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Analysis and Synthesis of Multiband Sierpinski Carpet Fractal Antenna Using Hybrid Neuro-Fuzzy Model

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Abstract—The paper presents the application of the hybrid neuro-fuzzy model for the analysis and synthesis of a square multiband Sierpinski carpet fractal antenna. For the analysis model, the antenna geometrical parameters were taken as the input, and the resonant frequencies were obtained as the output while for the synthesis model, the resonant frequencies were taken as the input, and geometrical parameters were obtained as the output. Also, a model was trained to obtain the return loss characteristics for the given set of geometrical parameters. The developed model was validated by comparing the resonant frequencies and radiation pattern of the simulated and fabricated antennas.

1. INTRODUCTION

Compact size, low profile, conformal and multiband characteristics are in high demand in both the military and the commercial sector. Several approaches have been proposed by researchers to fulfil one or more of these objectives [1–3]. One such antenna, which has the capability of achieving these goals, at least in parts, is a fractal antenna. Mandlebrot [4] coined the term 'Fractal' in 1975 to describe a family of complex shapes that possess an inherent self-similarity or self-affinity in their geometrical structure. The concept of fractals has been applied in many branches of science and engineering. One such area is 'Fractal Electrodynamics' [5–8]. In Fractal Electrodynamics, the fractal geometry is combined with Electromagnetic Theory to investigate a new class of radiation, propagation, and scattering problems. One of the most promising areas of the fractal electrodynamics research is in its application to the antenna theory and design. The self-similarity property of the fractal antenna helps it to resonant at a number of frequencies. The first application of fractals to the field of antenna theory was reported by Kim and Jaggard [9]. Ever since then many researchers have presented various shapes of fractal antenna like Sierpinski gasket, Koch, Hilbert curve, and others. In the present work, a square multiband Sierpinski carpet fractal antenna is taken into consideration.

The mathematical analysis of a fractal antenna is difficult due to its complex shapes and geometries. Also, it requires user expertise, more memory and time to design the antenna using any commercially available full wave solver. This motivated the authors to think of an alternative approach which is easier to implement, provides the solution in less time, requires minimum RAM and has accuracy almost similar to the commercially available tools. In recent years, researchers have explored many soft computing techniques, such as artificial neural networks, genetic algorithm, and fuzzy logic to predict the performance of various complicated antennas including fractal antenna [10–15]. Hybrid neuro-fuzzy system is one of such soft computing techniques which is successfully used in past few years for analyzing different types of antenna structures [16–22]. However, to the best of the authors' knowledge, it has not yet been explored to analyze and synthesize the Sierpinski carpet fractal antenna.

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The work aims at using the hybrid neuro-fuzzy system to analyze and synthesize the Sierpinski carpet fractal antenna. The results obtained through the proposed model are used to simulate the fractal antenna. A comparison of the simulated and measured return loss characteristics of the antenna is presented to validate the results. The paper is organized as follows: The Sierpinski carpet fractal antenna under consideration is discussed in Section 2, and the basics of hybrid neuro-fuzzy system are presented in Section 3. The detailed description of the proposed model for the analysis and synthesis of the fractal antenna is discussed in Section 4, and the results are presented in Section 5. Finally, the conclusions are outlined in Section 6.

2. SIERPINSKI CARPET FRACTAL ANTENNA

The design of a Sierpinski carpet fractal antenna begins with a square, as shown in Fig. 1(a), which is then divided into nine smaller congruent squares. Out of the nine squares, the central square is dropped as shown in Fig. 1(b). The remaining eight squares are further divided into nine small squares, and the process is repeated until the desired iteration is achieved. The geometry of the antenna under consideration is shown in Fig. 1, where a = length of the patch obtained in the 0th iteration, b = length of the patch removed in the 1st iteration and c = length of the patch removed in the 2nd iteration. Table 1 presents the geometrical and design parameters of the antenna.

