

Investigating Electron Beam Deflections by a Long Straight Wire Carrying a Constant Current Using Direct Action, Emission-Based and Field Theory Approaches of Electrodynamics

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Abstract—Results are presented for the transverse deflection of an electron beam by a long, straight wire carrying direct current. The experimental deflections are compared with three calculation methods based on the Lorentz force law (field theory) and both the Weber (direct action) and Ritz (emission) force formulae. The Lorentz force calculation is the conventional approach expressed in terms of electric and magnetic field components. By contrast the force formulae of Weber and Ritz do not contain any field vectors relating to \mathbf{E} or \mathbf{B} . The Weber force is based on direct action whereas the Ritz force expression is based on an emission/ballistic principle and is formulated in terms of a dimensionless constant, λ . The experimental beam deflections are for low speed (non-relativistic) electrons. Good agreement between experiment and theory is demonstrated for each approach. In fact, for the case of an infinitely long wire, all three calculation methods give identical results. Finally, the three approaches are contrasted when applied to the case of high speed electrons.

1. INTRODUCTION

Previous work [1] has demonstrated agreement between Lorentzian and Weberian approaches for the case of electron beam deflections when directed orthogonally across a direct current carrying solenoid. A more fundamental case to be investigated is that of a long straight wire carrying direct current. In terms of field theory, calculation of beam deflections is in terms of the well-known Lorentz force, $\mathbf{F} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ where \mathbf{B} is the magnetic flux density across the beam, \mathbf{v} is beam velocity and \mathbf{E} is the static electric field. In general, the Lorentz force accurately predicts the force exerted on a charged particle and is considered as foundational to classical electrodynamics. However, it has been noted by O’Rahilly [2, pp. 561] that, “*the two particular cases here combined are quite incompatible. In one case we have charges at rest, in the other charges are moving; they cannot both be stationary and moving*”. By contrast, Weber and Ritz formulations deal only with relative velocities and accelerations between charges. Both formulations reduce to Coulomb’s static force law for zero velocity.

It is of interest to note that Weber-Ritz formulations of electrodynamics have their origin in the work of Gauss [2, pp. 524–525]. As early as 1835, Gauss had commented: “*Two elements of electricity in relative motion repel or attract one another differently when in motion and when in relative rest* [3].” He based this view on non-instantaneous ballistic transmission. However it was Wilhelm Weber who first developed Gauss’s idea by correlating both Faraday’s induction and Ampere’s electrodynamics by means of the following two principles: (1) every action of a current element can be regarded as compounded of the actions of a positive and equal negative electrical particle which simultaneously traverse the same space element in opposite directions; (2) by this means the mutual action of two current elements can be represented, with the assumption that like charges attract if they are moving

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in the same sense and unlike when moving in opposite directions. Weber proposed his second order generalisation of Coulomb's law in an electrodynamic inter-particle force formula, between point charges e, e' distance, r , apart, in the form,

$$F = \frac{ee'}{4\pi\epsilon_0 r^2} \left[1 + \frac{r\ddot{\mathbf{r}}}{c^2} - \frac{\dot{\mathbf{r}}^2}{2c^2} \right] \quad (1)$$

Weber's electrodynamics has received extensive coverage in the works of both Assis [4] and Wesley [5–7]. Here the idea of direct action is contrasted with the Maxwellian approach of contact action. One of the main properties of the Weber force is that it obeys Newton's third law (action and reaction) for any state of motion of the charges. Moreover it implies conservation of linear and angular momentum.

Following Gauss, Ritz developed his emission theory of electrodynamics. An excellent review of this work has been given in [8]. Although Ritz's theory is generally not well known, it does receive extensive coverage in the comprehensive work of O'Rahilly [2, pp. 549–622]. It seems, however, that any objection to an emission theory of action tends to be based on optical considerations. In particular, that light emitted from a source with velocity, \mathbf{u} , combines with the velocity of light, \mathbf{c} , by the typical vector addition $(\mathbf{c} + \mathbf{u})$ or $(\mathbf{c} - \mathbf{u})$. Implicit in such an assumption, is that light behaves as 'crude bullet like particles' whereas Ritz explains the emission in terms of fictitious, infinitely small, virtual 'particles', all animated with the same radial velocity relative to the origin, where "*the particles are simply a concrete representation of kinematic and geometric data*" [9]. In any case, Ritz's approach stands as a theory of electrodynamics irrespective of any objections to its applications in optics.

2. THEORY: CALCULATION OF TRANSVERSE FORCE ON AN ELECTRON BEAM BY A LONG STRAIGHT WIRE CARRYING DIRECT CURRENT

2.1. Field Theory Based on Lorentz Force

Figure 1 shows the current carrying wire lying along the x axis while the e beam travels parallel to the z axis at speed v_e and distance b above the axis. The beam is intercepted by the circular lines of force, concentric with the axis of the wire. For an infinitely long wire, the deflecting magnetic field at a distance z along the beam is given by, $B_\varphi \sin \varphi$ where B_φ is the tangential field at radius ρ and $B_\varphi = \frac{\mu_0 I}{2\pi\rho} = \frac{I}{2\pi\epsilon_0 c^2 \rho}$.

