cosec^2$ pattern for predefined planes.

3. DIFFERENTIAL EVOLUTION ALGORITHM (DE)

Differential Evolution is a population-based stochastic evolutionary algorithm and was first introduced by Storn and Price in the year 2005. The main advantages of DE are lying in the fact that it has fast convergence time and is capable of determining true global minima utilizing fewer control parameters [10–13].

In D-dimensional search space, the individuals of population NP in generation G, generated by the algorithm, can be written as:

$$X_{i,G} = \{x_{1i,G}, x_{2i,G}, \ldots, x_{Di,G}\} \tag{5}$$

where, $i = 1, 2, \ldots, NP$. The entire search space is covered by the initial population.

The initialization of the jth parameter at a generation $G = 0$ can be described as:

$$x_{ji,0} = rand(0,1) \cdot (x_j^{up} - x_j^{low}) + x_j^{low} \tag{6}$$

where $i = 1, 2, \ldots, NP$, $j = 1, 2, \ldots, D$, and $rand(0,1)$ is uniformly distributed random variable within the range $(0,1)$; x_j^{low} and x_j^{up} are the lower and upper bounds of the jth parameter, respectively. The three main steps involved in DE algorithm are mutation, crossover and selection.

Mutation: In mutation operation, mutant vector $V_{i,G}$ is generated for each target vector $X_{i,G}$ based on the strategy of "$DE/best/1/bin$" which can be defined as follows:

$$V_{i,G} = X_{best,G} + F \cdot (X_{r1,G} - X_{r2,G}) \tag{7}$$

$r_1, r_2 \in [1, NP]$ and $r_1 \neq r_2 \neq i$. F is a real and constant factor and satisfies $F \in [0,2]$, and $X_{best,G}$ is the vector which has the best fitness at the Gth generation.

Crossover: In cross over operation, the trial vector $U_{i,G} = \{u_{1i,G}, u_{2i,G}, \ldots, u_{Di,G}\}$ is generated utilizing the target vector $X_{i,G}$ and mutant vector $V_{i,G}$ which can be described as follows:

$$u_{ji,G} = \begin{cases} v_{ji,G}, & \text{if } rand(0,1) \leq CR \\ x_{ji,G}, & \text{otherwise} \end{cases} \tag{8}$$

Selection: In this operation, for each trial vector $f(U_{i,G})$ and target vector $f(X_{i,G})$ the objective function values are compared, and the smaller fitness function value remains in the next generations. The selection operation can be described as follows:

$$X_{i,G+1} = \begin{cases} U_{i,G}, & \text{if } f(U_{i,G}) < f(X_{i,G}) \\ X_{i,G}, & \text{otherwise} \end{cases} \tag{9}$$

These steps are repeated till the predefined generation has been completed which results in the best vector in the current population $(X_{best,G})$ as the solution of the problem.

4. RESULTS

A planar array of 50 isotropic elements has been considered. $M = 10$ and $N = 5$ are chosen. The inter element spacing is considered as 0.5λ, i.e., $d_x = 0.5\lambda$ and $d_y = 0.5\lambda$. The population size, scale factor (F) and cross over rate (CR) in DE are taken as 50, 0.8 and 0.2, respectively. Here the applied DE scheme is "$DE/best/1/bin$". The termination condition is considered as a maximum iteration of 2500.

The design specification of the dual beam patterns and its corresponding obtained results are shown in Table 1. From Table 1, it has been observed that the obtained values of the peak SLL for the $cosec^2$ beam pattern in two different pre-specified planes are -14.53 dB and -16.45 dB corresponding to its desired value of -20.00 dB. The parameter $\triangle$ is introduced to measure the total deviation between the obtained and desired patterns within the angular region ($\theta = 0° - 30°$). The values of ripple ($\triangle$) are 28.48 and 21.99 for $\varphi = 10°$ and $\varphi = 20°$, respectively, whereas for pencil beam pattern the obtained values of peak SLL are -16.74 dB and -13.34 dB, respectively, for the same azimuth planes.

The obtained $cosec^2$ pattern along with the pencil beam for two predefine azimuth planes are shown in Figure 3. Figure 3(a) is for $\varphi = 10°$ plane and Figure 3(b) for $\varphi = 20°$ plane. The excitation amplitudes and phases of the array elements obtained using DE for generating the beam patterns are

Table 1. Desired and obtained values of design parameters.

