*Progress In Electromagnetics Research Letters, Vol. 63, 93–97, 2016*

# Temperature Coefficient Measurement of Microwave Dielectric Materials Using Closed Cavity Method

**Liangzu Cao[1, 2, *], Junmei Yan[1], and Lixia Yin[1]**

**Abstract**—The closed cavity method is proposed to measure the frequency temperature coefficient ($\tau_f$) of a dielectric resonator. The $\tau_f$ polynomial, which is linear combination of the temperature coefficient of relative dielectric constant and the linear expansion coefficient of the dielectric and cavity, is given. The coefficients of $\tau_f$ polynomial are discussed in detail. The intrinsic temperature coefficient of resonant frequency ($\tau_{f0}$) is introduced to improve the measurement precision. Resonators made of $BaO\text{-}TiO_2\text{-}Sm_2O_3$ and $(Zr_{0.8}Ti_{0.2})TiO_4$ ceramics with Teflon and alumina as supports were measured. The results show that the $\tau_f$ values of the same resonator with above supports are different, and the measured variation between them is more than 3 ppm/°C. Using the concept of $\tau_{f0}$, the variation is less than 2 ppm/°C.

## 1. INTRODUCTION

Microwave dielectric materials exhibiting high $Q$ value and very low temperature dependence of resonant frequency ($\tau_f$) have been developed since 70s [1]. Because of high permittivity ($\varepsilon_r$), more than 10, they promise to reduce the size and cost of waveguide cavities. They are widely used to manufacture resonators, filters and oscillators.

There are many methods for measuring $\tau_f$. The parallel plate type resonator method is used for $\tau_f$ measurement, but the smaller loaded $Q$ value lowers the accuracy of $\tau_f$ measurement. To increase the accuracy of $\tau_f$ measurement, Kobayashi et al. [2] proposed the shielded dielectric resonator of an image type to measure $\tau_f$ and introduced the intrinsic temperature coefficient of $f_o$, $\tau_{f0}$ to evaluate the temperature dependences of dielectric materials, but did not give a detailed procedure of getting the coefficients of $\tau_{f0}$ polynomial. Abramowicz and Modelski [3] developed a microwave integrated circuit (MIC) type resonator to measure $\tau_f$, and a similar $\tau_{f0}$ polynomial was obtained, but they did not explain how to get the coefficients of $\tau_{f0}$ polynomial. Nishikawa et al. [4] studied a ceramic cavity type resonator to measure $\tau_f$. The shielding cavity was made of metalized ceramic having the same thermal expansion coefficient ($\alpha$) as the dielectric resonator. The support was made of a ceramic tube which also had the same $\alpha$, but it is expensive to make metalized ceramic cavities with the measured materials. Nikawa and Guan [5] measured the temperature dependence of complex permittivity of alumina ceramics using $TM_{010}$ mode of a cylindrical cavity resonator in the range of from 25°C to 1150°C. Michael [6] utilized dual-mode frequency locked technique to accurately measure $\tau_f$ of sapphire, which is an anisotropic crystal, and the modes in the dielectric resonator are WGE and WGH. This technique is not suitable for ceramics, isotropic polycrystallines. Silva et al. [7] presented an alternative method for the measurement of $\tau_f$, based on the frequency variation with the temperature of the $HE_{11\delta}$ mode of a dielectric resonator antenna, and measured the $\tau_f$ value of $CaTiO_3$, $Al_2O_3$ and BTNO. In this paper, we use a resonant structure operating in the $TE_{01\delta}$ mode to measure $\tau_{f0}$, as shown in Figure 1. With respect to the former work on this measurement, we consider additional phenomena: (a) how to get the coefficients of $\tau_{f0}$ polynomial, (b) the effects of various supports on $\tau_{f0}$.

*Received 15 June 2016, Accepted 14 October 2016, Scheduled 2 November 2016*

* Corresponding author: Liangzu Cao (clz4233@aliyun.com).
[1] School of Mechanical and Electric Engineering of Jingdezhen Ceramic Institute, Jingdezhen 333403, P. R. China. [2] School of Electric and Optical Engineering of Nanjing University of Science & Technology, Nanjing 210014, P. R. China.