Since $\sin \varphi = \frac{z}{\rho}$, the deflecting force at right angles to the beam is, $F_x = ev_e B_\varphi \sin \varphi = \frac{ev_e I}{2\pi\epsilon_0 c^2} \frac{z}{(b^2+z^2)}$.

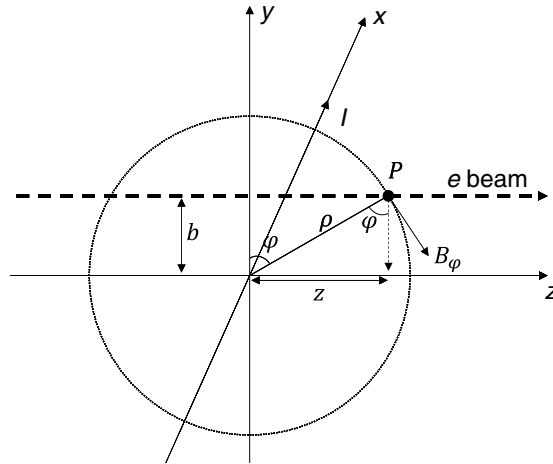


Figure 1. e beam travelling parallel to z axis and crossed by magnetic field lines concentric with the current carrying wire. The tangential magnetic field is shown at a point P along the e beam.

Writing $k^2 = b^2 + z^2$, the force in the x direction is given as,

$$F_x = \frac{ev_e I z}{2\pi\epsilon_0 c^2 k^2} \quad (2)$$

2.2. Direct Action Based on Weber Force

In the Weber calculation the aim is to calculate the force, δF_r between an element of moving charge e' , located between x and $x + \delta x$ at point Q on the wire and a point P on the e beam where $PQ = r$ (Fig. 2). The e beam, with velocity v_e , is directed parallel to the z axis. P has coordinates $(0, b, z)$ and $Q(x, 0, 0)$.

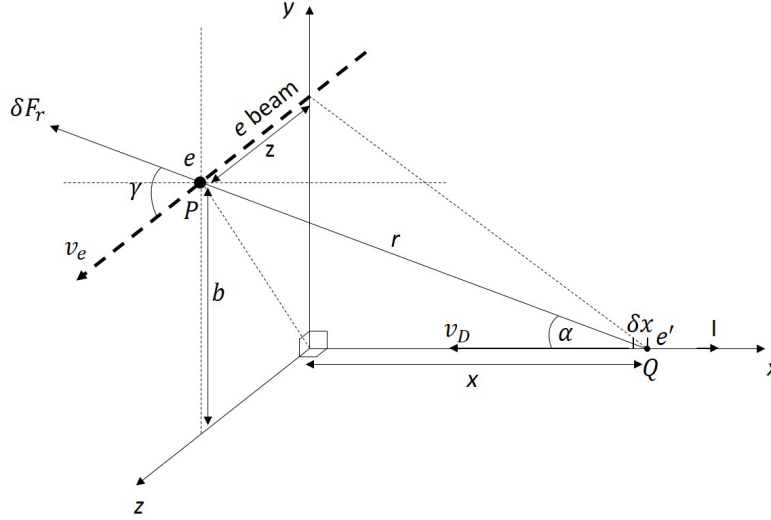


Figure 2. Geometry of Weber force between a current element on the wire and a point on the electron beam.

The Weber force expression is now expressed in terms of relative velocities between charges. If the charges have velocities, v, v' , where u_r is the relative velocity along, r and u is the actual relative velocity then, $\dot{r} = u_r = \frac{dr}{dt} = v_r - v'_r$ and $u^2 = \sum(v_x - v'_x)^2$. Also, $u_r = \sum(v_x - v'_x)(\frac{x-x'}{r})$ and $\ddot{r} = \frac{d^2r}{dt^2} = \frac{d}{dt} \sum(v_x - v'_x)(\frac{x-x'}{r}) = (\frac{u^2 - u_r^2}{r})(f_r - f'_r)$.

Substitution in Weber's force expression (1), the elemental force, δF_r along r between e and e' , ignoring acceleration terms, is given as,

$$\delta F_r (W) = \frac{ee'}{4\pi\epsilon_0 r^2} \left(1 + \frac{u^2}{c^2} - \frac{3u_r^2}{2c^2} \right) \quad (3)$$

where u is the actual relative velocity between e and e' , u_r the relative velocity along r , and v_d the electron drift velocity. Then, since $v_d \ll v_e$, $u^2 = v_d^2 + v_e^2 \cong v_e^2$, $u_r = v_e \cos \gamma + v_d \cos \alpha$ where $\cos \alpha = \frac{x}{r}$ and $\cos \gamma = \frac{z}{r}$, it follows, $u_r^2 = \frac{v_e^2 z^2}{r^2} + \frac{2v_e v_d x z}{r^2}$ and therefore Eq. (3) becomes,

$$\delta F_r (W) = \frac{ee'}{4\pi\epsilon_0 r^2} \left(1 + \frac{v_e^2}{c^2} - \frac{3}{2c^2} \left(\frac{v_e^2 z^2}{r^2} + \frac{2v_e v_d x z}{r^2} \right) \right) = \frac{ee'}{4\pi\epsilon_0 r^2} \left[1 + \frac{v_e^2}{c^2} \left(1 - \frac{3z^2}{2r^2} \right) - \frac{3v_e v_d x z}{c^2 r^2} \right]$$

