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Magnetically Tuned Two-Component Microwave Metamaterial

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Abstract—In this study, the effective magnetic response of magnetic metamaterial is considered in the microwave frequency range. The metamaterial is an infinite isotropic dielectric host medium with periodically embedded ferric cylindrical inclusions. It is assumed that the inclusions are partially magnetized by a dc bias magnetic field. The electromagnetic wave propagation is considered in the direction of bias magnetic field and transverse to bias magnetic field. It is shown that real part of the effective relative permeability can have $\mathrm{Re}(\mu_{eff}) < 0$ or $0 < \mathrm{Re}(\mu_{eff}) < 1$ or $\mathrm{Re}(\mu_{eff}) > 1$ depending on the value of bias field.

1. INTRODUCTION

Partially magnetized ferrites are widely used in microwave applications such as isolators [1], circulators [2], resonators [3], compact phase shifters [4], band-pass philters [5], band-stop philters [6] due to their large permeability tunable range, low or zero bias field, and fast tuning speed at micro-seconds. However, the relative permeability of partially magnetized conventional ferrites have $\mu_r > 1$ that can be accepted as a disadvantage taking into account a growing interest in antennas printed on metamaterial substrates with $0 < \mu_r < 1$ [7], and wireless power transfer systems fabricated with utilizing of mu-negative metamaterials $\mu_r < 0$ [8]. Magnetic metamaterials or metaferrites can avoid the mentioned disadvantage. Indeed, as shown in work [9], real part of the effective relative permeability of array of magnetic metallic wires embedded in a dielectric medium can span any of the above mentioned value range depending on the value of bias magnetic field. However, the study in [9] is purely based on numerical solvers while analytical expressions for the effective magnetic responses are currently only developed for the case of fully magnetized inclusions [10, 11].

In this paper, the microwave approximations of effective magnetic response of an infinite isotropic dielectric host medium with periodically embedded ferric cylindrical inclusions partially magnetized by a dc bias magnetic field is derived for the first time. Two directions of propagation of a plane monochromatic electromagnetic wave are considered in this study — the direction of bias magnetic field and the direction transverse to bias magnetic field.

2. EFFECTIVE PERMEABILITY TENSOR

Consider two-component metamaterial as an infinite host dielectric medium (matrix) with periodically embedded ferric cylindrical inclusions with the unit cell of square cross section, Fig. 1, where r is the radius of inclusions, a the dimension of unit cell, ϵ_m the relative permittivity of matrix, μ_m the relative permeability of matrix, ϵ_i the relative permittivity of inclusion material, μ_i the relative permeability of inclusion material, and $\overrightarrow{B}$ the vector of dc bias magnetic field.

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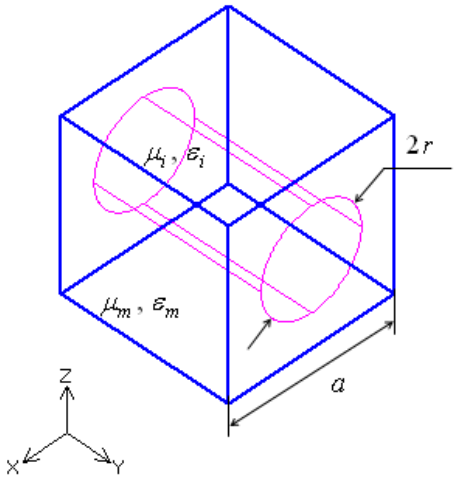

Figure 1. Unit cell of the metamaterial.

Let us assume that the bias magnetic field is applied along the cylinder axes. Then the microwave approximation for effective permeability tensor of the metamaterial is given by [12].

$$\hat{\mu}_{eff} = \mu_0 \begin{bmatrix} \mu & 0 & -ik \\ 0 & \mu_y & 0 \\ ik & 0 & \mu \end{bmatrix}, \tag{1}$$

$$\mu = \frac{1}{3} + \frac{2}{3}\left[\sqrt{\mu_+\mu_-}\left(1 - \langle\alpha_3\rangle^2\right) + \tilde{\mu}\langle\alpha_3\rangle^2\right], \tag{2}$$

$$\mu_+ = 1 + F\frac{\omega_\Sigma}{\omega_0 + i\omega F\alpha_{in} - \omega}, \tag{3}$$

$$\mu_- = 1 + F\frac{\omega_\Sigma}{\omega_0 + i\omega F\alpha_{in} + \omega}, \tag{4}$$