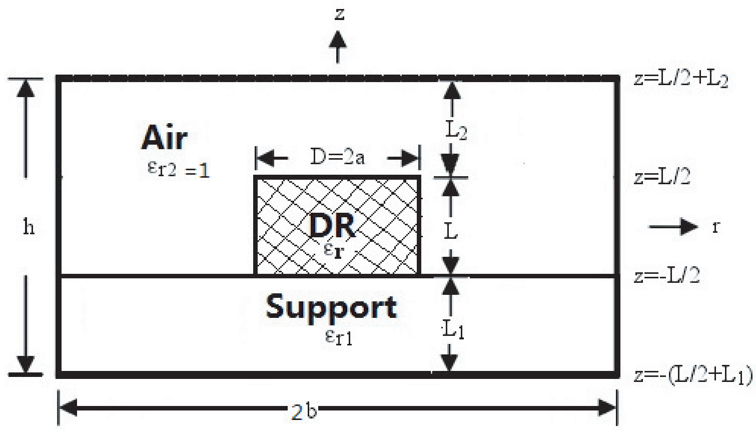


**Figure 1.** Side view of cylindrical dielectric resonator in a closed cavity.

## 2. RESONANT FREQUENCY TEMPERATURE COEFFICIENT

The Galerkin-Rayleigh-Ritz method [8] is one of several rigorous techniques used in computing resonant frequencies of a axially symmetric, multilateral dielectric structure. We have employed the technique to find a relationship between the permittivity and the $TE_{01\delta}$ resonant frequencies of a cylindrical cavity containing a cylindrical dielectric resonator (DR) and a dielectric support as shown in Figure 1. The magnetic field expressions of the $z$ component, radial magnetic field components and circular electric field in each region of the $TE_{01\delta}$ mode are solved using Galerkin-Rayleigh-Ritz method and expressed as in [9].

Known dimensions labeled in Figure 1 and resonant frequency ($f_0$), dielectric constant ($\varepsilon_r$) of the dielectric sample, operating in $TE_{01\delta}$ mode can be calculated by solving the characteristic equations [9]. Matlab package is required for this calculation.

We can generally express the resonant frequency temperature stability $\Delta f_0/(f_0 \cdot \Delta T)$ as $\tau_f$,

$$\tau_f = A_\varepsilon \tau_\varepsilon + A_d \alpha + A_c \alpha_c + A_t \alpha_t + A_s \tau_s \tag{1}$$

where $\tau_\varepsilon$ and $\tau_s$ are the temperature coefficients of $\varepsilon_r$ and $\varepsilon_{r1}$, respectively. $\alpha$, $\alpha_c$ and $\alpha_t$ are the linear expansion coefficients of the dielectric resonator, conducting cavity and dielectric support, respectively. The numerical constants, $A_\varepsilon$, $A_d$, $A_c$, $A_t$ and $A_s$, are given by [2]

$$A_\varepsilon = \frac{\varepsilon_r}{f_0}\frac{\Delta f_0}{\Delta \varepsilon_r} \quad A_d = \frac{D}{f_0}\frac{\Delta f_0}{\Delta D} + \frac{L}{f_0}\frac{\Delta f_0}{\Delta L} \quad A_c = \frac{h}{f_0}\frac{\Delta f_0}{\Delta h} + \frac{b}{f_0}\frac{\Delta f_0}{\Delta b} \quad A_t = \frac{L_1}{f_0}\frac{\Delta f_0}{\Delta L_1} \quad A_s = \frac{\varepsilon_{r1}}{f_0}\frac{\Delta f_0}{\Delta \varepsilon_{r1}} \tag{2}$$