Thus elemental force in the x direction = $\delta F_x = \delta F_r \cos \alpha$ where,

$$\delta F_x = \frac{ee'}{4\pi\epsilon_0 r^2} \frac{x}{r} \left[1 + \frac{v_e^2}{c^2} \left(1 - \frac{3z^2}{2r^2} \right) - \frac{3v_e v_d x z}{c^2 r^2} \right]$$

Now the positive ions (at rest) of the neutral current will exert an opposite force on e given by,

$$\delta F_x = \frac{ee'}{4\pi\epsilon_0 r^2} \frac{x}{r} \left[1 + \frac{v_e^2}{c^2} \left(1 - \frac{3z^2}{2r^2} \right) \right]$$

So the net force on e along the x direction is,

$$\delta F_x = \frac{e, e'}{4\pi\epsilon_0 r^2} \left(\frac{3v_e v_d x^2 z}{c^2 r^3} \right)$$

Given $e' = nAe\delta x$ and $I = nAv_d e$, where n = free electron density and A = wire cross-sectional area, we obtain,

$$\delta F_x = \frac{ev_e I z}{4\pi\epsilon_0 c^2} \left(\frac{3x^2}{r^5} \right) \delta x$$

at a given position, z , along the beam. So the deflecting force due to the complete wire length, $2L$, and carrying a steady current, I , at a distance z along the beam is,

$$F_x(W) = \frac{ev_e I z}{2\pi\epsilon_0 c^2} \int_0^L 3 \frac{x^2}{r^5} dx$$

Here, $r^2 = x^2 + b^2 + z^2$ and $k^2 = b^2 + z^2$. Substituting $x = k \tan \theta$, the integral is evaluated as $\frac{1}{k^2} \sin^3(\arctan(\frac{L}{k}))$ which for an infinite straight wire (i.e., as $L \rightarrow \infty$) becomes, $\frac{1}{k^2}$. Finally,

$$F_x(W) = \frac{ev I z}{2\pi\epsilon_0 c^2} = \frac{ev_e I z}{2\pi\epsilon_0 c^2} \frac{1}{k^2} \quad (4)$$

which is identical to the force expression of Eq. (2).

2.3. Emission Theory Based on Ritz Force

Following O'Rahilly's development of Ritz's electrodynamics [2], the x component of force between point particles e, e' distance r apart and moving without acceleration is given as:

$$\delta F_x(R) = \frac{ee'}{4\pi\epsilon_0 r^2} \left[\left(1 + \frac{(3-\lambda)u^2}{4c^2} - \frac{3(1-\lambda)u_r^2}{4c^2} \right) \cos \alpha - \frac{(1+\lambda)u_x u_r}{2c^2} \right] \quad (5)$$

Here λ is a constant to be determined by experiment. Clearly when $\lambda = -1$, the Ritz formula becomes identical to the Weber force of Eq. (3). However, in general, the experimental data suggests a value of $\lambda = 3$ in which case the second term of Eq. (5) containing the product of $u_x u_r$ becomes significant. With the value $\lambda = 3$, the Ritz force becomes,

$$\delta F_x(R) = \frac{ee'}{4\pi\epsilon_0 r^2} \left[\left(1 + \frac{3u_r^2}{2c^2} \right) \cos \alpha - 2 \frac{u_x u_r}{c^2} \right]$$

Proceeding as in *Section 2.2* with $u_x = v_d$, $u_r = v_e \cos \gamma + v_d \cos \alpha$, $\cos \alpha = \frac{x}{r}$, $\cos \gamma = \frac{z}{r}$, $u_x u_r = v_e v_d \frac{z}{r}$.

Finally, doubling up for both sides of the wire, we obtain the Ritz force as,

$$F_x(R) = \frac{ev_e I z}{2\pi\epsilon_0 c^2} \int_0^L \left(\frac{2}{r^3} - \frac{3x^2}{r^5} \right) dx$$

In comparison with the Weber force, the Ritz formulation contains the additional integral, $I_2 = \int_0^L \frac{2}{r^3} dx$

and this is evaluated as, $\frac{2}{k^2} (\sin(\arctan(\frac{L}{k}))) = \frac{2}{k^2}$ when $L \rightarrow \infty$. Since the second term of the integral has the value $-\frac{1}{k^2}$ it follows that $I_1 + I_2 = \frac{2}{k^2} - \frac{1}{k^2} = \frac{1}{k^2}$ which is the same result as given by the Weber formulation (i.e., when $\lambda = -1$) in Eq. (4) and the Lorentz force in Eq. (2).

