Progress In Electromagnetics Research M, Vol. 54, 1–7, 2017

Higher Radial Modes of Azimuthal Surface Waves in Cylindrical Waveguides without External Magnetic Field

Igor O. Girka[1, *], Igor V. Omelchenko[1], and Richard D. Sydora[2]

Abstract—The properties of higher order radial modes of electromagnetic azimuthal surface-type waves (ASW) which propagate in partially plasma-filled cylindrical waveguides without external magnetic field are analyzed using analytical and numerical techniques. For a waveguide with plasma surrounded by dielectric material and encased in metal, the eigenfrequencies for higher order radial modes are obtained. It is found that the ASW higher radial modes propagate with shorter vacuum wavelength than the zero-th order radial modes and that the more favourable conditions for higher order radial mode propagation are for ASW's with larger azimuthal wavenumber in waveguides with wider dielectric layer and larger dielectric constant. A further salient feature of ASW higher radial modes is that a change in plasma waveguide parameters causes a drastic change in ASW eigenfrequency in contrast to the zero-th order modes which have a smoother frequency variation with effective wavenumber.

1. INTRODUCTION

The properties of electromagnetic surface waves (SW) which propagate along a planar boundary between two different media and those that propagate along a boundary with a finite value of curvature radius differ substantially [1]. For instance, in the case of the planar plasma boundary shape, most of the SWs are slow waves; they are either potential ones or at least can be considered in the potential approach (see, e.g., [2, 3]), unlike the case of SW propagating along the plasma boundary with finite value of curvature radius [1].

Theoretical results from studying the SWs properties are widely used in different branches of radio-physics and electronics, plasma technologies and nano-physics. One of the actively developing spheres of SWs application is the evaluation and construction of plasma-antenna systems. Utilization of antenna with plasma coating has been known for some time [4–6]. It has undisputed advantages, namely, application of a plasma allows for an increase in the electromagnetic radiation and control of the frequency spectrum. Experimental results presented in [5, 6] have illustrated an essential enhancement of the plasma-antenna efficiency for transmission and reception of electromagnetic signals. At the present time the main efforts of scientists which are working in this area are directed towards solving the following problems: (i) expansion of the working frequency range; (ii) making the plasma antenna invisible; (iii) protection from electronic warfare; (iv) operation in the regime of shielding; and (v) reduction of electromagnetic noise in the antenna system. Generally speaking, it has been shown that plasma-antenna systems can now work at least as well as metal antennas.

The other area of utilization of SW propagation is gas discharges and their practical applications (see, e.g., monograph [7] and references therein). Comparative analysis of different types of plasma production methods has shown the advantages of the plasma source sustained by SWs in comparison with other high-density plasma sources. It has allowed the authors of [8] to conclude that the regime of low-pressure discharge sustained by azimuthally non-symmetric SWs is attractive for sustaining a high density, large-scale non-magnetized plasma that can be utilized for various solids processing technologies.

Received 21 November 2016, Accepted 5 January 2017, Scheduled 1 February 2017

* Corresponding author: Igor O. Girka (igorgirka@karazin.ua).

[1] V. N. Karazin Kharkiv National University, Kharkiv, Ukraine. [2] University of Alberta, Edmonton, AB, Canada.

One other sphere of SW utilization under current active development is in nano-physics. The possibility to build up nano-plasma materials makes them attractive for applications such as antenna systems and elements of millimetre wave electronic devices [9]. Reference [10] presents the material devoted to various technologies for metal plasma production, especially on the formation of nano-structures using the metal-plasmas. These techniques allow one to obtain a trench-filling and a conformal coating of a substrate surface down to the 100 nm size; to fabricate super-hard and tough nano-structures by depositing the titanium-aluminium and titanium-zirconium films and even to reduce the friction coefficient between nano-structures by incorporating the ions of yttrium or vanadium.

