# A Clutter Suppression Method Based on Improved Principal Component Selection Rule for Ground Penetrating Radar

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Abstract—Principal component analysis is usually used for clutter suppression of ground penetrating radar, but its performance is influenced by the selection of main components of target signal. In the paper, an improved principal component selection rule is proposed for selecting the main components of target signal. In the method, firstly difference spectrum of singular value is used to extract direct wave and strong target signal, and then, Fuzzy-C means clustering algorithm is used to determine the weights of principal component of weak target signal. Finally, the principal components of strong target signal are reconstructed to obtain target signal. Experimental results show that the proposed method can effectively remove the clutter signals and reserve more target information.

# 1. INTRODUCTION

Ground Penetrating Radar (GPR) is an effective tool for underground detection [1, 2] with convenient operation, high resolution and non-destructiveness, and it is widely used in various fields such as civil engineering, archaeology, geology and military. GPR data consist of target and clutter (direct wave, underground clutter and other noise signals) [3–5]. It is difficult to extract target signal especially in the shallow subsurface. Therefore, clutter suppression is an important task for target detection of GPR.

Several methods have been applied to clutter suppression in GPR, and among them are clutter model-based methods [6–8], transform domain methods [9–12] and subspace projection methods [13– 17]. Principal Component Analysis (PCA) is a major subspace projection method [14, 15, 18–22], and the main problem of PCA is how to select the principal components of target. An auto-selected rule based on PCA is proposed in [20], and the defect of the method is that threshold selection depends on experiences, which restricts the application of the method. A method based on SVD and Fuzzy-C Means (FCM) is proposed in [22], but FCM is sensitive to single extreme point which causes the degradation of clutter suppression performance.

To solve the problem of extracting weak target signal for point target detection, an improved principal component selection rule based on difference spectrum of singular value and FCM is proposed in the paper. Firstly, difference spectrum of singular value is used to determine the principal component range of direct wave and strong target signal, and then, FCM is applied to the remaining singular values to obtain the principal component weights of weak target signal. Finally, the target signal is reconstructed with principal components of strong target signal and weighted principal components of weak target signal. Some experiments in the case of near field have been executed to evaluate the performance of the proposed method. The experimental results show that the proposed method can remove clutter effectively and reserve more target information.

Received 29 October 2016, Accepted 24 December 2016, Scheduled 6 January 2017

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#### 2. PRINCIPLE OF CLUTTER SUPPRESSION BASED ON PCA

PCA is a linear transformation algorithm based on least mean square error, and it gets orthogonal principal component of signal by special orthogonal matrix. SVD is usually used to implement orthogonal decomposition in PCA, and the signal component corresponding to different singular value is called principal component. Therefore, the selection of the principal component is actually the selection of its corresponding singular value.

B-scan data of GPR can be represented by matrix  $\mathbf{X} \in \mathbb{R}^{M \times N}$ , where M and N are sample numbers for space and time, respectively. According to the composition of GPR data,  $\mathbf{X}$  can be expressed as:

$$\mathbf{X} = \mathbf{X}_D + \mathbf{X}_S + \mathbf{X}_C \tag{1}$$

where  $\mathbf{X}_D$  is the direct wave,  $\mathbf{X}_S$  the target signal, and  $\mathbf{X}_C$  the interference signal (underground clutter and other noise signals).

The SVD of  $\mathbf{X}$  can be expressed as:

$$\mathbf{X} = \mathbf{U}\mathbf{S}\mathbf{V}^T \tag{2}$$

where  $\mathbf{U} \in \mathbb{R}^{M \times M}$  and  $\mathbf{V} \in \mathbb{R}^{N \times N}$  are unitary matrix.  $\mathbf{S} = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_r), \sigma_i$  represents the singular value of  $\mathbf{X}$  with  $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_r \geq 0$ , and r is the smaller value of M and N.  $\mathbf{u}_i$  is the  $M \times 1$  vector and  $\mathbf{v}_i$  the  $N \times 1$  vector. Therefore,

$$\mathbf{X} = \mathbf{u}_1 \sigma_1 \mathbf{v}_1^T + \mathbf{u}_2 \sigma_2 \mathbf{v}_2^T + \ldots + \mathbf{u}_r \sigma_r \mathbf{v}_r^T$$
(3)