3. EXPERIMENTAL INVESTIGATION

The experimental setup (Fig. 3) shows the general arrangement of the Teltron tube and long straight wire AB carrying direct current. An important feature is the graticule which is angled to the beam and provides visualisation of the beam over 10 cm horizontal travel. In any experiment, it was found necessary to cancel any residual deflection of the beam (due to Earth’s magnetic field) by applying a vertical electric field across the beam; in practice the magnitude of this was of the order of 0.5 kVm^{-1} . The e gun voltage (V) was set at 2.00 kV providing a beam velocity of approx. $2.7 \times 10^7 \text{ m/s}$. A range of direct current magnitudes is passed through the wire ($5.00, 10.00, \dots, 30.00 \text{ A}$) and held constant for several seconds during measurement. The copper wire (AB) has a diameter of 1 mm and length of 1 metre . Several minutes is allowed to pass between each measurement to minimise error due to residual thermal heating. The corresponding results and comparison of experiment and theory are now presented.

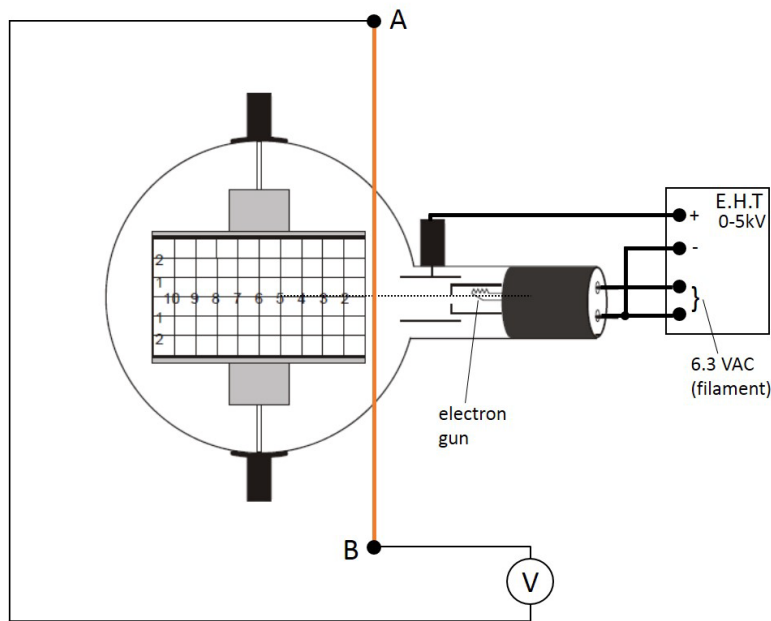


Figure 3. Experimental arrangement for e beam deflection by long straight direct current carrying wire (AB).

4. RESULTS AND COMPARISON OF EXPERIMENT AND THEORY

The standard parameters used in all calculations include: gun voltage = 2 kV yielding a beam speed $= \sqrt{\frac{2ev}{m_e}} = 2.65 \times 10^7 \text{ m/s}$, $I = 10.00 \text{ A}$, electron charge, $e = 1.6 \times 10^{-19} \text{ C}$, $b = 0.02 \text{ m}$. Assuming an infinitely long wire all three methods of calculation give the x force component at a distance z along the beam as, $F_x = \frac{ev_e I}{2\pi\epsilon_0 c^2} \frac{z}{b^2+z^2}$.

Starting with Eq. (2), electron acceleration in the x direction, is $= \frac{d^2x}{dt^2} = \frac{F_x}{m_e} = \frac{ev_e I}{2\pi m_e \epsilon_0 c^2} \frac{z}{b^2+z^2}$.

For the standard data, $K = \frac{ev_e I}{2\pi m_e \epsilon_0 c^2} = 9.30 \times 10^{12} (S.I.)$.

So velocity in the x direction at time, t , is $v_x(t)$ and setting $z = v_e t$ gives,

$$v_x(t) = \frac{dx}{dt} = K \int_0^{t_f} \frac{v_e t}{v_e^2 t^2 + b^2} dt$$

$$v_x(t) = \frac{K}{2v_e} [\ln(v_e^2 t^2 + b^2) - \ln b^2]_0^{t_f}$$

where the beam time of flight is given by $t_f = \frac{0.1}{2.65 \times 10^7} = 3.77 \times 10^{-9}$ (s). The definite integral for the maximum deflection in the x direction can then be calculated (by parts):

$$I_1 = \int_0^{3.77 \times 10^{-9}} \ln(v_e^2 t^2 + b^2) - \ln b^2 dt = 1.72 \times 10^{-8} - 2 \left[t - \frac{t}{v_e} \arctan\left(\frac{tv_e}{b}\right) \right]_0^{3.77 \times 10^{-9}}$$

And with $b = 0.02$ m, $v_e = 2.65 \times 10^7$ m/s, then $I_1 = -2.27 \times 10^{-8}$. Also, $I_2 = -[t \ln b^2]_0^{3.77 \times 10^{-9}} = 2.95 \times 10^{-8}$. So, $(I_1 + I_2) = 6.8 \times 10^{-9}$ and finally, $x(\max) = \frac{9.30 \times 10^{12}}{2 \times 2.65 \times 10^7} \times 6.8 \times 10^{-9}$ [m] = 1.2 mm.