$$\tilde{\mu} = 1 + F\frac{\omega_\Sigma\left(\omega_0 + i\omega F\alpha_{in}\right)}{\left(\omega_0 + i\omega F\alpha_{in}\right)^2 - \omega^2}, \tag{5}$$

$$\mu_y = 1 + F\frac{\omega_\Sigma\left(\omega_0 + i\omega F\alpha_{in}\right)}{\left(\omega_0 + i\omega F\alpha_{in}\right)^2 - \omega^2}\left(1 - \langle\alpha_3\rangle^2\right), \tag{6}$$

$$k = \langle\alpha_3\rangle F\frac{\omega_\Sigma}{\omega_0 + i\omega F\alpha_{in} - \omega}, \tag{7}$$

$$\langle\alpha_3\rangle = \begin{cases} 1, & M = M_s, \\ \dfrac{M}{M_\Sigma}, & M \neq M_s, \end{cases} \tag{8}$$

where μ_0 is the permeability of vacuum, γ the gyromagnetic ratio, $F = \pi r^2/a^2$ the metal volume fraction, ω the angular frequency of electromagnetic wave, α_{in} the damping factor of inclusion material, M the magnetization, M_s the magnetization saturation of inclusion material, $\omega_0 = \mu_0\gamma H_0$ the Larmor or precession frequency, H_0 the dc magnetic field strength on the surface of inclusions, $\omega_\Sigma = \gamma\mu_0 M_\Sigma$ the intrinsic precession frequency, and M_Σ the effective magnetization of the metamaterial given by [11]

$$M_\Sigma = \frac{\omega_0}{\gamma}\frac{\mu_\xi^2 - 3\mu_\xi - 2 - \left\{1 + 16\mu_\xi^4 + 2\mu_\xi^3 - 3\mu_\xi^2 + (\omega/\omega_0)^2\left(3 - 15\mu_\xi^4 - 8\mu_\xi^3 + 8\mu_\xi^2 + 12\mu_\xi\right)\right\}^{-1/2}}{1 + 3\mu_\xi}, \tag{9}$$

where

$$\mu_\xi = \mu_m\left(1 - \frac{F}{1 + i/\pi\mu_0\mu_m r^2\sigma\omega}\right), \tag{10}$$

where σ is the conductivity of inclusion material.

Equation (7) is, in fact, the high frequency approximation of k. It means that Eq. (7) is valid for $\omega_\Sigma/\omega << 1$ and $\omega_0/\omega << 1$. In order to obtain the expression of k valid for whole of the considered frequency range, it is necessary to take into account that:

$$\lim_{M\to M_s} k = F\frac{\omega\gamma\mu_0 M_\Sigma}{(\omega_0 + i\omega F\alpha_{in})^2 - \omega^2}, \tag{11}$$

where the right part of Eq. (11) is the microwave approximation of k for the case of fully magnetized inclusions obtained in work [12]. Taking into account Eq. (11), it is logically to conclude:

$$k = \langle\alpha_3\rangle F\frac{\omega_\Sigma\omega}{(\omega_0 + i\omega F\alpha_{in})^2 - \omega^2}. \tag{12}$$

It is important to mention that Eq. (12) is irrespective of the shape of inclusions, and Eq. (12) is obtained for the first time.

On the analogy of work [9], in our study we consider air host medium to prevent losses due to the dielectric ($\mu_m = 1 = \epsilon_m$).

Taking into account 2-D symmetry of the considered metamaterial medium, we can write the tensor of effective permeability in the form:

$$\hat{\epsilon}_{eff} = \epsilon_0 \begin{bmatrix} \epsilon & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon \end{bmatrix}, \tag{13}$$

where ϵ is obtained in [11], and ϵ_{yy} is given in [13] for the case of very thin PEC cylinders.

Throughout this study, three magnetization modes are considered: 1) the case of maximal value of relative permeability of the inclusions ($H_0 = 120A/m$, $\mu_i = 5500$, $M = 6.5988\cdot 10^5 A/m$); 2) the case of fully magnetized inclusions ($H_0 = 1.6\cdot 10^5 A/m$, $\mu_i = 500$, $M = 1.7189\cdot 10^6 A/m$); 3) the intermediate case ($H_0 = 600A/m$, $\mu_i = 1880$, $M = 3.661\cdot 10^5 A/m$). It is also assumed that $a = 0.001m$, $r = 0.00034m$, $\alpha_{in} = 0.5$. The above values of a and r are chosen to reach the maximum permissible value of metal volume fraction defined in the Effective Medium Theory of work [11]. The second magnetization mode is also considered to take the appropriate results from work [11].