Different signal components are reflected by different singular values. The greater singular value corresponds to the signal component with higher energy. The clutter suppression based on PCA firstly determines the singular values of target signal, and then the target signal is reconstructed with its corresponding singular values. In GPR data, the energy of  $\mathbf{X}_D$  is larger, the energy of  $\mathbf{X}_S$  in the middle and the energy of  $\mathbf{X}_C$  smaller, so the reconstructed target signal can be given by:

$$\mathbf{X}_S = \sum_{i=k_1}^{k_2} \mathbf{u}_i \sigma_i \mathbf{v}_i^T \tag{4}$$

where  $k_1$  to  $k_2$  are the subscripts of singular value  $(1 \le k_1 \le k_2 \le r)$ , and the singular values from  $k_1$  to  $k_2$  belong to target signal.

# 3. IMPROVED PRINCIPAL COMPONENT SELECTION RULE

## 3.1. FCM Algorithm

FCM is a clustering algorithm based on partition. In FCM algorithm, the objects belonging to same cluster have the largest similarity, and different clusters have the smallest similarity by minimizing the objective function [23–25]. If the sample  $\mathbf{A} = (a_1, a_2, \ldots, a_n)$  is divided into c clusters  $(T_1, T_2, \ldots, T_c)$  by FCM algorithm, the objective function  $J_{FCM}^m(\mathbf{E}, \mathbf{P})$  is:

$$J_{FCM}^{m}(\mathbf{E}, \mathbf{P}) = \sum_{i=1}^{n} \sum_{j=1}^{c} e_{ij}^{m} d_{ij}^{2}$$
(5)

where  $\mathbf{E} = \{e_{ij}\}_{n \times c}$  is the membership matrix, and  $e_{ij}$  represents membership of specimen  $a_i$  to cluster  $T_j$  and satisfies  $\sum_{j=1}^{c} e_{ij} = 1, e_{ij} \geq 0, \forall i = 1, 2, ..., n$ .  $\mathbf{P} = (p_1, p_2, ..., p_c)$  is the cluster center,  $m \in [1, \infty)$  the weighted exponent, and the optimal range is (1.5, 2.5).  $d_{ij} = ||a_i - p_j||$  is Euclidean distance of specimen  $a_i$  and cluster center  $p_j$ .

The aim of clustering is to obtain the minimum value of the function  $J_{FCM}^m(\mathbf{E}, \mathbf{P})$ . Two optimal iterative formulas resulting from Lagrange multiplier theory are given by:

$$e_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{d_{ij}}{d_{ik}}\right)^{\frac{2}{m-1}}}$$
(6)

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$$p_{j} = \frac{\sum_{i=1}^{n} e_{ij}^{m} \times a_{i}}{\sum_{i=1}^{n} e_{ij}^{m}} \qquad j = 1, 2, \dots, c$$
(7)

If the two adjacent iterative objective functions satisfy  $|J_t - J_{t+1}| < \varepsilon$ , the iteration will stop.  $\varepsilon$  is the allowable error, and it is a small positive number.

#### 3.2. Principal Component Selection Rule Based on Difference Spectrum and FCM

Since the singular value is in descending order, the energy of its corresponding signal component is also in descending order. Direct wave has strong horizontal correlation and greater energy, and it corresponds to the first few larger singular values. The energy of target decreases as the distance between target and antenna increases. When target and antenna are far apart, the energy of target is close to the energy of underground clutter and interference signals, and it is difficult to extract the weak target signal from echo signal. In order to solve this problem, an improved principle component selection rule based on difference spectrum and FCM is proposed in the paper.