Finally, we remark on another SW application such as elaboration of bio-sensors which utilize surface plasmon resonance for their operation [11, 12]. Devices based on these bio-sensors are utilized for important operations that include: diagnostics of animal and plant pathogens in order to organize a quarantine zone, gene analysis, remote rapid testing of a malarial strain, water purity analysis, and detecting an unexpected chemical processes in laboratories. Thus, the presence of such a variety of important practical applications of the SWs' properties is the motivation to investigate the properties of azimuthally non-symmetric SWs, which can propagate in isotropic plasma-filled waveguides.

In a previous paper [13] we have studied the dispersion properties of transverse surface waves propagating across the axis of a cylindrical metal waveguide that is partially filled with isotropic plasma, so-called azimuthal SW (ASW). The configuration has been restricted to the assumption of a thin dielectric layer, $(b - a) \ll a$, where a and b refer to the inner and outer radii. This assumption invoked to ensure that just the zero-th order radial mode was considered. In this case, the authors expected that this was the main influence of plasma column parameters on the dispersion properties of the electromagnetic waves. On the other hand, practical applications of the waves need the highest possible frequency of the waveguide's eigenmodes. The waves under consideration can propagate in the frequency range below the Langmuir frequency, $\omega < \Omega_e$ [1, 14]. In the present paper we continue studying these waves and focus on the problem of the dispersion properties of their higher radial modes. In the waveguides with small values of effective wave number $k_{ef} = |m|\delta/a$ ($\delta = c/\Omega_e$ is skin-depth), higher radial modes of ASW are found to propagate with the frequencies higher than those of the zero-th order radial modes.

The paper is arranged as follows. Section 2 is devoted to the formulation of the problem, in particular, the description of the geometry of the plasma waveguide and boundary conditions applied for the derivation of the set of equations for harmonics of the tangential electric field of these waves. Section 3 is devoted to a theoretical analysis of the dispersion relation of these modes. Results of numerical analysis of the dispersion relation of these ASW are discussed in Section 4. Section 5 summarizes the main results.

2. FORMULATION OF THE PROBLEM

Let us consider a uniform plasma cylinder of radius a, being placed concentrically inside the metal chamber of radius b (see Fig. 1). The plasma cylinder is separated from the metal by a dielectric with dielectric constant ε_d. We assume the waveguide to be uniform along the axis: $\partial/\partial z = 0$. Plasma electromagnetic properties are described by the permittivity, $\varepsilon_p = 1 - \Omega_e^2/\omega^2$.

First, we consider the propagation of electromagnetic waves along the small azimuth, such that the wave fields depend on time and spatial coordinates as follows: $f(\vec{r}) = f_1(r)\exp(im\varphi - i\omega t)$. In this case Maxwell's set of equations is separated into two subsets which describe independent propagation of two waves: that of ordinary polarisation with the components E_z, B_r, B_φ and that of extraordinary polarisation with the components E_r, E_φ, B_z. We study here just the surface-type extraordinary wave. This means that the amplitudes of the wave fields decrease when going from the plasma boundary to the axis. Such waves are called azimuthal surface waves (ASW) and in addition to being surface-type in the plasma column, they are of volumetric nature in the dielectric layer.

Next we discuss the restrictions of our model. Propagation of the ordinary ASW was studied in [15] and the influence of a weak radial plasma density inhomogeneity on the dispersion properties of ASW was studied in [16]. If the plasma density has a linear profile near the boundary, then the ASW dispersion properties are similar to those of a uniform plasma by replacing the penetration depth of these waves into the plasma with an "effective" value of this parameter [17]. The propagation of

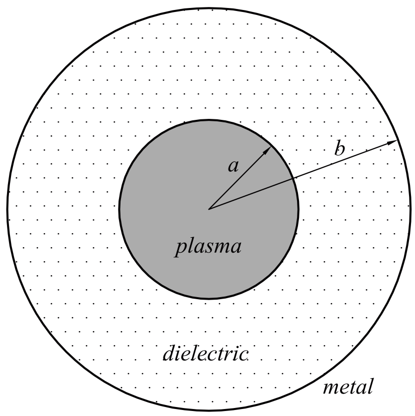

Figure 1. Schematic of the geometry.