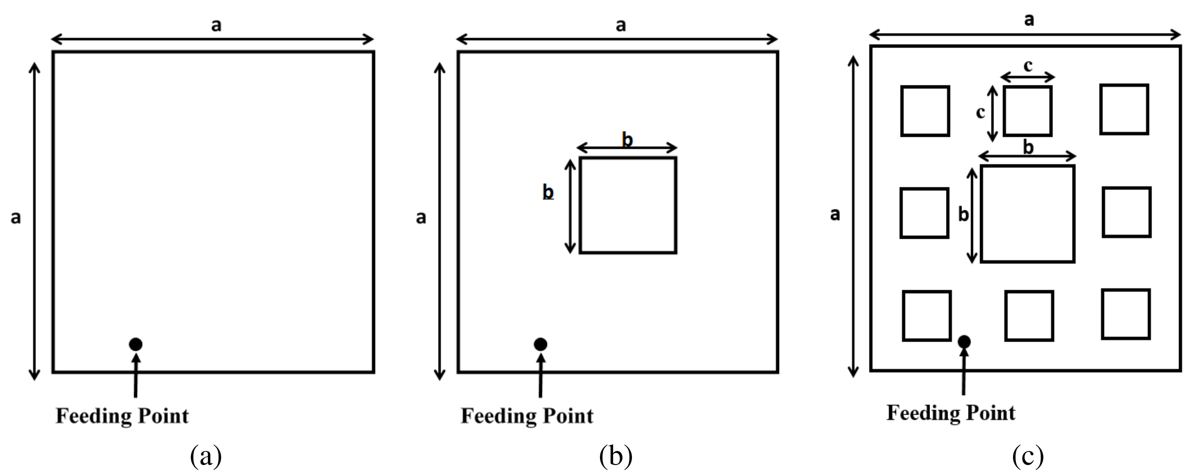

Figure 1. Sierpinski carpet fractal antenna geometry under consideration. (a) Zeroth Iteration. (b) First Iteration. (c) Second Iteration.

Table 1. Parameters of the antenna under consideration.

Parameters	**Value**
Sierpinski carpet length (a)	77 mm
Type of the Substrate	FR4
Dielectric constant of the substrate	4.4
Scaling factor	3
Operating frequency	0.5–3 GHz

3. HYBRID NEURO-FUZZY SYSTEM

The hybrid neuro-fuzzy system was proposed by Jang [23]. Basically, it is a neural network based on Takagi-Sugeno fuzzy inference system. It integrates the powerful smoothening characteristic of a fuzzy system (FS) with the adaptability of artificial neural networks (ANN) [20]. Ultimately, this combination makes the hybrid system to learn faster and accurately. Due to these inherent advantages, the system performs better than the individual ANNs or FS.

The basic hybrid neuro-fuzzy system architecture is shown in Fig. 2. It consists of five layers excluding the input and output layers. The inputs are given to the Fuzzy Layer which converts them to fuzzy value. On these fuzzy values, the *AND* operation is performed in the Product Layer to determine the weight factor of each rule. The weights are then normalized in a Normalized Layer. After normalization, the output rules are constructed in the De-fuzzy Layer. Finally, all the rules are summed up in the Summation Layer, and the output is made available.

For the development of any hybrid neuro-fuzzy model, first the dataset is generated using mathematical analysis, simulation or measurement. The dataset is divided into three parts: training, testing and validation. The dataset is provided to the hybrid neuro-fuzzy model. The model tunes itself using parameters such as type of membership function, number of membership functions and number of epoches. A feedback kind of loop is present in the model which helps obtain the optimized parameters. Once the model is trained, it is validated and tested to obtain the least error as per the requirement.

4. DEVELOPMENT OF THE PROPOSED MODELS

In this section, the development of the hybrid neuro-fuzzy models is discussed for the analysis and synthesis of the square multiband Sierpinski carpet fractal antenna. The proposed models were developed on a MATLAB platform (version 2011) using a 4 GB, 2.40 GHz machine. The necessary data for training and testing the hybrid neuro-fuzzy model were generated using High Frequency Structure Simulator (HFSS) version 15 [24]. For the analysis of the Sierpinski carpet fractal antenna, the length of the carpet (a) and scaling factor (s) were taken as the input while the first five resonant frequencies were considered as the output. The scaling factor is the ratio of the length of patch obtained in nth iteration to the length of patch obtained in $(n+1)$th iteration. For the synthesis of the Sierpinski carpet fractal antenna, the resonant frequencies were taken as the input while the antenna geometrical parameters were taken as the output. Fig. 2 presents the analysis and synthesis hybrid neuro-fuzzy model. The effectiveness of both the models was assessed by calculating the error as per the following expression:

$$\text{Error} = |\text{Result obtained through Simulation} - \text{Result obtained through the proposed model}| \quad (1)$$

