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Fast Synthesis of Planar, Maximally Thinned Arrays

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Abstract—In this paper a method for a fast synthesis of planar, maximally thinned and steerable arrays is proposed and tested on several benchmarks available in literature. The method optimizes simultaneously the weight coefficients and sensor positions of a planar array without using global optimization schemes, properly exploiting convex optimization based algorithms. The resulting arrays are able to radiate a steerable beam pattern, satisfying a prescribed power mask and avoid to constraint the fitting of any a priori assigned reference field pattern. Although such a method takes into account the general case of sparse arrays, this letter is focused on the case of thinned arrays as a special case of sparse ones, since the initial grid to thin on has only half-wavelength distances. Such a feature allows a faster synthesis than in the general case of sparse arrays.

1. INTRODUCTION

The design of the sparse Active Electronically Scanned Arrays (AESAs) radiating far-field pattern, satisfying a prescribed upper bound power mask, with as few elements as possible has a number of significant applications, and interests.

Starting from the results in [1] and extending them in [2] to the case of non-superdirective (according to the definition given by Hansen[†] [3]), steerable and planar arrays, in the following we are focusing on thinned arrays. Thinning an array means turning off some elements in a periodic array while maintaining the radiation properties of the original array. In order to switch off as many elements as possible, i.e., by satisfying the constraints of interest, several methods related to different approaches can be applied. Global or evolutionary optimization schemes, such as Genetic Algorithms (GA) (and advanced versions) [11, 12], Simulated Annealing (SA) [13, 14] and Ant Colony Optimization (ACO) [15, 16], can be found in a great amount of works regarding array thinning.

Another typical approach to find the sparse solution is the Bayesian framework [17–20], which is especially well suited for undetermined problems or compressive sensing ones. In such cases, some a priori limited knowledge in probabilistic terms is assumed to seek for the sparse solution.

Another way to formulate the thinned array design problem is searching for a sparse solution of an ℓ_0-(pseudo)norm minimization problem, which is known to be of combinatorial complexity (NP-hard) and likely intractable by brute-force methods even for moderate-sized arrays. Our synthesis scheme refers to the latter approach. To reduce the computational burden we consider two approaches [2]: one based on a relaxation of the aforementioned norm that solves a sequence of ℓ_1-norm minimization problems [4] by means of convex optimization, and the other on fast Branch-and-Bound (B&B) algorithms that fall in the context of Mixed Integer Programming (MIP) [6–10].

In particular, a synthesis method for achieving ***maximally thinned*** (i.e., with the minimum number of elements), ***planar***, ***steerable***, arrays is described. The reason for selecting thinned arrays

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[†] *<<An array is superdirective if its directivity is higher than the one obtained from the isophoric case (uniform excitations with constant amplitude and linear phase)>>*

as a particular application of the general formulation of [2] is that they may result in great interest when both designing AESAs of very large size and requiring a fast synthesis. In fact, in such cases application of the general, full approach of [2], which starts from initial grids sampled with distances smaller than half-wavelength, can lead to unacceptably long simulation time. Sparsifying on a grid with distances greater than or equal to half-wavelength turns out to be faster than the general case discussed in [2] because no constraint must be applied to achieve non-superdirectivity and the solution is sought on a smaller set of unknowns. Moreover, a thinned approach returns a greater number of elements with respect to a full, sparse one, which can be an advantage when higher antenna gains are expected.

Therefore, the benchmarks that we selected in [5] for the particular application of the formulation in [2] to the case at hand are of thinned type.

The paper is organized as follows. Section 2 introduces the problem formulation. Section 3 describes the numerical examples. Finally, a conclusion section ends the paper.

2. PROBLEM FORMULATION

Once upper bounds have been given to the desired power pattern, the problem at hand aims at finding the excitation and location distributions of the radiating elements such that the radiated field satisfies the prescribed power mask with the minimum number of antennas for a desired steering solid angle.

To this end, let us consider a planar, rectangular dense grid composed of $N \times M$ sources and assume the inter-element spacing being d_x and d_y along the x-axis and y-axis, respectively. The array factor of such a distribution of antennas is given by

$$AF(\theta, \phi) = \sum_{n=1}^{N} \sum_{m=1}^{M} w(x_n, y_m)\, e^{j(2\pi/\lambda)[(u-u_0)x_n+(v-v_0)y_m]} \tag{1}$$

where (θ, ϕ) is the observation direction; $u = \sin\theta\cos\phi$, $v = \sin\theta\sin\phi$, (u_0, v_0) is the angular orientation of the main beam (or "steering direction"); (x_n, y_m) are the antenna locations counted per columns and expressed in wavelengths; $w(x_n,\ y_m) = w_{nm}$ is the (n, m)th complex weight. The schematic representation and relative notation are reported in Fig. 1.