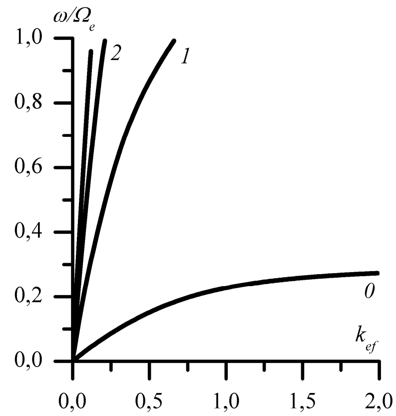

Figure 2. Dependence of ASW eigenfrequency normalised by electron Langmuir frequency versus effective wave number. Parameters used are: $\varepsilon_d = 4$, $\Delta = 0.5$, $m = 1$.

surface-type electromagnetic waves with small axial wavenumber k_z was studied in [18]. It was shown that the presence of finite axial wavenumber results in a decreasing of the eigenfrequency as compared to the case of ASW, and onset of the new slow mode solution that does not exist in the case of ASW. The eigenfrequency of the slow mode is almost proportional to k_z. The presence of the small $k_z < |m|/a$ weakly effects the eigenfrequency of the fast mode which becomes the ASW in the limit $k_z \to 0$. The correction to the eigenfrequency caused by the small k_z is proportional to k_z^2.

To get the ASW dispersion relation one applies the following boundary conditions: wave fields should be of finite value inside the waveguide, in particular, at its axis; tangential components of electric and magnetic fields E_φ and B_z should be continuous at the boundary plasma-dielectric; tangential electric field E_φ should be equal to zero at the ideal metal wall. The dispersion relation was derived in [14] (see also [1, 13]):

$$\frac{J'_m(x_1)\,N'_m(x_2) - J'_m(x_2)\,N'_m(x_1)}{J'_m(x_2)\,N_m(x_1) - J_m(x_1)\,N'_m(x_2)} = \frac{x_1}{x_3}\frac{I'_m(x_3)}{I_m(x_3)}. \tag{1}$$

In Eq. (1), $J_m(x)$ and $N_m(x)$ are Bessel functions of the first and second kinds, respectively; $I_m(x)$ is modified Bessel function of the first kind [19]; prime denotes the derivative with respect to the argument; $x_1 = (\omega/c)a\sqrt{\varepsilon_d}$; $x_2 = (\omega/c)b\sqrt{\varepsilon_d}$; $x_3 = (\omega/c)a\sqrt{-\varepsilon_p}$.

3. THEORETICAL ANALYSIS OF THE DISPERSION RELATION

Since we deal with higher radial modes of ASW we should operate with large arguments in the cylindrical functions $J_m(x)$ and $N_m(x)$ in the dispersion relation (1):

$$x_1 = \frac{\omega}{\Omega_e}\frac{|m|\sqrt{\varepsilon_d}}{k_{ef}}. \tag{2}$$

Taking into account the asymptotic expressions for cylindrical functions $J_m(x)$ and $N_m(x)$ of large argument [19], one can derive the asymptotic expansion of the left-hand side of the dispersion relation in Eq. (1):

$$\frac{J'_m(x_1)\,N'_m(x_2) - J'_m(x_2)\,N'_m(x_1)}{J'_m(x_2)\,N_m(x_1) - J_m(x_1)\,N'_m(x_2)} \approx -\tan(x_2 - x_1). \tag{3}$$

Let us assume the waveguide to be wide, i.e., its radius is larger than the radial wave penetration depth,

$$x_3 = x_1\sqrt{\frac{|\varepsilon_p|}{\varepsilon_d}} \gg 1. \tag{4}$$

This assumption is true for the case of low frequencies, $\omega \ll \Omega_e$: $x_3 \approx |m|/k_{ef}$. Then the right-hand side of the dispersion relation is approximately equal to

$$\frac{x_1}{x_3}\frac{I'_m(x_3)}{I_m(x_3)} \approx \frac{\omega}{\Omega_e}\sqrt{\varepsilon_d}. \tag{5}$$