A study of the effect of change in the model parameters on the return loss characteristics for $a = 77$ mm and $s = 3$ was also carried out. The hybrid neuro-fuzzy system has a limitation of providing one output, so 51 networks were used to obtain the return loss characteristics of the antenna under consideration from 0.5 to 3 GHz with a step size of 0.05 GHz. A step size of 0.025, 0.1 and 0.2 GHz was also taken into consideration, but it was observed that good results are obtained at 0.05 GHz step size. The antenna geometrical parameters were considered as the input of the models while the return loss at a particular frequency was obtained as the output as shown in Fig. 3.

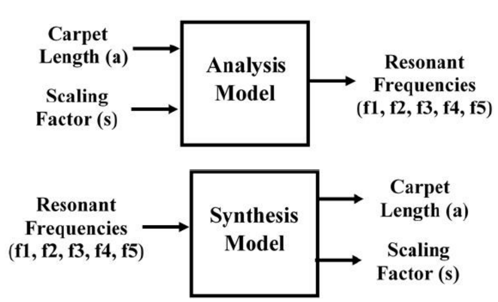

Figure 2. Proposed analysis and synthesis model.

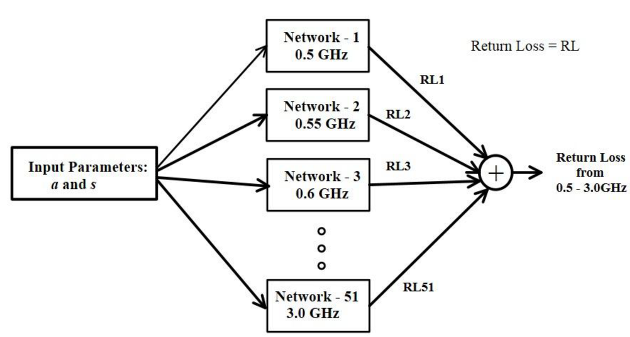

Figure 3. Proposed model for the diagnosis of the return loss characteristics.

The effectiveness of the proposed model was assessed using two statistical parameters, namely Root Mean Square Error (RMSE) and Coefficient of Determination ($R2$). The Coefficient of Determination can be obtained as [25]:

$$R^2 = \left(\frac{\sum_{i=1}^{n} (a_{it} - a_{mt})(a_{io} - a_{mo})}{\sqrt{\sum_{i=1}^{n} (a_{it} - a_{mt})^2 (a_{io} - a_{mo})^2}} \right)^2 \quad (2)$$

whereas, the Root Mean Square Error is defined as:

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (a_{it} - a_{io})^2} \quad (3)$$

In Eqs. (2) and (3), a_{it} and a_{io} denote the target values of the return loss obtained from full wave solver and from ANFIS model, respectively. a_{mt} is the mean of the targeted values and a_{mo} the mean of the observed values.

5. RESULTS AND ANALYSIS

Table 2 shows the comparison of the results obtained from the HFSS and the proposed model for the analysis of the antenna under consideration. It can be observed that the results obtained from the proposed model resemble the one obtained through simulation.

Table 2. Comparison of the results for the analysis model of the Sierpinski carpet fractal antenna.

Input	**Output**					
f1, f2, f3, f4, f5 (GHz)	**Simulation**		**Proposed Model**		**Error**	
	a (mm)	**s**	**a (mm)**	**s**	**a (mm)**	**s**
0.95, 1.40, 1.95, 2.025, 2.20	71.75	5.50	71.75	5.52	**0.00**	**0.02**
0.80, 1.25, 1.70, 2.00, 2.50	77.00	3.0	76.99	2.95	**0.01**	**0.05**
0.725, 1.075, 1.50, 1.575, 1.675	93.25	5.0	93.65	5.16	**0.40**	**0.16**

Table 3 shows the comparison of the results obtained from the HFSS and the proposed model for the synthesis of the antenna under consideration. It can be seen that the results are in close agreement.