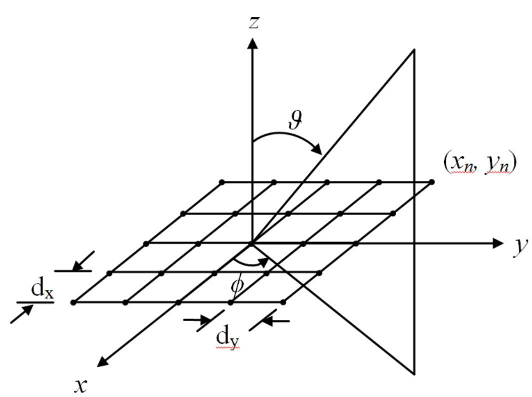

Figure 1. Geometry and notation of the considered planar, rectangular grid.

The synthesis problem can be formulated as one of constrained optimization type, in which the cost function, acting on the elements excitations, is chosen to return the solution that exhibits the minimum cardinality, i.e., the ℓ_0-pseudo-norm.

However, we can make the cost function affine by introducing a vector $\mathbf{w}_b$ of binary variables belonging to $B = (0, 1)^{NM}$ that accounts for the presence or absence of actual radiating elements. In fact, in such a context the ℓ_0-norm of a binary vector is the sum of its ones, i.e., its cardinality. With respect to relations (4) in [2], we can formally define the synthesis process in solving the following optimization problem:

$$\begin{aligned}
&\min_{\substack{\mathbf{w}\in C^{NM}\\ \mathbf{w}_b\in B}} \|\mathbf{w}_b\|_0 \\
&s.t. \quad |\mathbf{w}| \le \mathbf{w}_b \\
&\qquad\; AF(u_0, v_0) = 1 \\
&\qquad\; |AF(u,v)|_{(u,v)\in Sidelobe} \le UB(u,v)
\end{aligned} \tag{2}$$

where the ℓ_0-norm operator $\|\|_0$ represents the number of non-zero entries of its argument, $\mathbf{w} = \{w_{nm}\}\forall n = 1,\ldots,N$ and $\forall m = 1,\ldots,M$, $UB(u,v)$ is the mask function defining the upper bounds for the sidelobes, and *Sidelobe* represents the region in which the upper bound constraint is applied. More in detail, *Sidelobe* is the region out of the assigned mainbeam (whose extension is considered at the maximum SLL) such that $u^2+v^2 \le [1+\sin(\vartheta_M)]^2$, where ϑ_M defines the desired steering cone, whose axis lies along the broadside direction, so to extend the constraint beyond the visible region in the (u,v) domain and automatically suppress possible grating lobes (when doing linear steering). Clearly, the point density of the grid is smaller in the case of distances greater than or equal to half-wavelength with respect to the same grid sampled at, say, $\lambda/10$ or $\lambda/100$, as we usually do when adopting the full, sparse approach of [2]. Therefore, the full-grid array will be considered as reference for the reduction of the number of elements.

As discussed in [2], the problem in Eq. (2) can be solved by using two types of approaches, a sequential one based on the iteratively weighted ℓ_1-norm [4] and a parallel one based on a fast Branch&Bound (B&B) [6, 7] technique which falls in the general context of Mixed Integer Programming (MIP) [8–10]. In particular, since the problem in Eq. (2) is an NP-hard one, if no relaxations are applied (like, for instance, the reweighted ℓ_1-norm) it intrinsically involves MIP, and the B&B is tailored for the latter. The B&B algorithm is a general method for finding the optimal solution to discrete problems, which enumerates candidate solutions by means of a rooted-tree space search. According to its implementation and the upper and lower bounds of the cost function, which in our case are easily definable (the cardinality of $\mathbf{w}_b$ is apparently limited), it can prune huge amounts of branches, thus massively shortening the global solution search. At the execution stage, for each binary entry of the vector $\mathbf{w}_b$ the fast B&B algorithm divides the associated convex set [0, 1] in a set of intervals in which the search for the solution is carried out according to a parallel computation (for instance, an ensemble of such intervals is assigned to a core of the processor, whereas another ensemble is assigned to another core and so on, so that parallel computing can be efficiently exploited).