Searching for the eigenfrequency of the l-th radial mode, $x_2 - x_1 \approx \pi l - (\omega/\Omega_e)\sqrt{\varepsilon_d}$, we derive the following approximate expression:

$$\frac{\omega}{\Omega_e} \approx \frac{\pi l}{\sqrt{\varepsilon_d}}\frac{k_{ef}}{|m|\Delta}, \tag{6}$$

where $\Delta = (b-a)/a$ is not a small value. By inspection of formula (6) one can predict the almost linear dependence of the ASW higher radial modes eigenfrequency versus effective wave number k_{ef}, inverse proportionality of ω to the width of dielectric layer Δ and to the square root of dielectric constant ε_d. This rough evaluation does not indicate any dependence of the ASW eigenfrequency from neither ASW azimuthal wave number m (since $k_{ef} \propto |m|$) nor plasma density n_e (since $k_{ef} \propto \Omega_e^{-1}$).

It should be noted that when we say "l-th radial mode" we mean that the wave magnetic field has l zeros in the range $0 < r < b$ except for $r = 0$. The zero-th ASW radial mode has only one zero of its magnetic wave's field for $r = 0$, and its radial derivative is equal to zero at the metal wall of the chamber at $r = b$.

4. NUMERICAL ANALYSIS

In this section we make a numerical analysis of the dispersion relation (1) and plot the dependence of the ASW eigenfrequency (normalized by the Langmuir frequency, Ω_e) versus the effective wave number k_{ef} for the first three azimuthal wave numbers: $m = 1$, 2, 3, and for the lowest four radial modes: zero-th, first, second and third (see Figs. 2–4). Numerical analysis makes it possible also to study spatial distribution of the waves under consideration (see Figs. 5–7).

To distinguish the branches corresponding to different radial modes we checked the phase incursion for each root of the dispersion relation. Numerals on the Figs. 2–4 denote the numbers of the radial modes. Higher radial modes are pronounced in the range of small values of k_{ef}, that is for large radius a of plasma column and high plasma density n_e. Numerical calculations define the dependence of the ASW eigenfrequency on plasma density n_e or more accurately, $\omega = C_1 - C_2 n_e^{-1/2}$, the constants being positive: $C_1 > 0$, $C_2 > 0$.

Numerical analysis confirms that the range of effective wave numbers, within which the ASW higher radial modes propagate, increases in the direction of higher k_{ef} with increase of azimuthal wave number m, width of the dielectric layer Δ and dielectric constant ε_d. This is the reason for choosing

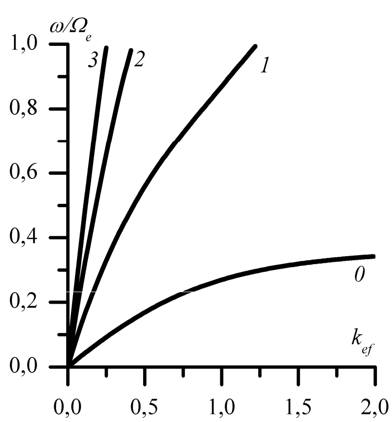

Figure 3. The same as in Fig. 8, but for $m = 2$.

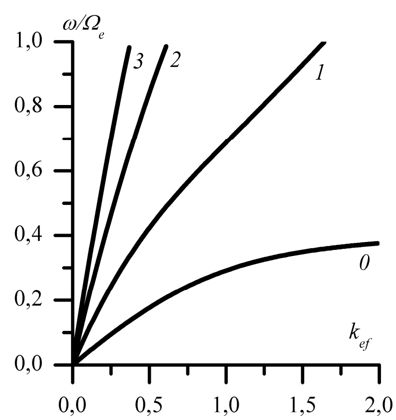

Figure 4. The same as in Fig. 8, but for $m = 3$.

the following plasma waveguide parameters. Figs. 2–4 demonstrate the dependence of the normalized ASW eigenfrequency ω/Ω_e on the effective wave number k_{ef} for a rather narrow dielectric layer $\Delta = 0.5$ and a moderate dielectric constant $\varepsilon_d = 4$. We restrict the ordinate axes of our plots on Figs. 2–4 by the value $\omega/\Omega_e < 1$ since the waves under consideration are those of surface-type just in this frequency range.