φ **in degree**	**Design Parameters**	$cosec^2$ **Pattern**		**Pencil beam Pattern**	
		Desired	**Obtained**	**Desired**	**Obtained**
$\varphi = 10°$	Peak SLL (dB)	−20.00	−14.53	−20.00	−16.74
	$\triangle$ (dB)	0.00	28.48	−	−
$\varphi = 20°$	Peak SLL (dB)	−20.00	−16.45	−20.00	−13.34
	$\triangle$ (dB)	0.00	21.99	−	−
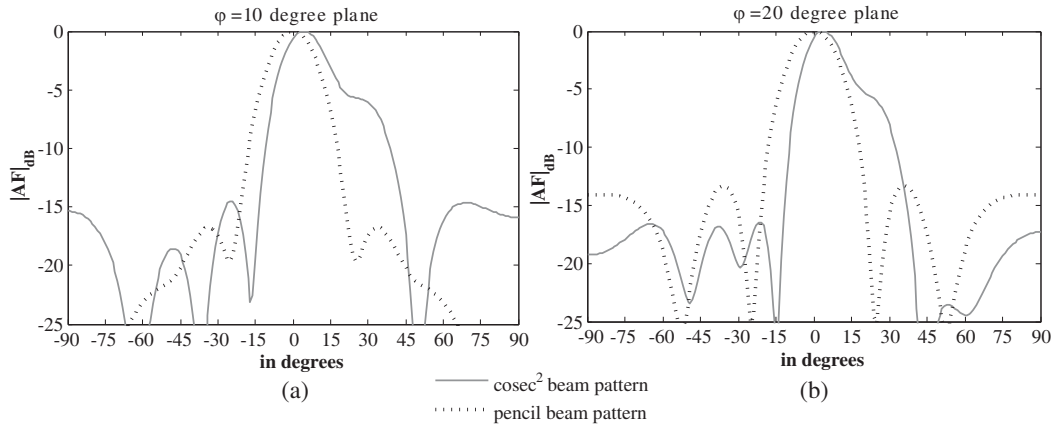
Figure 3. Obtained dual beam patterns for (a) $\varphi = 10°$ and (b) $\varphi = 20°$ plane.
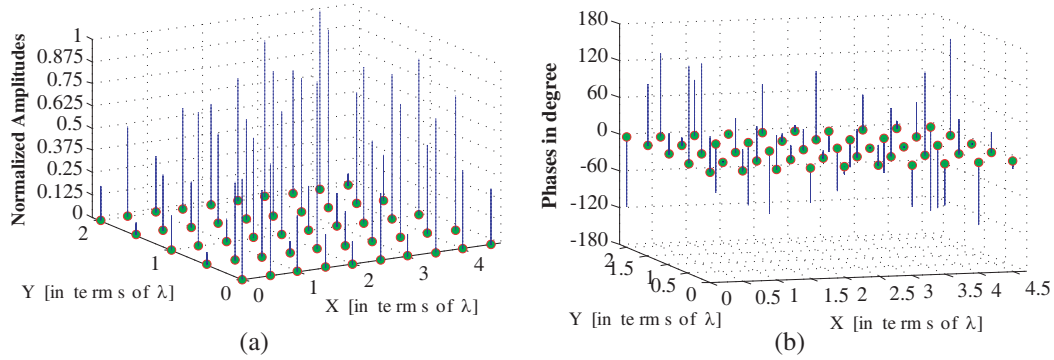
Figure 4. Excitations of the array elements: (a) Normalized Amplitudes, (b) phases in degrees.

shown in Figure 4. Figure 4(a) shows the 4-bit discrete amplitudes, and Figure 4(b) shows the 5-bit discrete phases.

In Figure 5, the beam patterns in three arbitrarily chosen azimuth planes for the same excitations with some minor variation in pattern have been achieved. In Figures 5(a), (b) and (c), the arbitrary azimuth angles are chosen as 7.5 degrees ($< 10°$, below the pre-specified φ plane), 15 degrees (within the pre-specified φ plane) and 22.5 degrees ($> 20°$, beyond the predefined azimuth plane), respectively. Figure 3 and Figure 5 clearly show that the obtained $cosec^2$ beam patterns follow the desired beam pattern shown in Figure 2 within the coverage range of elevation angle (0°–30°). The obtained values of design parameters for arbitrarily selected φ planes have been presented in Table 2. Figure 6 shows the convergence curve of DE algorithm. The convergence time taken to optimize the array pattern is 1 hr and 36 min. Computations have been done in MATLAB 2010a with core 2 duo processor, 3 GHz, 2 GB RAM.

Table 2. Obtained results for arbitrary φ planes.