3. WAVE PROPAGATION IN DIRECTION OF BIAS

Consider a propagation of plane monochromatic electromagnetic wave in the direction of bias magnetic field that is parallel to the axis y. In this way, y-dependence of electromagnetic field components is $e^{-i\beta y}$ ($\partial/\partial x = 0 = \partial/\partial z$):

$$\left.\begin{array}{ll} \overrightarrow{E} = \overrightarrow{E_0}e^{-i\beta y}, & \overrightarrow{E_0} = E_{0x}\overrightarrow{x_0} + E_{0y}\overrightarrow{y_0} + E_{0z}\overrightarrow{z_0}, \\ \overrightarrow{H} = \overrightarrow{H_0}e^{-i\beta y}, & \overrightarrow{H_0} = H_{0x}\overrightarrow{x_0} + H_{0y}\overrightarrow{y_0} + H_{0z}\overrightarrow{z_0}, \end{array}\right\} \tag{14}$$

where β is the propagation constant, and $\overrightarrow{x_0}$, $\overrightarrow{y_0}$ and $\overrightarrow{z_0}$ are the unit vectors of 3-D Cartesian coordinate system.

Putting Eq. (1) and Eqs. (13)–(14) into Faraday's law ($\nabla\times\overrightarrow{E} = -i\omega\hat{\mu}_{eff}\overrightarrow{H}$) finally gives

$$\left.\begin{array}{l} \beta E_{0z} = \omega\left(\mu H_{0x} - ikH_{0z}\right), \\ 0 = -i\omega\mu_y H_{0y}, \\ \beta E_{0x} = -\omega\left(ikH_{0x} + \mu H_{0z}\right). \end{array}\right\} \tag{15}$$

Putting Eq. (1) and Eqs. (13)–(14) into Ampere's law ($\nabla\times\overrightarrow{H} = i\omega\hat{\epsilon}_{eff}\overrightarrow{E}$) finally gives

$$\left.\begin{array}{l} -\beta H_{0z} = \omega\epsilon E_{0x}, \\ 0 = -i\omega\epsilon_{yy}E_{0y}, \\ \beta H_{0x} = \omega\epsilon E_{0z}. \end{array}\right\} \tag{16}$$

Putting first and third equalities of Eqs. (16) into first and third equations of Eqs. (15) finally gives:

$$\left.\begin{aligned}\left(\beta^2-\omega^2\epsilon\mu\right)E_{0z}-ik\omega\epsilon E_{0x}=0,\\ \left(\beta^2-\omega^2\epsilon\mu\right)E_{0x}-ik\omega\epsilon E_{0z}=0.\end{aligned}\right\} \tag{17}$$

The system of Equations (17) with respect to the unknowns E_{0x} and E_{0z} has a solution if its determinant is equal to zero:

$$\omega^4\epsilon^2k^2-\left(\beta^2-\omega^2\epsilon\mu\right)^2=0. \tag{18}$$

Solving Eq. (18) with respect to the unknown β gives

$$\beta_{\pm}=\omega\sqrt{\epsilon\left(\mu\pm k\right)}. \tag{19}$$

Putting $\beta_{\pm}$ into any equality of Eqs. (17) finally gives: $E_{0z}=\mp iE_{0x}$. Last equality corresponds to a circularly polarized plane EM wave where the minus sign stands for right-hand circularly polarized (RHCP) plane wave, and the plus sign stands for left-hand circularly polarized (LHCP) plane wave. Then the expression of effective relative permeability is given by

$$\mu_{eff}=\left(\mu\pm k\right)/\mu_0=\left(1+2\left[\sqrt{\mu_+\mu_-}\left(1-\langle\alpha_3\rangle^2\right)+\tilde{\mu}\langle\alpha_3\rangle^2\right]\pm 3k\right)/3\mu_0, \tag{20}$$

where the plus sign stands for RHCP plane wave while the minus sign stands for LHCP plane wave.

It should be noted that the condition $H_{0y}=0=E_{0y}$ implies that only TEM plane wave can propagate in the considered metamaterial medium in the direction of bias.