A prototype of the fractal antenna was fabricated from the results obtained through the synthesis model as shown in Fig. 4(a). Fig. 4(b) shows the comparison of the simulated and measured return

Table 3. Comparison of the results for the synthesis model of the Sierpinski carpet fractal antenna.

Input			**Output (GHz)**				
a (mm)	s		$f1$	$f2$	$f3$	$f4$	$f5$
60.25	5.5	Simulation	1.125	1.675	2.325	2.375	2.600
		Proposed Model	1.127	1.661	2.316	2.409	2.533
		Error	**0.002**	**0.014**	**0.009**	**0.034**	**0.067**
77	3	Simulation	0.800	1.250	1.700	2.000	2.500
		Proposed Model	0.798	1.251	1.695	1.941	2.644
		Error	**0.002**	**0.001**	**0.005**	**0.059**	**0.144**
84.25	5	Simulation	0.800	1.200	1.675	1.750	1.875
		Proposed Model	0.786	1.196	1.655	1.742	1.899
		Error	**0.014**	**0.004**	**0.020**	**0.008**	**0.024**

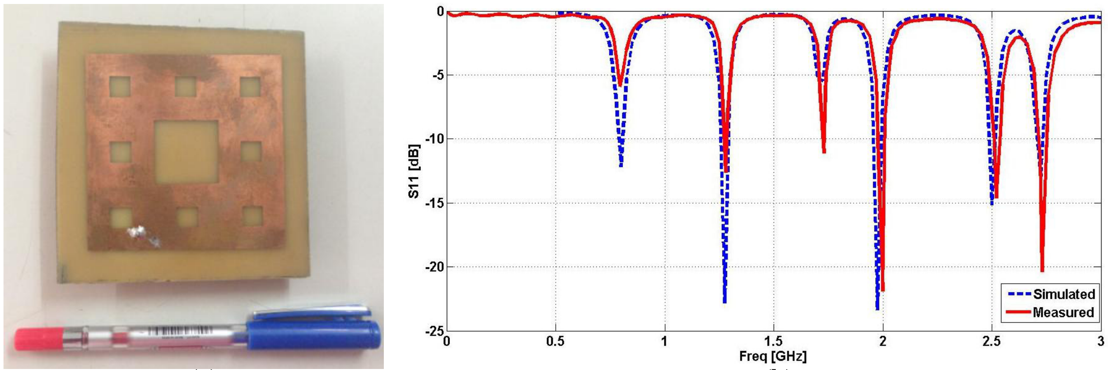

Figure 4. (a) Prototype of the square Sierpinski carpet fractal antenna. (b) Comparison of the simulated and measured return loss characterisitcs of the Sierpinski carpet fractal antenna designed using the proposed model.