In order to improve the measurement precision, we introduce the intrinsic temperature coefficient of $f_0$ and $\tau_{f0}$ defined by $\tau_f$ for an assumed dielectric resonator in which all the stored energy is confined. In this case, Equation (1) yields the following relation [10],

$$\tau_{f0} = -0.5\tau_\varepsilon - \alpha \tag{3}$$

Eliminating $\tau_\varepsilon$ from Equations (1) and (3), we can rewrite $\tau_f$ as follows:

$$\tau_f = -2A_\varepsilon \tau_{f0} + (A_d - 2A_\varepsilon)\alpha + A_c \alpha_c + A_t \alpha_t + A_s \tau_s \tag{4}$$

Usually, the magnitude of $A_d - 2A_\varepsilon$ is about 0.1 as much as that of $2A_\varepsilon$, thus the effects of $\alpha$ on $\tau_f$ can be reduced by 0.1, compared to the estimation from Equation (1).

## 3. MEASUREMENT OF $\tau_{f0}$

Two types of cylindrical dielectric resonators, whose diameters and heights are listed in Table 1, are made of dielectric ceramic with composite of $BaO$-$TiO_2$-$Sm_2O_3$ and $(Zr_{0.8}Ti_{0.2})TiO_4$ with dielectric constants of 74 and 38, respectively. The supports are Teflon with dielectric constant of 2.55 and alumina ceramics with dielectric constant of 9.4. A metal cavity is made of aluminum, whose dimensions are listed in Table 1. The resonant frequencies and frequency changes in the range from 25°C to 85°C are measured by using Agilent E570B network analyzer, the measurement setup, consisting of a instrument, a pair of semi-rigid cables, a aluminum cavity, a heater and a refractory slab (SiC), is shown in Figure 2, and the measurement results are listed in Table 2.

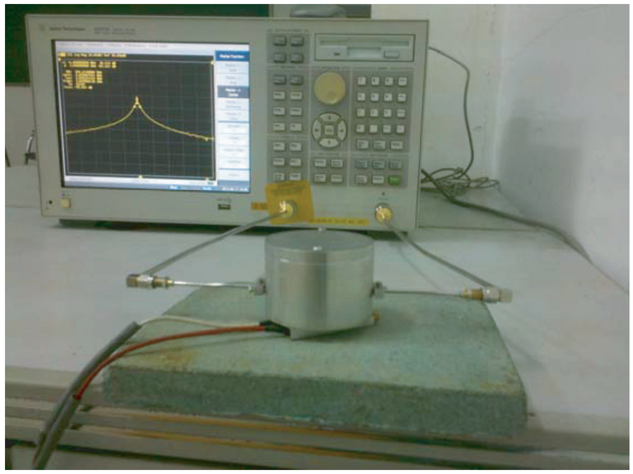

**Figure 2.** Experiment setup.

**Table 1.** Dimensions of resonators and cavity (unit: mm).

| Material | Diameter | Height |
|---|---|---|
| BaO-$TiO_2$-$Sm_2O_3$ | 13 | 6.48 |
| $(Zr_{0.8}Ti_{0.2})TiO_4$ | 12.9 | 7.38 |
| Aluminum cavity | 45 | 30 |
| Teflon support | 10 | 6.3 |
| Alumina support | 10 | 3 |

**Table 2.** Measurement results.

| No. | Resonator | Cavity | Support | $f_0$ (GHz) | $\Delta f$ (MHz) | $\tau_f$ (ppm/°$C$) |
|---|---|---|---|---|---|---|
| 1 | BaO-$TiO_2$-$Sm_2O_3$ | Aluminum | Teflon | 2.75597 | 1.6 | 9.676 |
| 2 | | | Alumina | 2.8108 | 2.2 | 13.04 |
| 3 | $(Zr_{0.8}Ti_{0.2})TiO_4$ | Aluminum | Teflon | 3.8355 | −1.1 | −4.78 |
| 4 | | | Alumina | 3.9061 | −0.4 | −1.71 |

The procedure which analyzes the variation of the resonant frequency with each physical parameter is represented as follows.

First, we calculate dielectric constant ($\varepsilon_r$) of the dielectric resonator by using Matlab package under the condition of knowing dimensions and resonant frequency ($f_0$) [9].