The spectrum of $\mathrm{Re}(\mu_{eff})$ is plotted in Fig. 2, Fig. 4, and the spectrum of magnetic loss δ_m is plotted in Fig. 3, Fig. 5 for all the above mentioned magnetization modes.

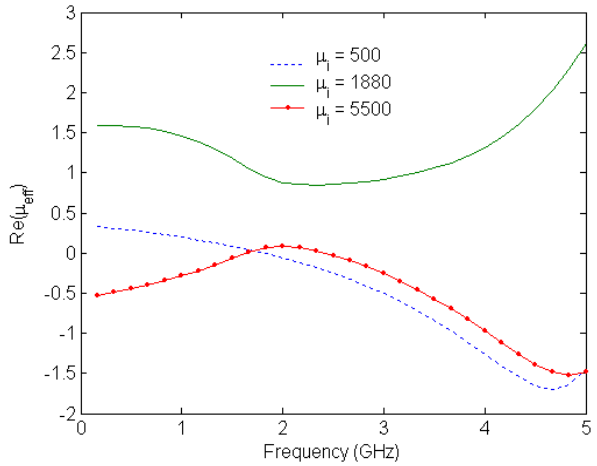

Figure 2. Real part of μ_{eff} of metamaterial versus frequency ω of RHCP wave propagated in direction of bias.

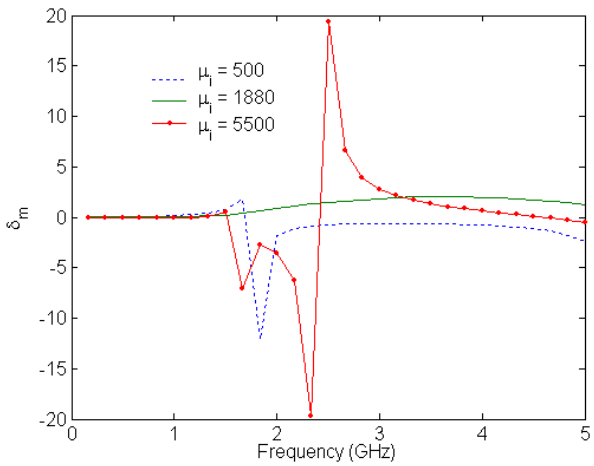

Figure 3. Magnetic loss δ_m of metamaterial versus frequency ω of RHCP wave propagated in direction of bias.

As observed from Fig. 2–Fig. 5, real part of the effective relative permeability of metamaterial with partially magnetized inclusions can span any of the possible value ranges: $\mathrm{Re}(\mu_{eff})<0$ and $0<\mathrm{Re}(\mu_{eff})<1$ or $\mathrm{Re}(\mu_{eff})>1$, and its imaginary part can have values $\sim 10^{-1}\div 10^{1}$ for the case of wave propagation in the direction of bias. Moreover, the value of effective relative permeability and its value range are tuned by the value of bias magnetic field.

4. WAVE PROPAGATION TRANSVERSE TO BIAS

Consider a propagation of plane monochromatic electromagnetic wave transverse to bias magnetic field that is parallel to the axis x (x-dependence of electromagnetic field components is $e^{-i\beta x}$:

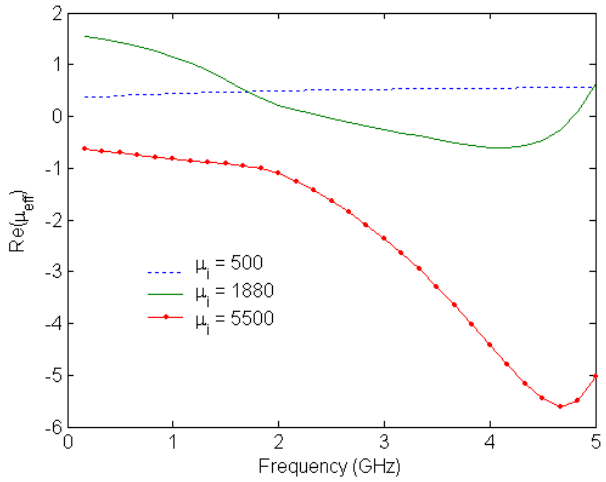

Figure 4. Real part of μ_{eff} of metamaterial versus frequency ω of LHCP wave propagated in direction of bias.