Figures 2–4 confirm the analytical prediction that larger dielectric constant $\varepsilon_d = 4$ provides better conditions for higher radial mode propagation even for the case of moderate dielectric width of $\Delta = 0.5$. The lines in these figures are very similar to those calculated for the case of twice larger width of the dielectric layer, $\Delta = 1.0$, and four times less dielectric constant, $\varepsilon_d = 1$. Analytical estimations (approximate formula (6)) predict the same values for the eigenfrequencies of the ASW higher radial modes in these two cases. Indeed, eigenfrequencies of ASW first radial mode in these two cases coincide with the accuracy of about 11%. For instance, in the middle of the range of k_{ef} where the first ASW

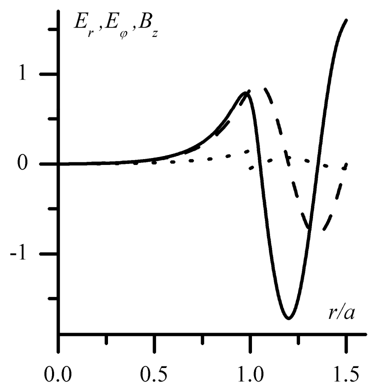

Figure 5. Radial distribution of ASW electromagnetic fields amplitudes: E_r (dotted line), E_φ (dashed line), B_z (solid line). $\varepsilon_d = 4$, $\Delta = 0.5$, $m = 2$, $l = 2$, $k_{ef} = 0.25$, $\omega/\Omega_e = 0.659$.

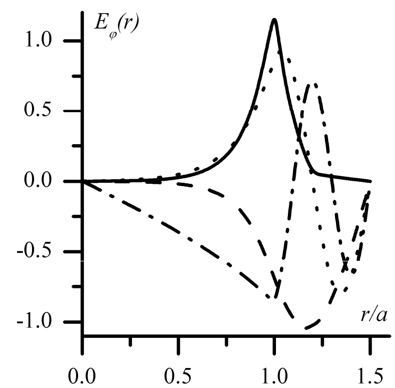

Figure 6. Radial distribution of ASW azimuthal electric field E_φ amplitudes for four radial modes: $l = 0$ (solid line), $l = 1$ (dashed line), $l = 2$ (dotted line), $l = 3$ (dash-dotted line). $\varepsilon_d = 4$, $\Delta = 0.5$, $m = 2$, $k_{ef} = 0.25$.

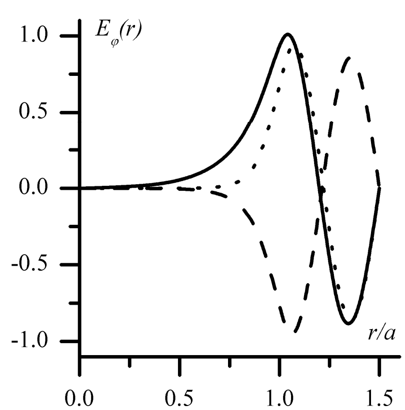

Figure 7. Radial distribution of ASW azimuthal electric field E_φ amplitudes for three azimuthal wave numbers: $m = 2$ (solid line), $m = 3$ (dashed line), $m = 4$ (dotted line). $\varepsilon_d = 4$, $\Delta = 0.5$, $m = 2$, $k_{ef} = 0.25$, $l = 2$.

radial modes exist, namely, for $k_{ef} = 0.33$, the eigenfrequency of the ASW is $\omega/\Omega_e = 0.771$ for the case $\Delta = 1.0$ and $\varepsilon_d = 1$, and $\omega/\Omega_e = 0.683$ for the conditions of the Fig. 2.

Spatial distribution of ASW fields in azimuthal direction is determined by the factor $\exp(im\varphi)$. The phases of the axial magnetic field B_z and radial electric field E_r coincide with each other and lag behind the phase of azimuthal electric field E_φ by $\pi/2$.