Second, another computer program using Matlab package is required to compile for analyzing the variation of the resonant frequency with each physical parameter, such as the height of the resonator.

Finally, set the value of $\Delta x_i/x_{i0}$ for only one parameter with other parameters unchanged ($\Delta x_j = 0(j \neq i)$), for example $\Delta L/L_0 = 1\%$, then run the above compiled program and display a figure about the relationship between the resonant frequency and the physical parameter.

Figure 3 shows the variation of the resonant frequency of the $(Zr_{0.8}Ti_{0.2})TiO_4$ resonator with the height of the resonator while other parameters are unchanged.

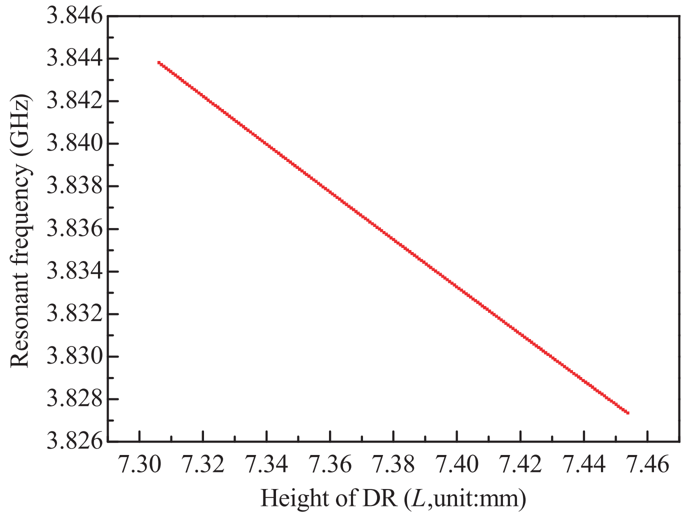


**Figure 3.** Frequency variation with the height of the $(Zr_{0.8}Ti_{0.2})TiO_4$ resonator.

It is obvious that the variation ratio of the resonant frequency resonator to the height of the resonator is equal to the slope ($k$) of the line in Figure 3, i.e., $\Delta f_0/\Delta L = k$, so is $\Delta f_0/\Delta x_i = k_i$.

The numerical constant $A_\varepsilon$, $A_d$, $A_c$, $A_t$ and $A_s$ are obtained by using Equation (2). The computed values of the coefficients are listed in Table 3.

From Equation (4), we obtain the expression to estimate $\tau_{f0}$, as shown in Equation (5),

$$\tau_{f0} = -\frac{1}{2A_\varepsilon}[\tau_f - (A_d - 2A_\varepsilon)\alpha - A_c\alpha_c - A_t\alpha_t - A_s\tau_s] \tag{5}$$

In the estimation of $\tau_{f0}$, it is known that $\alpha = 6.5\,\text{ppm}/^\circ\text{C}$ for resonators, $\alpha_c = 23.8\,\text{ppm}/^\circ\text{C}$ for aluminum cavity, $\alpha_t = 100\,\text{ppm}/^\circ\text{C}$ and $\tau_s = 19\,\text{ppm}/^\circ\text{C}$ for Teflon support, $\alpha_t = 7.2\,\text{ppm}/^\circ\text{C}$ and $\tau_s = 110\,\text{ppm}/^\circ\text{C}$ for alumina support. The estimated $\tau_{f0}$ values are also listed in Table 3.

**Table 3.** Values of the numerical constants.

| No. | $A_\varepsilon$ | $A_d$ | $A_c$ | $A_t$ | $A_s$ | $\tau_{f0}$ (ppm/°C) |
|---|---|---|---|---|---|---|
| 1 | −0.4929 | −0.9288 | −0.0529 | −0.01344 | −1e-4 | 10.73 |
| 2 | −0.4912 | −0.9184 | −0.06 | −0.0303 | 1.78e-3 | 11.26 |
| 3 | −0.4794 | −0.9238 | −0.04536 | −0.01369 | −1e-4 | −2.52 |
| 4 | −0.474 | −0.8789 | −0.0567 | −0.0256 | −1e-4 | −0.66 |