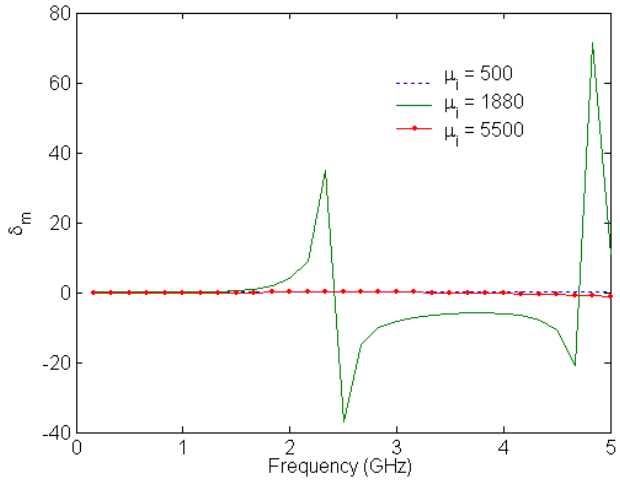

Figure 5. Magnetic loss δ_m of metamaterial versus frequency ω of LHCP wave propagated in direction of bias.

$\partial/\partial y = 0 = \partial/\partial z$) or parallel to the axis z (z-dependence of electromagnetic field components is $e^{-i\beta z}$: $\partial/\partial x = 0 = \partial/\partial y$).

Consider initially the wave propagation along the axis x. Electromagnetic wave in the metamaterial is defined by:

$$\left.\begin{array}{ll} \overrightarrow{E} = \overrightarrow{E_0} e^{-i\beta x}, & \overrightarrow{E_0} = E_{0x}\overrightarrow{x_0} + E_{0y}\overrightarrow{y_0} + E_{0z}\overrightarrow{z_0}, \\ \overrightarrow{H} = \overrightarrow{H_0} e^{-i\beta x}, & \overrightarrow{H_0} = H_{0x}\overrightarrow{x_0} + H_{0y}\overrightarrow{y_0} + H_{0z}\overrightarrow{z_0}. \end{array}\right\} \tag{21}$$

Putting Eq. (1), Eq. (13) and Eqs. (21) into Faraday's law finally gives

$$\left.\begin{array}{l} 0 = -i\omega\left(\mu H_{0x} - ikH_{0z}\right), \\ i\beta E_{0z} = -i\omega\mu_y H_{0y}, \\ -i\beta E_{0y} = -i\omega\left(ikH_{0x} + \mu H_{0z}\right). \end{array}\right\} \tag{22}$$

Putting Eq. (1), Eq. (13) and Eqs. (21) into Ampere's law finally gives

$$\left.\begin{array}{l} 0 = i\omega\epsilon E_{0x}, \\ i\beta H_{0z} = i\omega\epsilon_{yy} E_{0y}, \\ -i\beta H_{0y} = i\omega\epsilon E_{0z}. \end{array}\right\} \tag{23}$$

Putting first equality of Eqs. (22), second and third equalities of Eqs. (23) into second and third equalities of Eqs. (22) finally gives:

$$\left.\begin{array}{l} \left(\beta^2 - \omega^2\epsilon\mu_y\right) E_{0z} = 0, \\ \left(\beta^2 - \omega^2\epsilon_{yy}\dfrac{\mu^2 - k^2}{\mu}\right) E_{0y} = 0. \end{array}\right\} \tag{24}$$

We impose $E_{0y} = 0$ in Eqs. (24) so long as the wave propagation in the metamaterial is almost absent for $F > 0.2$ (the inclusions act as a shield for large values of the metal volume fraction). That is why the solution of Eqs. (24) is given by:

$$\beta = \omega\sqrt{\epsilon\mu_y}. \tag{25}$$

Then the expression of effective relative permeability is given by

$$\mu_{eff} = \mu_y/\mu_0. \tag{26}$$

Consider now the wave propagation along the axis z. Electromagnetic wave in the metamaterial is given by:

$$\left.\begin{array}{ll} \overrightarrow{E} = \overrightarrow{E_0}e^{-i\beta z}, & \overrightarrow{E_0} = E_{0x}\overrightarrow{x_0} + E_{0y}\overrightarrow{y_0} + E_{0z}\overrightarrow{z_0}, \\ \overrightarrow{H} = \overrightarrow{H_0}e^{-i\beta z}, & \overrightarrow{H_0} = H_{0x}\overrightarrow{x_0} + H_{0y}\overrightarrow{y_0} + H_{0z}\overrightarrow{z_0}. \end{array}\right\} \tag{27}$$