Generally, the singular value of direct wave is much larger than that of target and interference signals, so there is an abrupt change in the singular value curve, and here the difference spectrum of singular value is used to describe the change and defined as:

$$q = \sigma_i - \sigma_{i+1}$$
  $i = 1, 2, \dots, r-1$  (8)

The curve describes the variation between two adjacent singular values, and a peak will occur at the position of abrupt of singular values. If the position of peak is  $k_1$ , the reconstructed direct wave can be given by:

$$\mathbf{X}_D = \sum_{i=1}^{k_1} \mathbf{u}_i \sigma_i \mathbf{v}_i^T \tag{9}$$

For the remaining  $r - k_1$  singular values, target signal can be obtained through the combination of difference spectrum and FCM. For two adjacent values  $(q_k, q_{k+1})$  in the remaining difference spectrum curve, the position relationship between k in the remaining difference spectrum curve and  $k_2$  in the whole difference spectrum is:

$$k_2 = k + k_1 \tag{10}$$

When  $|q_k - q_{k+1}|$  reaches the maximum value, the singular values from the position  $k_1 + 1$  to  $k_2$  is considered to correspond to strong target signal, then the strong target signal can be given by:

$$\mathbf{X}_{S_{strong}} = \sum_{i=k_1+1}^{k_2} \mathbf{u}_i \sigma_i \mathbf{v}_i^T \tag{11}$$

Finally, FCM is used to divide the remaining singular values  $(\sigma_{k_2+1}, \sigma_{k_2+2}, \ldots, \sigma_r)$  into two clusters. One cluster is the weak target, and the other is interference signal. According to Equation (5), the objective function is:

$$J_{FCM}^{m}(\mathbf{E}, \mathbf{P}) = \sum_{i=k_{2}+1}^{r} \sum_{j=1}^{2} e_{ij}^{m} d_{ij}^{2}$$
(12)

The two iterative formulas are:

$$e_{ij} = \frac{1}{\sum_{k=1}^{2} \left(\frac{d_{ij}}{d_{ik}}\right)^{\frac{2}{m-1}}}$$
(13)

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$$p_{j} = \frac{\sum_{i=k_{2}+1}^{r} e_{ij}^{m} \times \sigma_{i}}{\sum_{i=k_{2}+1}^{r} e_{ij}^{m}} \qquad j = 1, 2$$
(14)

The weak target signal is given by weighted sum of signal components:

$$\mathbf{X}_{S_{weak}} = \sum_{i=k_2+1}^{T} e_{i1} \mathbf{u}_i \sigma_i \mathbf{v}_i^T$$
(15)

Hence the total target signal can be given by:

$$\mathbf{X}_{S} = \mathbf{X}_{S_{strong}} + \mathbf{X}_{S_{weak}} = \sum_{i=k_{1}+1}^{k_{2}} \mathbf{u}_{i} \sigma_{i} \mathbf{v}_{i}^{T} + \sum_{i=k_{2}+1}^{r} e_{i1} \mathbf{u}_{i} \sigma_{i} \mathbf{v}_{i}^{T}$$
(16)

## 4. EXPERIMENTAL VERIFICATION

Simulation data and real GPR data in the case of near field have been applied to evaluate the performance of the proposed method, auto-selected rule based on PCA [20] and FCM based on SVD [22]. Meanwhile, image entropy is used to estimate the clutter suppression performance of the three methods. The image entropy is defined as:

$$Q = \frac{\left(\sum_{i=1}^{M} \sum_{j=1}^{N} X^{2}(i,j)\right)^{2}}{\sum_{i=1}^{M} \sum_{j=1}^{N} X^{4}(i,j)}$$
(17)

Image entropy represents the information capacity of image, smaller entropy represents better clutter suppression performance.

#### 4.1. Simulation Data

Simulation data are generated by GPRmax simulator based on finite difference time domain (FDTD) method. The center frequency of antenna is 900 MHz, and the target is a perfectly conducting rebar with 0.05 m diameter and buried at the depth of 0.22 m. The medium is a homogenous medium ( $\varepsilon_r = 3.0$ ,  $\sigma = 0.01$  S/m). The B-scan data contain 82 A-scans, and each A-scan contains 1696 samples.

Figure 1 shows the original GPR data in ideal condition, and Figure 2 shows the difference spectrum curve and membership curve of the GPR data. For the proposed method, it can be seen that the strong target signal corresponds to the second and third singular values from Figure 2(a) and Figure 2(b). The fourth and later singular values are clustered by FCM. Figure 2(c) shows the membership function curve. The curve of data1 represents the membership of target signal, and the curve of data2 represents the membership of interference signals.