This deflection is for $I = 10.00$ A and since $x \propto I$ this can be readily contrasted with experimental data (Fig. 4).

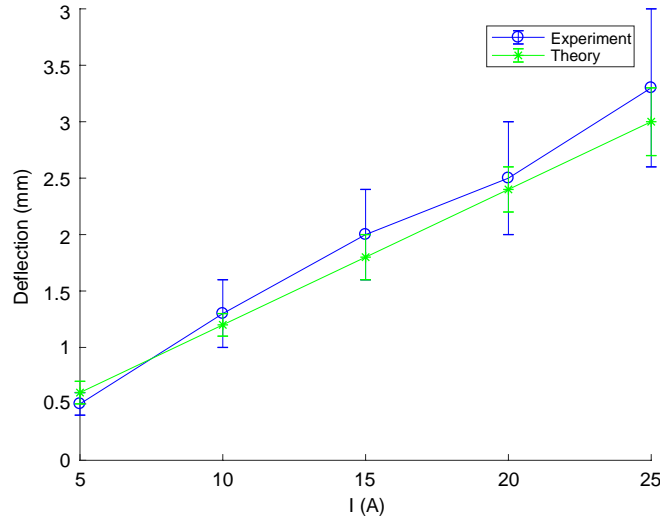


Figure 4. Experiment versus theory for e beam deflections by a long straight wire carrying direct current.

5. HIGH SPEED ELECTRONS, KAUFMANN-BUCHERER EXPERIMENTS AND RITZ'S THEORY

Regarding high speed (relativistic) electrons, the question arises as to whether it is the Ritz formulation ($\lambda = 3$) or Weber ($\lambda = -1$) which agrees with experiment. In terms of established field theory, the altered trajectory of high speed electrons require a correction factor, $\gamma = (1 - \frac{v^2}{c^2})^{-1/2}$ in order to agree with experiment. This factor has received significant experimental verification in particle accelerator experiments. We investigate whether the direct-action theories of Weber-Ritz can predict an electron trajectory at high speed without invoking a correction factor directly. Therefore we follow O'Rahilly's adaptation of the Ritz formula for the case of electrons moving between charged metal plates [2, pp. 621].

The general expression for the force between two point charges e , e' distance r apart in terms of the dimensionless constant, λ , and ignoring acceleration terms is,

$$\delta F_r(R) = \frac{ee'}{4\pi\epsilon_0 r^2} \left(\left[1 + \frac{(3-\lambda)u^2}{4c^2} - \frac{3(1-\lambda)u_r^2}{4c^2} \right] \hat{r} - \frac{(1+\lambda)uu_r}{2c^2} \right) \quad (6)$$

where u is the actual relative velocity between e and e' , u_r the relative velocity along r , and \hat{r} the unit vector along r . The formula is first applied to the case of a point charge, e , moving with velocity, v , in the x direction between two infinite plates ($y = \pm h$) parallel to xz and charged to density $\pm\sigma$ (Fig. 5).