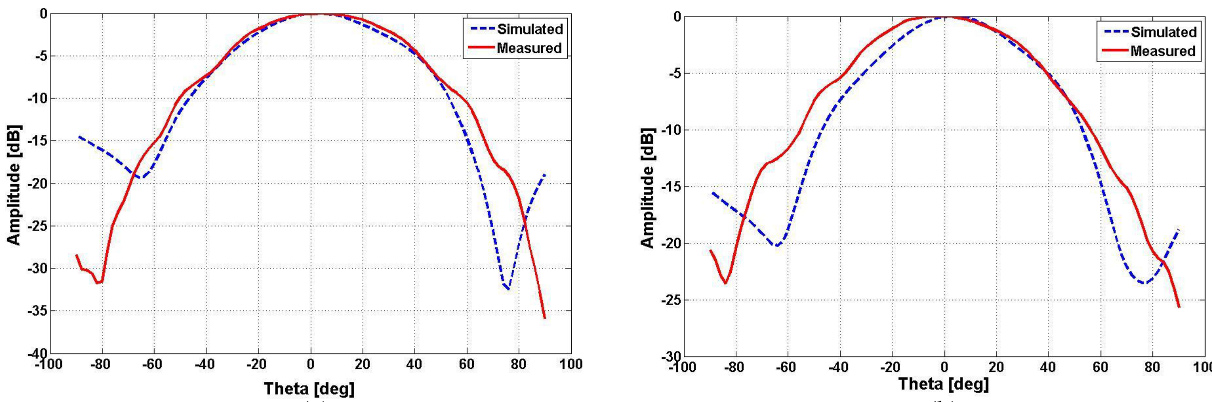

Figure 5. Comparison of the simulated and measured normalized radiation pattern of the Sierpinski carpet fractal antenna in (a) phi = 0° plane and (b) phi = 90° plane.

Table 4. Effect of change in the type and number of membership functions on the return loss characteristics of Sierpinski carpet fractal antenna considering $a = 77$ mm and $s = 3$.

No. of MF	Generalized Bell			Gaussian			Triangular			Trapezoidal		
	RMSE	R^2	Time (sec)	RMSE	R^2	Time (sec)	RMSE	R^2	Time (sec)	RMSE	R^2	Time (sec)
2	3.12	0.67	9.13	3.14	0.66	6.38	3.14	0.66	5.32	3.11	0.67	5.73
3	3.18	0.66	16.64	3.13	0.67	14.41	2.84	0.73	12.85	3.20	0.65	13.30
4	2.80	0.74	30.30	2.80	0.74	30.01	2.81	0.73	27.92	2.80	0.74	28.85
5	2.59	0.78	62.74	2.58	0.78	61.99	2.55	0.78	59.95	2.60	0.77	60.39
6	2.41	0.81	116.11	2.42	0.81	115.49	2.51	0.79	111.99	2.45	0.81	113.19
7	2.49	0.80	209.10	2.52	0.79	211.51	2.49	0.79	205.43	2.49	0.80	205.84
8	2.31	0.83	345.78	2.33	0.83	417.40	2.49	0.80	342.71	2.27	0.84	415.26
9	2.36	0.84	541.68	2.30	0.84	653.62	2.24	0.84	535.09	2.43	0.83	649.59
10	2.23	0.85	810.02	2.23	0.85	981.63	2.18	0.85	808.37	2.22	0.84	803.43

loss performances of the antenna. Comparisons of the measured and simulated normalized radiation patterns of the antenna in phi = 0° and 90° planes are given in Figs. 5(a) and 5(b), respectively. A close resemblance can be observed in all the figures.

The effect of change in number and type of membership function on the model statistical parameters is presented in Table 4. It can be observed that increase in the number of membership functions gives better result but at the cost of time taken to train the network. Also, change in the type of the membership function does not have any significance effect on the result in the present case.

6. CONCLUSIONS

In the present work, a hybrid neuro-fuzzy system is developed to analyze and synthesize the square multiband Sierpinski carpet fractal antenna. It is observed that the results obtained through the proposed method resemble closely to that obtained through the simulation. Moreover, based on the parametric study, the following conclusions can be drawn:

(i) The model is able to analyze and synthesize the Sierpinski carpet fractal antenna operating from 0.5 to 3 GHz band.
(ii) The error obtained for the analysis and synthesis models is negligible.
(iii) For obtaining the entire return loss characteristics, one model is not enough, hence 51 models are developed. Also, the value of RMSE is approximately 3, and that of R^2 is 0.8.
(iv) As the number of membership functions increases, the accuracy increases, but the time taken to train the model also increases. However, the time taken to train the model is approximately 14 mins while the commercial software like HFSS takes couple of hours to give the final results.
(v) For the antenna geometry under consideration, Gaussian, generalized bell, trapezoidal and triangular membership functions give similar results.
(vi) The comparison of the return loss characteristics and radiation pattern show that the results of the antenna simulated from the parameters obtained through the hybrid-neuro fuzzy model resemble closely to the measured results.
(vii) The model is able to provide the results with less error (less than 0.5%) and less computation time (few tens of minutes).

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