Figure 3(a) shows auto-selected rule based on PCA produces obvious underground clutter. Figure 3(b) shows FCM based on SVD eliminates clutter to a certain extent. Figure 3(c) shows that the proposed method obtains least clutter on the top of the hyperbola, and the edge of the curve is clear. Table 1 shows that the proposed method obtains the smallest entropy under ideal condition. Therefore, the proposed method achieves better clutter suppression performance than other two methods under ideal condition.

In practical application, GPR signal is inevitably influenced by various noises and interferences. In order to evaluate the performance of the proposed method under different noise levels, the experiments are also implemented for the simulation data with white Gaussian noise. Figure 4, Figure 7 and Figure 10 show the image of GPR data under different noise levels.

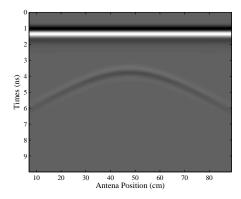
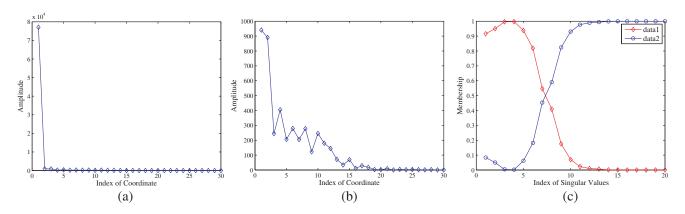
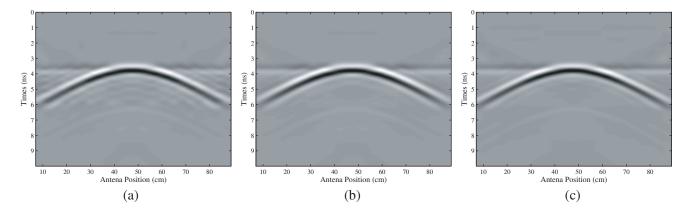


Figure 1. Original image of simulation data (ideal condition).



**Figure 2.** Difference spectrum curve and membership curve of simulation data (ideal condition). (a) Difference spectrum curve of singular value. (b) Difference spectrum curve of singular value without direct wave. (c) Membership function curve.



**Figure 3.** Experimental results of simulation data (ideal condition). (a) Auto-selected rule based on PCA. (b) FCM based on SVD. (c) Proposed method.

Figure 5, Figure 8 and Figure 11 show difference spectrum curves and membership curves of simulation data under different noise levels. Figure 5(b) and Figure 8(b) show that strong target signal corresponds to the second and third singular values. However, Figure 11(b) shows that strong target signal corresponds to the second singular value, which indicates the loss of strong target signal components under low SNR.

| Methods                         | simulation data |              |              |             |
|---------------------------------|-----------------|--------------|--------------|-------------|
| Methods                         | ideal condition | SNR = 30  dB | SNR = 10  dB | SNR = 3  dB |
| Auto-selected rule based on PCA | 6707.0          | 6878.1       | 44278        | 46019       |
| FCM based on SVD                | 6588.4          | 6632.4       | 42042        | 44756       |
| Proposed Method                 | 6421.8          | 6616.1       | 40851        | 44737       |

Table 1. Entropy comparison of simulation data.

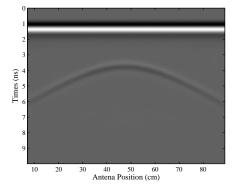


Figure 4. Original image of simulation data with noise (SNR = 30 dB).

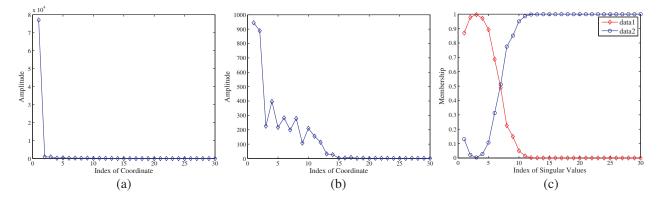


Figure 5. Difference spectrum curve and membership curve of simulation data with noise (SNR = 30 dB). (a) Difference spectrum curve of singular value. (b) Difference spectrum curve of singular value without direct wave. (c) Membership function curve.