Putting Eq. (1), Eq. (13) and Eqs. (27) into Faraday's law finally gives

$$\left.\begin{array}{l} i\beta E_{0y} = -i\omega\left(\mu H_{0x} - ikH_{0z}\right), \\ -i\beta E_{0x} = -i\omega\mu_y H_{0y}, \\ 0 = -\omega\left(ikH_{0x} + \mu H_{0z}\right). \end{array}\right\} \tag{28}$$

Putting Eq. (1), Eq. (13) and Eqs. (27) into Ampere's law finally gives

$$\left.\begin{array}{l} i\beta H_{0y} = i\omega\epsilon E_{0x}, \\ -i\beta H_{0x} = i\omega\epsilon_{yy} E_{0y}, \\ 0 = i\omega\epsilon E_{0z}. \end{array}\right\} \tag{29}$$

Constituting third equality of Eqs. (28), first and second equalities of Eqs. (29) into first and second equalities of Eqs. (28) finally gives:

$$\left.\begin{array}{l} \left(\beta^2 - \omega^2\epsilon\mu_y\right) E_{0x} = 0, \\ \left(\beta^2 - \omega^2\epsilon_{yy}\dfrac{\mu^2 - k^2}{\mu}\right) E_{0y} = 0. \end{array}\right\} \tag{30}$$

We impose $E_{0y} = 0$ in Eqs. (30) so long as the wave propagation in the metamaterial is almost absent for $F > 0.2$ (the inclusions act as a shield for large values of the metal volume fraction). That is why the solution of Eqs. (30) is given by Eq. (25). Then the expression of effective relative permeability is given by Eq. (26).

It should be noted that the conditions $E_{0x} = 0 = E_{0y}$ for the wave propagation along the axis x and $E_{0y} = 0 = E_{0z}$ for the wave propagation along the axis z imply that only TE plane wave can propagate in the considered metamaterial in the direction transverse to bias.

The spectrum of $\mathrm{Re}(\mu_{eff})$ is plotted in Fig. 6, and the spectrum of magnetic loss δ_m is plotted in Fig. 7 for all the above mentioned magnetization modes.

As observed from Fig. 6-Fig. 7, real part of the effective relative permeability of the metamaterial with partially magnetized inclusions has only $\mathrm{Re}(\mu_{eff}) > 1$, and its imaginary part can have values

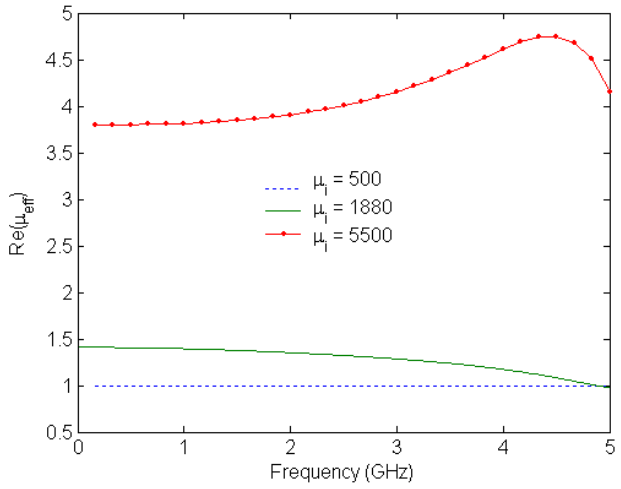

Figure 6. Real part of μ_{eff} of metamaterial versus frequency ω of wave propagated transverse to bias.

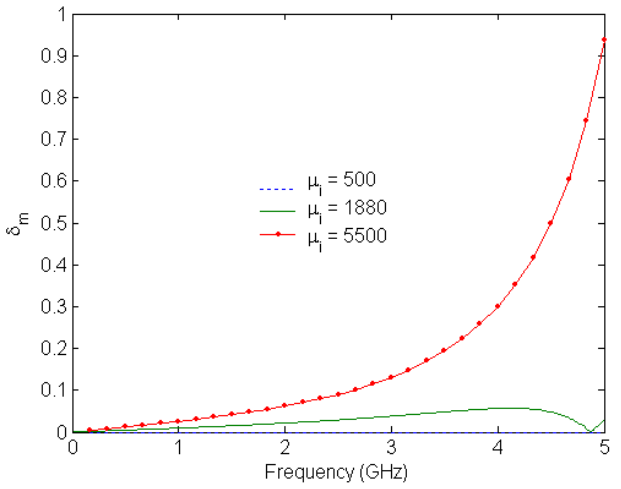

Figure 7. Magnetic loss δ_m of metamaterial versus frequency ω of wave propagated transverse to bias.