Typical radial distribution of the ASW fields is given in Figs. 5–7 in arbitrary units. In Fig. 5, one can see the radial distribution of the amplitudes of the fields E_r (dotted line), E_φ (dashed line), B_z (solid line) for second radial mode. The other plasma waveguide parameters are as follows: $\varepsilon_d = 4$, $\Delta = 0.5$, $m = 2$, $k_{ef} = 0.25$. In this case ASW propagates with the frequency $\omega = 0.659\Omega_e$, and its fields weakly penetrates into the plasma, $x_3 = 6.017$ (this means that plasma radius is 6.017 times larger than the field's penetration depth). The amplitude of the radial electric field E_r is discontinuous at the plasma-dielectric interface, $r = a$. The amplitude of the azimuthal electric field E_φ is equal to zero at the internal surface of the metal chamber, $r = b$, which is accompanied with the derivative of the amplitude of the axial magnetic field B_z being equal to zero there. Note that the phase incursion of the E_φ is just 2π as it should be for the second radial mode being under the consideration.

Figure 6 presents the radial distribution of the first four radial modes of azimuthal ASW electric field E_φ, other plasma waveguide parameters being the same as in Fig. 5. The zeroth radial mode is given by the solid line ($\omega/\Omega_e = 0.092$), the first mode — by the dashed line ($\omega/\Omega_e = 0.33$), the second mode — by the dotted line ($\omega/\Omega_e = 0.659$), and the third mode — by the dash-dotted line ($\omega/\Omega_e = 0.989$). Let us underline the increase of the phase incursion in the dielectric region, $a < r < b$: $x_2 - x_1 = 0.736$ in the case $l = 0$, $x_2 - x_1 = 2.64$ in the case $l = 1$, $x_2 - x_1 = 5.273$ in the case $l = 2$, and $x_2 - x_1 = 7.912$ in the case $l = 3$. The surface nature of the ASW is less pronounced for the higher radial modes: $x_3 = 7.966$ (this means that plasma radius is 7.966 times larger than the field's penetration depth) for the zeroth radial mode, $l = 0$, $x_3 = 7.552$ for $l = 1$, $x_3 = 6.017$ for $l = 2$, and $x_3 = 1.183$ for $l = 3$.

The radial distribution of the amplitude of the second radial mode of azimuthal electric field E_φ for ASWs with azimuthal wave numbers $m = 2$ is given in Fig. 7 by the solid line, the plasma waveguide parameters being the same as in Fig. 5. In this case ASW propagates with the frequency $\omega = 0.659\Omega_e$. The radial distribution of the amplitude of the same radial mode of E_φ for $m = 3$ is given there by the dashed line ($\omega/\Omega_e = 0.465$), and for $m = 4$ — by the dotted line ($\omega/\Omega_e = 0.364$). ASW with $m = 1$ does not propagate in the form of the second radial mode for the chosen plasma-waveguide parameters. An increase in azimuthal wave number m causes a decrease in both ASW frequency and the penetration depth.

5. CONCLUSIONS

Both analytical and numerical studies of the dispersion properties of higher radial modes of electromagnetic surface-type waves which propagate in cylindrical waveguides, partially filled by plasma, along the small azimuth are carried out. These mode branches complement previous results [1, 13, 14] obtained for the zero-th order radial mode. The most favourable conditions for these higher radial modes propagation are observed for waves with larger azimuthal wave numbers in the waveguides with wider dielectric layer, and larger dielectric constant. An approximate expression (6) satisfactorily describes the eigenfrequency of the ASW higher radial modes. The possibility of ASW higher radial mode propagation is demonstrated for small values of the effective wavenumber, i.e., in the waveguides with large radius of plasma column and plasma density. One particular merit of the ASW higher radial modes dispersion properties is that a change in plasma waveguide parameters causes a drastic change in ASW eigenfrequency. This is in contrast to the possibility of obtaining smoother frequency tuning for the ASW zero-th order radial mode.