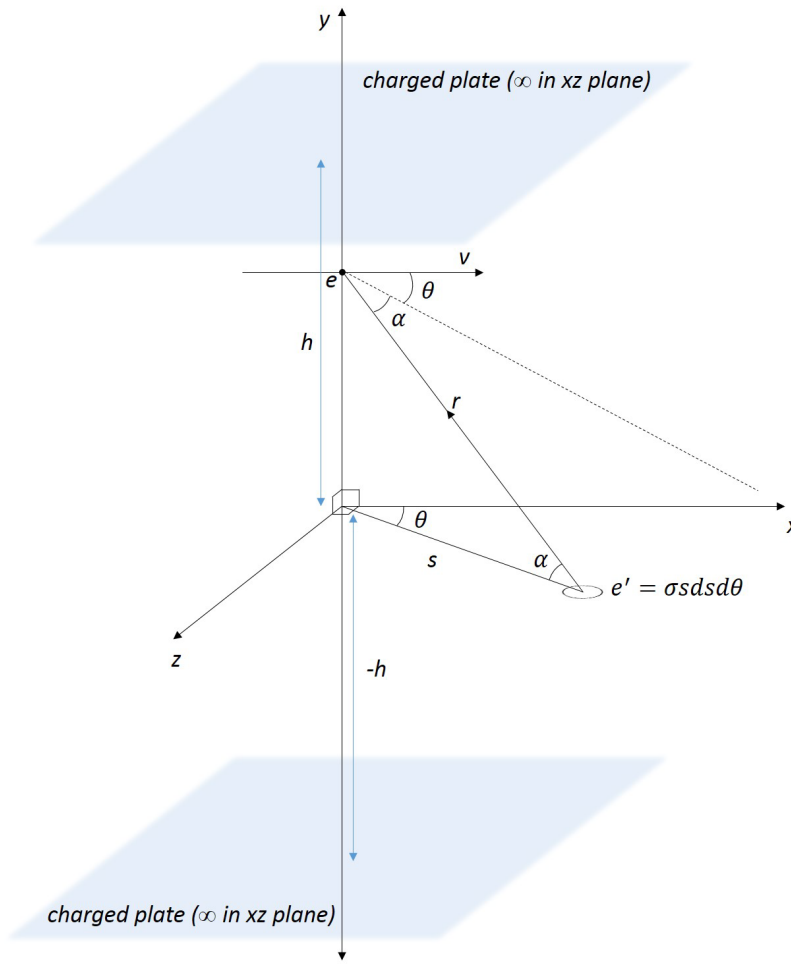


Figure 5. Point charge, e , moving with velocity, v , between infinite charged metal plates, parallel to the xz plane. Adapted from [2, pp. 813].

Given that, $u = v_x = v$, $v_y = 0$ and $u_r = -v \cos \theta \cos \alpha$, then considering the y force component, Eq. (6) becomes,

$$\delta F_y(R) = \frac{ee'}{4\pi\epsilon_0 r^2} \sin \alpha \left[1 + \frac{(3-\lambda)v^2}{4c^2} - \frac{3(1-\lambda)v^2}{4c^2} \cos^2 \theta \cos^2 \alpha \right]$$

where $\cos \alpha = \frac{s}{(h^2+s^2)^{1/2}}$, $r^2 = h^2 + s^2$, $\sin \alpha = \frac{h}{(h^2+s^2)^{1/2}}$.

It follows that the force in the y direction is given as,

$$F_y(R) = \frac{e\sigma}{2\pi\epsilon_0} \left[\int_0^{2\pi} d\theta \int_0^\infty ds s \frac{h}{(h^2+s^2)^{3/2}} \left[1 + \frac{(3-\lambda)v^2}{4c^2} - \frac{3(1-\lambda)v^2}{4c^2} \cos^2 \theta \frac{s^2}{(h^2+s^2)} \right] \right]$$

The s integrals are,

$$[I_1] = \int_0^\infty \frac{hs}{(h^2+s^2)^{3/2}} ds = 1 \quad \text{and} \quad [I_2] = \int_0^\infty \frac{hs^3}{(h^2+s^2)^{5/2}} ds = \frac{2}{3}$$

Then,

$$F_y(R) = \frac{e\sigma}{2\pi\epsilon_0} \left[\int_0^{2\pi} \left(1 + \frac{(3-\lambda)v^2}{4c^2} \right) d\theta - \int_0^{2\pi} \left(\frac{(1-\lambda)v^2}{2c^2} \cos^2 \theta \right) d\theta \right]$$

and,

$$F_y(R) = \frac{e\sigma}{\epsilon_0} \left[1 + \left(\frac{(3-\lambda)}{4} - \frac{(1-\lambda)}{4} \right) \frac{v^2}{c^2} \right]$$

Finally,

$$F_y(R) = \frac{e\sigma}{\epsilon_0} \left[1 + \frac{v^2}{2c^2} \right] \quad (7)$$

This may be re-written as, $F_y(R) = Ee\gamma$, where $\gamma = (1 + \frac{v^2}{2c^2})$, corresponds to the Lorentz factor $(1 - \frac{v^2}{c^2})^{-1/2}$ to the second order and electric intensity, E , is $= \frac{\sigma}{\epsilon_0}$. Note that $F_y(R)$ does not depend on the value of λ for this particular case; so it is clear the Weber formula ($\lambda = -1$), would give the same $F_y(R)$. Eq. (7) can now be applied to the crossed field experiments originating with Kaufmann-Bucherer [10]. A typical Kaufmann-Bucherer experiment is illustrated in Fig. 6.

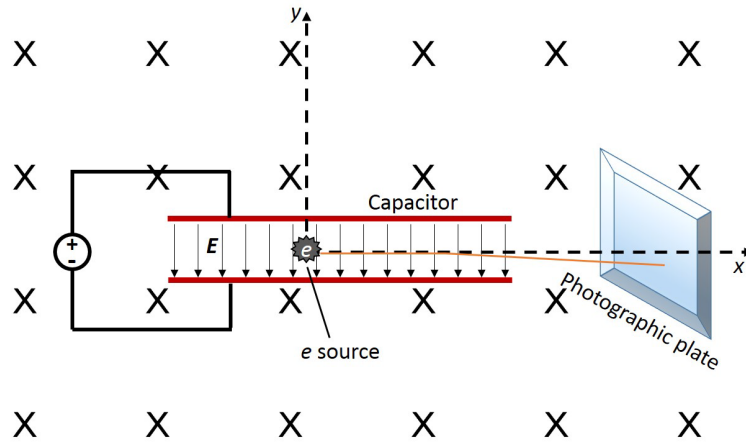


Figure 6. Illustration of the Kaufmann-Bucherer type cross-field experiment. The magnetic field is directed into the paper.