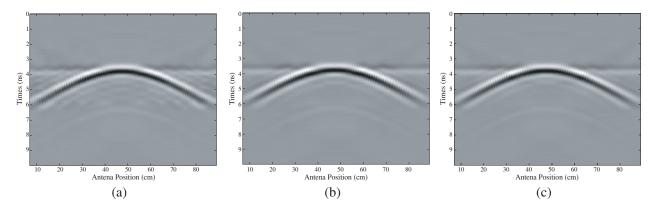


Figure 6. Experimental results of simulation data with noise (SNR = 30 dB). (a) Auto-selected rule based on PCA. (b) FCM based on SVD. (c) Proposed method.

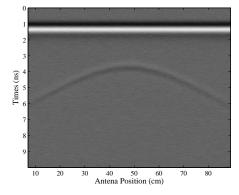


Figure 7. Original image of simulation data with noise (SNR = 10 dB).

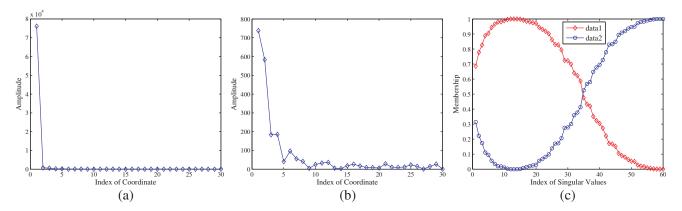


Figure 8. Difference spectrum curve and membership curve of simulation data with noise (SNR = 10 dB). (a) Difference spectrum curve of singular value. (b) Difference spectrum curve of singular value without direct wave. (c) Membership function curve.

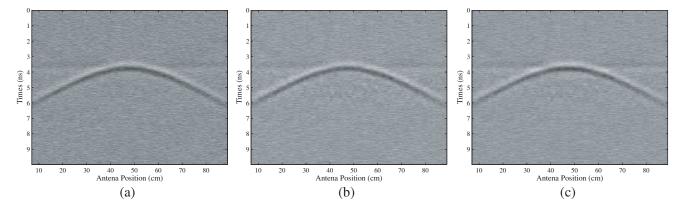


Figure 9. Experimental results of simulation data with noise (SNR = 10 dB). (a) Auto-selected rule based on PCA. (b) FCM based on SVD. (c) Proposed method.

Figure 6 and Figure 9 show that the proposed method can effectively suppress clutter under higher SNR. When SNR is 30 dB and 10 dB, the results obtained by the proposed method are better than the results obtained by other two methods. The proposed method also obtains the smallest entropy among the three methods in Table 1. The clutter suppression performance of the proposed method drops with the decrease of SNR. Figure 12 shows that the proposed method is unable to extract the target signal when SNR decreases to 3 dB, and the other two methods also fail to detect the target signal under lower SNR.

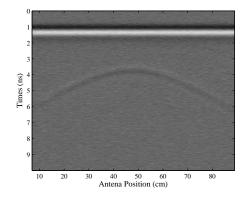


Figure 10. Original image of simulation data with noise (SNR = 3 dB).

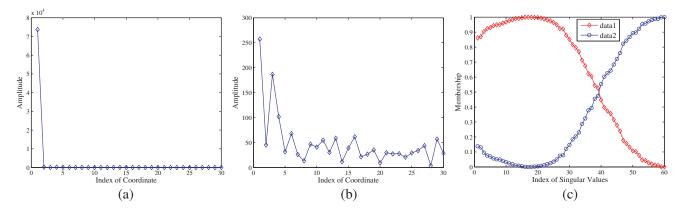


Figure 11. Difference spectrum curve and membership curve of simulation data with noise (SNR = 3 dB). (a) Difference spectrum curve of singular value. (b) Difference spectrum curve of singular value without direct wave. (c) Membership function curve.