The advantage of ASW higher radial modes is that their eigenfrequencies are larger than those of the zero-th radial modes effectively studied earlier. In other words, ASW higher radial modes propagate with shorter vacuum wavelength than the zero-th modes. The results presented here are of interest for the purposes of plasma electronics. The possibility of ASW irradiation from the narrow axial slot in the waveguide metal wall was demonstrated in [13].

REFERENCES

1. Girka, V., I. Girka, and M. Thumm, *Surface Flute Waves in Plasmas: Theory and Applications*, Springer, Switzerland, 2014.
2. Gradov, O. M. and L. Stenflo, "Linear theory of a cold bounded plasma," *Physics Reports — Review Section of Phys. Lett.*, Vol. 94, 111–137, 1983.
3. Gradov, O. M. and L. Stenflo, "Theory of nonlinear plasma surface waves," *Journal of Plasma Physics*, Vol. 65, 73–77, 2001.
4. Kudrin, A. V., E. Y. Petrov, G. A. Kyriacou, and T. M. Zaboronkova, "Insulated cylindrical antenna in a cold magnetoplasma," *Progress In Electromagnetics Research*, Vol. 53, 135–166, 2005.
5. Alexeff, I., T. Anderson, E. Farshi, et al., "Recent results for plasma antennas," *Physics of Plasmas*, Vol. 15, 057104, 2008.
6. Anderson, T., *Plasma Antennas*, Artech House, Boston, London, 2011.
7. Aliev, Y. M., H. Schluter, and A. Shivarova, *Guided-wave-produced Plasmas*, Springer, New-York, 2000.
8. Sugai, H., I. Ghanashev, and M. Nagatsu, "High-density flat plasma production based on surface waves," *Plasma Sources Science and Technology*, Vol. 7, 192–205, 1998.
9. Ederra, I., J. C. Iriarte, R. Gonzalo, and P. de Maagt, "Surface waves of finite size electromagnetic band gap woodpile structures," *Progress In Electromagnetics Research B*, Vol. 28, 19–34, 2011.
10. Anders, A., "Metal plasmas for the fabrication of nanostructures," *Physics D: Applied Physics*, Vol. 40, 2272–2284, 2007.
11. Morrow, R., D. R. McKenzie, and M. M. Bilek, "Electric field effects on adsorption/desorption of proteins and colloidal particles on a gold film observed using surface plasmon resonance," *Physica B: Condensed Matter*, Vol. 394, 203–207, 2007.
12. Feltis, B. N., B. A. Sexton, F. L. Glenn, et al., "A hand-held surface plasmon resonance biosensor for the detection of ricin and other biological agents," *Biosensors and Bioelectronics*, Vol. 23, 1131–1136, 2008.
13. Girka, V. O., I. O. Girka, and R. D. Sydora, "Azimuthally non-symmetric surface waves propagating in metal waveguides filled with isotropic plasma," *Progress In Electromagnetics Research B*, Vol. 61, 87–98, 2014.
14. Girka, V. O., I. O. Girka, A. N. Kondratenko, and V. I. Tkachenko, "Azimuthal surface waves of magnetoactive plasma wavequides," *Soviet Journal of Communications Technology and Electronics*, Vol. 33, 37–41, 1988.
15. Girka, V. O. and I. O. Girka, "Coupled azimuthal surface waves in a nonuniform current — carrying plasma cylinder," *Soviet Journal of Communications Technology and Electronics*, Vol. 37, 23–29, 1992.
16. Girka, V. O. and I. O. Girka, "Influence of plasma inhomogeneity on the spectra of azimuthal surface waves," *Radiophysics and Quantum Electronics*, Vol. 33, 516–517, 1990.
17. Girka, V. O. and I. O. Girka, "Azimuthal surface waves in a nonuniform plasma cylinder," *Radiophysics and Quantum Electronics*, Vol. 34, 324–328, 1991.
18. Girka, V. O. and I. O. Girka, "Asymmetric long-wavelength surface modes of isotropic plasma waveguides," *Plasma Physics Reports*, Vol. 28, 682–689, 2002.
19. Abramowitz, M. and I. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards, Applied Mathematics Series 55, Washington, 1972.