When the electrical and magnetic forces balance, the electron will move in a straight line to pass through the narrow gap between the capacitor plates and since the magnetic field is not changed by the velocity of e , we have,

$$\gamma E e = B e v \quad \text{or} \quad v = \frac{\gamma E}{B}$$

When e emerges between the plates it is subject only to the magnetic field, B , and describes a circular trajectory with radius, R , where $B e v = \frac{m v^2}{R}$.

$$\text{Giving, } R = \frac{m v}{B e} = \frac{\gamma E m}{B^2 e} = \frac{E m}{B^2 e} \left(1 + \frac{v^2}{2c^2} \right).$$

Again following O'Rahilly [2], the motion of an electron travelling at right angles to the plates along the y axis is now considered. For this case (Fig. 7), given $u_r = v \sin \alpha$ and $u_y = v$, to obtain the y force component Eq. (6) becomes,

$$\delta F_y(R) = \frac{e e'}{4\pi \epsilon_0 r^2} \sin \alpha \left[1 + \frac{(3-\lambda) v^2}{4 c^2} - \frac{3(1-\lambda) v^2}{4 c^2} \sin^2 \alpha - \frac{(1+\lambda) v^2}{2 c^2} \right]$$

where $\sin \alpha = \frac{h}{(h^2 + s^2)^{1/2}}$, $r^2 = h^2 + s^2$ and $e' = \sigma s ds d\theta$. The force on e due to both charged plates is

then,

$$F_y = \frac{2e\sigma}{4\pi\epsilon_0} \left[\int_0^{2\pi} d\theta \int_0^\infty \left[\left(1 + \left(\frac{3-\lambda}{4} \right) \frac{v^2}{c^2} \right) \frac{hs}{(h^2+s^2)^{3/2}} - \frac{3}{4} (1-\lambda) \frac{v^2}{c^2} \frac{h^3s}{(h^2+s^2)^{5/2}} - \frac{(1+\lambda)v^2}{2} \frac{hs}{c^2 (h^2+s^2)^{3/2}} \right] ds \right]$$

$$= \frac{e\sigma}{2\pi\epsilon_0} \left[\int_0^{2\pi} d\theta \int_0^\infty \left[\left(1 + \left(\frac{1-3\lambda}{4} \right) \frac{v^2}{c^2} \right) \frac{hs}{(h^2+s^2)^{3/2}} - \frac{3}{4} (1-\lambda) \frac{v^2}{c^2} \frac{h^3s}{(h^2+s^2)^{5/2}} \right] ds \right]$$

The s integrals are $[I]_1 = \int_0^\infty \frac{hs}{(h^2+s^2)^{3/2}} ds$ and $[I]_2 = \int_0^\infty \frac{h^3s}{(h^2+s^2)^{5/2}} ds$.

With the substitution $s = h \tan \phi$, $[I]_1 = 1$ and $[I]_2 = \frac{1}{3}$.

Finally, $F_y = \frac{e\sigma}{\epsilon_0} (1 - \frac{\lambda v^2}{2c^2}) = eE(1 - \frac{\lambda v^2}{2c^2})$ where $E = \frac{\sigma}{\epsilon_0}$ is the electric intensity between the plates.

Then applying Newton's second law, $mv \frac{dv}{dy} = F_y = Ee(1 - \frac{\lambda v^2}{2c^2})$ and rearranging, $edV = mv dv(1 + \frac{\lambda v^2}{2c^2})$ where $E = \frac{dV}{dy}$ and V is the potential difference between the plates.

Integrating, $eV = m \int_0^v v(1 + \lambda \frac{v^2}{2c^2}) dv = m(\frac{v^2}{2} + \lambda \frac{v^2}{8})$.