$\sim 10^{-2} \div 10^{-1}$ for the case of wave propagation transverse to bias. Moreover, the value of effective relative permeability and its value range are tuned by the value of bias magnetic field.

Last results are in a good agreement with those of work [9]. The difference between the permeability values obtained here and those reported in [9] does not exceed 5%.

As seen from Figs. 2–7, the case of wave propagation in the direction of bias can "provide" us with any of possible value range of the effective relative permeability of the considered metamaterial depending on the value of bias field. At the same time, the metamaterial behaves as a conventional ferrite in the case of wave propagation transverse to bias.

5. APPLICATION HINTS

Let us define a rough range of possible applications for the considered metamaterial. In order to do that, consider a normal incidence of plane electromagnetic wave on an infinitely long metamaterial layer of thickness d. The full reflection R and transmission T coefficients are defined by [14]

$$\left.\begin{aligned} R &= \rho\frac{1-e^{-2ikd}}{1-\rho^2 e^{-2ikd}}, \\ T &= \frac{1-\rho^2}{1-\rho^2 e^{-2ikd}}e^{-ikd} \end{aligned}\right\} \tag{31}$$

where $\rho = \left(\sqrt{\mu_{eff}/\epsilon_{eff}} - 1\right) / \left(\sqrt{\mu_{eff}/\epsilon_{eff}} + 1\right)$ is the effective Fresnel coefficient, and $k = k_0 n_{eff} = \sqrt{\epsilon_{eff}\mu_{eff}}\omega/c$ is the effective wavenumber, c is velocity of speed in vacuum.

As seen from Eqs. (31), if $\mathrm{Re}(\mu_{eff}) < 0$, then the coefficients R and T are complex. It means that electromagnetic wave in the microwave frequency range is absorbed by the layer. That is why such a layer can be used for designing a microwave filter or microwave absorber. At the same time, as mentioned in Introductory Section, such a layer can also be used for designing the transfer element of microwave power transfer systems, [8].

As seen from Eqs. (31): $\lim_{\mu_{eff}\to+0} R = -1$ and $\lim_{\mu_{eff}\to+0} T = 0$. It means that if $0 < \mathrm{Re}(\mu_{eff}) < 1$, then electromagnetic wave in the microwave frequency range is entirely reflected by the layer changing its phase for π. It means, in turn, that the metamaterial layer can be used for designing a microwave phase inverter. At the same time, such a layer can also be used for designing the transfer element of microwave power transfer systems, [15].

As seen from Eqs. (31): $\lim_{\mu_{eff}\to+\infty} R = 1$ and $\lim_{\mu_{eff}\to+\infty} T = 0$. It means that if $\mathrm{Re}(\mu_{eff}) > 1$, then electromagnetic wave in the microwave frequency range is entirely reflected by the layer. It means, in turn, that the metamaterial layer can be used for designing a microwave transponder. At the same time, magnetically enhanced metamaterials ($\mathrm{Re}(\mu_{eff}) > 1$) can also be used for fabricating the substrates of miniaturized microwave patch antennas [16]. Indeed, antennas printed on magnetized ferrite substrates can offer beam steering even with a single element as well as the ability of electronic tuning, [17].

6. CONCLUSION

The real part of effective relative permittivity μ_{eff} of an infinite isotropic dielectric host medium with periodically embedded partially magnetized ferric cylindrical inclusions can have $\mathrm{Re}(\mu_{eff}) < 0$ or $0 < \mathrm{Re}(\mu_{eff}) < 1$ or $\mathrm{Re}(\mu_{eff}) > 1$, and its imaginary part can have values $\sim 10^{-2} \div 10^{1}$ depending on the value of dc bias magnetic field. The ability to control the value of complex effective permeability by varying the value of bias magnetic field enables to use the metamaterial for designing the transfer element of microwave power transfer systems, microwave phase inverters, microwave absorbers, microwave filters, microwave transponders, substrates of miniaturized patch antennas.

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