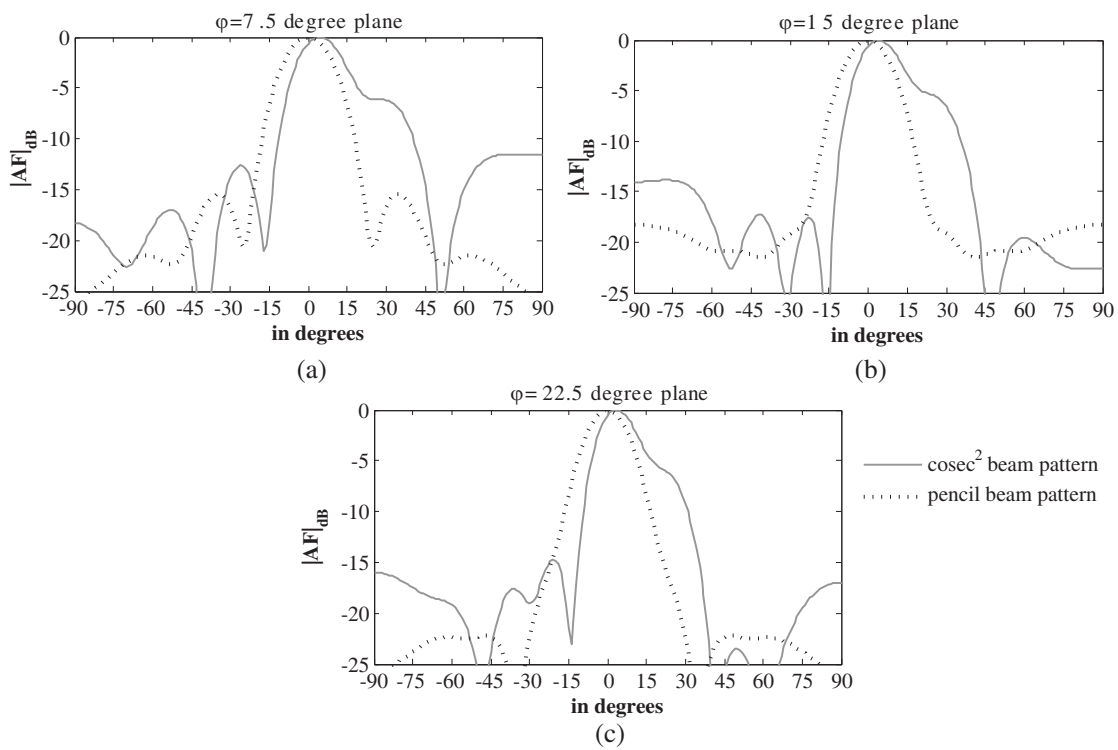| φ in degree	Design Parameters	$cosec^2$ Pattern	Pencil beam Pattern
$\varphi = 7.5°$	Peak SLL (dB)	−11.50	−16.15
	$\triangle$ (dB)	28.15	–
$\varphi = 15°$	Peak SLL (dB)	−13.85	−18.32
	$\triangle$ (dB)	26.77	–
$\varphi = 22.5°$	Peak SLL (dB)	−14.73	−13.69
	$\triangle$ (dB)	27.89	–

(a)

(b)

(c)

Figure 5. Dual beam patterns for three arbitrarily chosen φ planes.
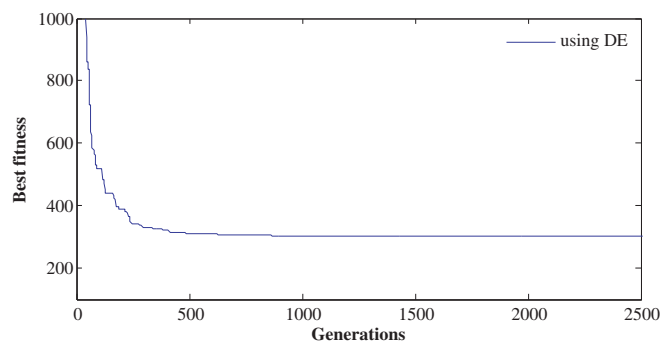

Figure 6. Convergence curve of Differential Evolution Algorithm (DE).

5. CONCLUSION

A dual-beam planar array antenna of two different azimuth angles has been synthesized. A shaped beam ($cosec^2$ beam) and a pencil beam are generated using 4-bit discrete amplitudes and 5-bit discrete phases for keeping low dynamic range ratio (DRR). The peak side-lobe level and ripple are also reduced by finding the optimum set of array excitations using DE algorithm. This also ensures the desired patterns within a range of azimuth plane rather in a pre-specified φ plane. A good agreement between the desired and obtained results validates the proposal. The presented method can also be applied to synthesize other array configurations.

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