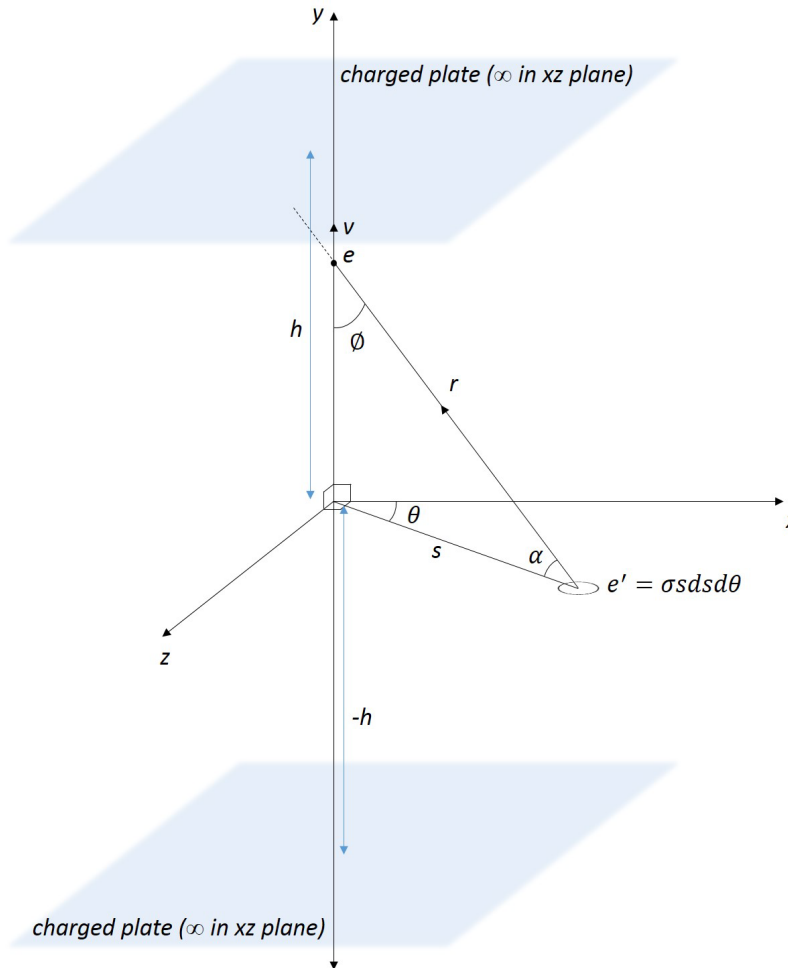


Figure 7. Point charge e is moving along the y axis between two infinite plates at $y = \pm h$.

Making,

$$eV = m \frac{v^2}{2} \left(1 + \lambda \frac{v^2}{4c^2} \right) \quad (8)$$

Now consider the analysis based on the modified Lorentz theory, $m = m_0\gamma$ where $\gamma = (1 - \frac{v^2}{c^2})^{-1/2} = 1 + \frac{v^2}{2c^2}$ to the second order.

Then,

$$eV = \int_0^v \frac{d}{dv} (\gamma m_0 v)$$

Here, $\frac{d}{dv} (\gamma m_0 v) = m_0 \frac{d}{dv} (v + \frac{v^3}{2c^2}) = m_0 (1 + \frac{3}{2} \frac{v^2}{c^2})$.

So,

$$eV = m_0 \int (v + \frac{3}{2} \frac{v^3}{c^2}) dv.$$

Therefore,

$$eV = m_0 \left(\frac{v^2}{2} + \frac{3}{8} \frac{v^4}{c^2} \right) = m_0 \frac{v^2}{2} \left(1 + \frac{3}{4} \frac{v^2}{c^2} \right) \quad (9)$$

It follows that when, $\lambda = 3$, (8) and (9) coincide as far as second order terms. For the case of electrons travelling close to the speed of light, the second order approximation is clearly invalid; higher speeds would require additional treatment (i.e., as $v/c \rightarrow 1$).

6. CONCLUSION

In the deflection of electron beams by a long, straight wire carrying direct current, the electrodynamic theories of both Weber and Ritz have been tested against the field-based approach of Maxwell-Lorentz. For ‘non-relativistic’ beams all three theories give identical results which agree with experimental observations. It is of interest that field theory, direct action and emission theories of electrodynamics, each based on essentially different assumptions, give the same result.

In the case of high speed electrons, it is well known that electron trajectories must be corrected, according to the Lorentz factor, $\gamma = (1 - \frac{v^2}{c^2})^{-1/2}$. In typical crossed-field experiments for high speed electrons, it is shown that Weber-Ritz provides the same result as the established theory. Recall that crossed-field experiments involve transverse electron beam forces and in this case, Ritz’s theory indicates that the force is independent of the value of λ . Interestingly, when an electron moves parallel to the electric field, a distinction is made between Weber and Ritz. That is, the experimental data is satisfied by a value of $\lambda = 3$ (Ritz), rather than $\lambda = -1$ (Weber). Assis [4, pp. 245] has commented that when calculating the force from a closed circuit on a current element of another circuit, Ritz’s theory yields the same result as Ampere or Grassmann, independent of λ . The general pattern of Ritz’s electrodynamic theory suggests two distinct cases, one for metallic current elements and low speed electrons (independent of λ) and the other for high speed electrons ($\lambda = 3$) [11].

In contrast to field-based theory, the direct action theories of Weber-Ritz can offer a more physical insight into electrodynamics. Moreover direct-action is not opposed to the established field-based theory originating with Maxwell-Lorentz, but rather it is complementary. It has been demonstrated [12] that from Weber’s force formula, Faraday’s law, vector potential and mutual inductance can be derived and that a significant degree of compatibility exists between Weber’s force and Maxwell’s equations. It is uncertain whether direct-action theories can describe the variety of electromagnetic phenomena as field theory. However, they have a number of advantageous features. For example they conform to a philosophy of science consistent with Ockham’s principle that physical entities should not be multiplied unnecessarily. In direct-action theories, electric and magnetic fields are not necessary; there are only forces between electrical charges in relative motion. From a pedagogical perspective, direct action formulations can provide a clearer understanding of certain phenomena. For example, the forces between steady currents and motional electromagnetic induction are identified with the velocity terms in the Weber force formula, while transformer induction is explained through the acceleration

terms [13]. Finally, such an approach has potential advantages in a number of fields such as applied electromagnetics [14], and charge particle dynamics [15, 16].

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