## 4. DISCUSSIONS

From Table 2, $\tau_f$ values of either $BaO\text{-}TiO_2\text{-}Sm_2O_3$ resonators or $(Zr_{0.8}Ti_{0.2})TiO_4$ resonators are varied with the supports, and the variation is more than $3\,\text{ppm}/^\circ\text{C}$. From Table 3, we obtain that $A_d \cong -0.9$ and $A_\varepsilon \cong -0.5$. This indicates that the main factor affecting the resonator frequency stability is the change rate of dimension of a dielectric resonator. The second factor is the change of resonator dielectric constant. Both sides of the resonator are terminated by metals, and the equation [10]

$$\tau_{f0} = -0.5\tau_\varepsilon - \alpha \tag{6}$$

holds in a good approximation. For the resonator made of BaO-$TiO_2$-$Sm_2O_3$, $A_d - 2A_\varepsilon$ is very small, about 0.05, and this indicates that the surfaces of a resonator with high dielectric constant are as magnetic walls, with which the stored energy is entirely confined in the resonator. From Table 3, we find that $\tau_{f0}$ variation with supports is less than 2 ppm/°C.

## 5. CONCLUSION

The closed cavity method is introduced to measure the frequency temperature coefficient of resonator, $\tau_f$. The measured values vary with the support, and the variation is more than 3 ppm/°C. The intrinsic temperature coefficients of $f_0$, $\tau_{f0}$ can be used conveniently to evaluate the temperature dependences of dielectric materials. The variation of $\tau_{f0}$ with different supports is less than 2 ppm/°C.

## ACKNOWLEDGMENT

This work was in part supported by the National Natural Science Foundation of Jiangxi Province [No. 20151BAB207014].

## REFERENCES

1. Fiedziuszko, S. J., I. C. Hunter, T. Itoh, Y. Kobayashi, T. Nishikawa, S. N. Stitzer, and K. Wakino, "Dielectric materials, devices and circuits," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 50, No. 3, 706–720, Mar. 2002.
2. Kobayashi, Y., Y. Kogami, and M. Katoh, "A method of evaluating the temperature dependence of dielectric resonators and materials," *Proc. of 23th European Microwave Conference*, 562–563, Sep. 1993.
3. Abramowicz, A. and J. Modelski, "Accurate method of measuring the temperature coefficients of dielectric resonator materials," *21st European Microwave Conference*, 1391–1396, 1991.
4. Nishikawa, T., K. Wakino, H. Tamura, H. Tanaka, and Y. Ishikawa, "Precise measurement method for temperature of dielectric resonator material," *IEEE MTT-S Int. Microwave Symp. Digest*, No. J-6, 277–280, 1987.
5. Nikawa, Y. and Y. Guan, "Dynamic measurement of temperature dependent complex permittivity of material by microwave heating using cylindrical cavity resonator," *Proceedings of Asia-Pacific Microwave Conference*, 1–4, 2007.
6. Tobar, M. E., G. L. Hamilton, J. G. Hartnett, E. N. Ivanov, D. Cros, and P. Guillon, "Accurate characterization of the temperature coefficients of permittivity of sapphire utilizing the dual-mode frequency locked technique," *Proceedings of the 2003 IEEE International Frequency Control Symposium*, 365–370, 2013.
7. Silva, M. A. S., T. S. M. Fernandes, and A. S. B. Sombra, "An alternative method for the measurement of the microwave temperature coefficient of resonant frequency ($\tau f$)," *Journal of Applied Physics*, Vol. 112, 074106-1 ∼ 7,2012.
8. Krupka, J., "Resonant modes in shielded cylindrical ferrite and single-crystal dielectric resonators," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 37, 691–697, 1989.
9. Sheen, J., "Microwave measurements of dielectric properties using a closed cylindrical cavity dielectric resonator,"*IEEE Transactions on Dielectrics and Electrical Insulation*, Vol. 14, No. 5, 1139–1144, 2007.
10. Kafjez, D. and P. Guillon, *Dielectric Resonators*, Artech House, 1986.