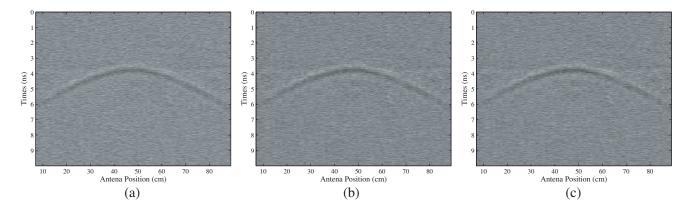


Figure 12. Experimental results of simulation data with noise (SNR = 3 dB). (a) Auto-selected rule based on PCA. (b) FCM based on SVD. (c) Proposed method.

## 4.2. Real GPR Data

Real GPR data come from foreign anti-landmine research center. A PMN-2 Antipersonnel Landmine with 12 cm in diameter and 5.3 cm in height is buried at the depth of 10 cm. An impulse GPR is used to detect the landmine. The antenna is a shielding antenna, and its operation frequency is 1 GHz. The B-scan contains 98 A-scans, and each A-scan contains 512 samples.

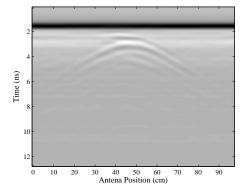
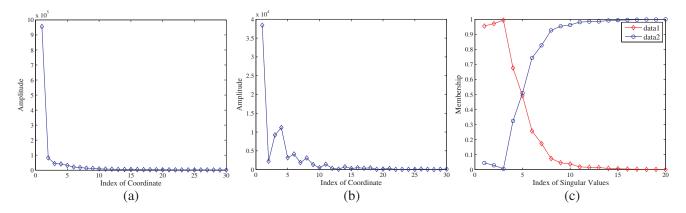
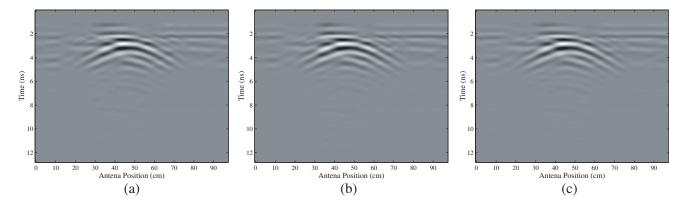


Figure 13. Original image of real GPR data.



**Figure 14.** Difference spectrum curve and membership curve of real GPR data. (a) Difference spectrum curve of singular value. (b) Difference spectrum curve of singular value without direct wave. (c) Membership function curve.



**Figure 15.** Experimental results of real GPR data. (a) Auto-selected rule based on PCA. (b) FCM based on SVD. (c) Proposed method.

Figure 13 shows the original image of real GPR data. Figure 14(a) shows that the first singular value represents direct wave. Figure 14(b) shows that the second singular value represents strong target. Figure 14(c) shows the membership function curve of the remaining singular values. Curve of data1 represents the membership of target signal, and curve of data2 represents the membership of interference signals. Figure 15 shows that the proposed method can eliminate direct wave and other clutter effectively, and it gets clearer target hyperbola than other two methods. Table 2 also shows that

| ĺ | Methods                         | Entropy |
|---|---------------------------------|---------|
|   | Auto-selected rule based on PCA | 1818.8  |
|   | FCM based on SVD                | 1886.8  |
|   | Proposed Method                 | 1742.3  |

Table 2. Entropy comparison of real GPR data.

the proposed method obtains lowest entropy among the three methods, which further demonstrates that the proposed method can improve the performance of clutter suppression.

## 5. CONCLUSION

PCA is an effective method for clutter suppression of GPR, and its performance is influenced by the selection of target principal component. In order to solve the problem of extracting the weak target signal for point target detection, an improved principal component selection rule based on difference spectrum of singular value and FCM is proposed in the paper. The principal components of strong target signal and the weight of principal components of weak target signal are determined by difference spectrum and FCM respectively. The target signals are obtained by reconstructing principal components of strong target signal and weighted principal components of weak target signal. The experimental results show that the proposed method exhibits better clutter suppression performance than other two principal component selection rules.

# ACKNOWLEDGMENT

The work is supported by the National Natural Science Foundation of China (Nos. 61401409, 61503351, 61503350) and Hubei province Natural Science Foundation of China (No. 2015CFB520).

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