3. SYNTHESIS RESULTS

We are showing numerical results derived from the multipattern synthesis of benchmarks discussed in [5] by resorting to the algorithms proposed in [2] and applied to the case at hand. In particular, we consider a rectangular-grid array with a half-wavelength spacing of 14×14 elements for the joint synthesis of two shaped patterns, a circular and a diamond-like one, which in [5] have been obtained with 150 radiating elements with SLLs of -25.85 dB and -24.3 dB, respectively. For the first pattern, the mainlobe region is specified as a circular-shaped one, with equation $\left\{(u,v): u^2+v^2 \le 0.2^2\right\}$, whereas the sidelobe one is given by $\left\{(u,v): u^2+v^2 \ge 0.4^2\right\}$. For the second pattern, which was originally synthesized in [14] with a 14×14 array with half-wavelength spacing, the mainlobe region is diamond-shape like and is defined by $\{(u,v): |u-0.2|+|v-0.2| \le 0.2\}$, whereas the sidelobe one is given by $\{(u,v): |u-0.2|+|v-0.2| \ge 0.4\}$. Furthermore, a circular-shaped null region is required too and given by $\left\{(u,v): (u+0.5)^2+(v+0.5)^2 \le 0.1^2\right\}$, with a null of -50 dB. In both cases, the required ripple is ≤ 1 dB. All the sidelobe regions are within the visible circle. We would like to remark, in the present discussion, that the aforementioned SLL for the diamond-shaped pattern makes the joint synthesis unfeasible unless the (u,v) spectrum is sampled with 81×81 points, as apparently done in [5]. With the (u,v) spectrum sampled with 128×128 points, the minimum possible SLL is slightly higher, and in particular, it is -23.67 dB. With such sampling and SLL for the diamond-shape pattern, all the constraints are met even when plotting the radiation pattern on more points. With our proposed algorithms, we obtain the joint synthesis with only 119 radiating elements, and recalling the definition

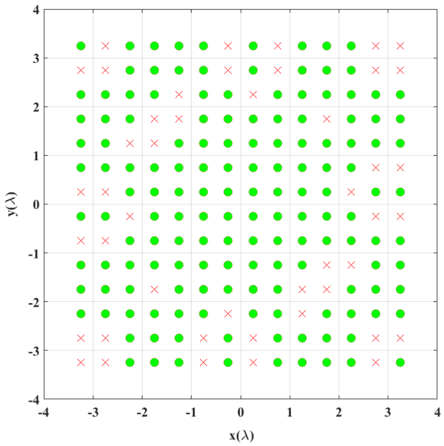

Figure 2. Array of 150 elements obtained in [5].

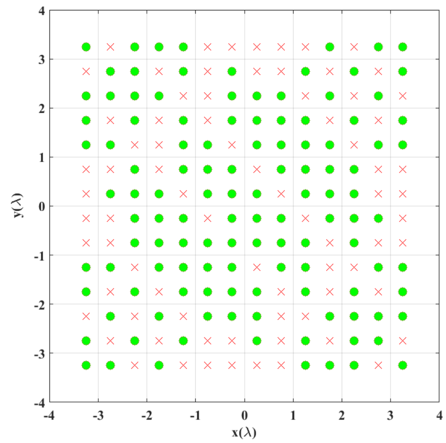

Figure 3. Array of 119 elements obtained by means of our proposed algorithms.

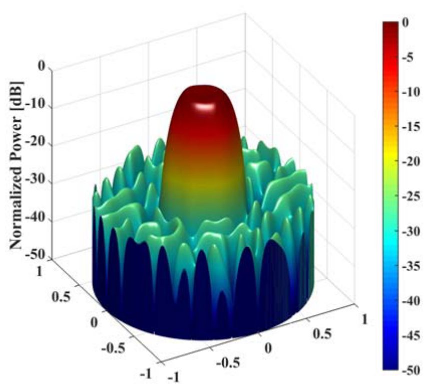

Figure 4. Circular shape 3D pattern.

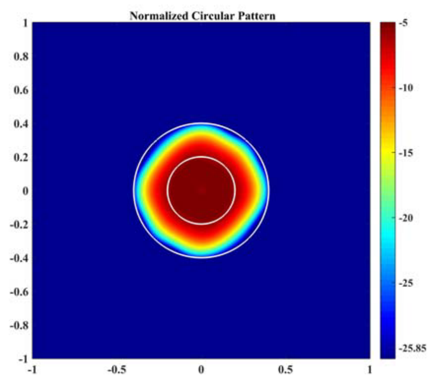

Figure 5. 2D circular pattern. The two white circles represent the mainlobe and sidelobe boundaries respectively.

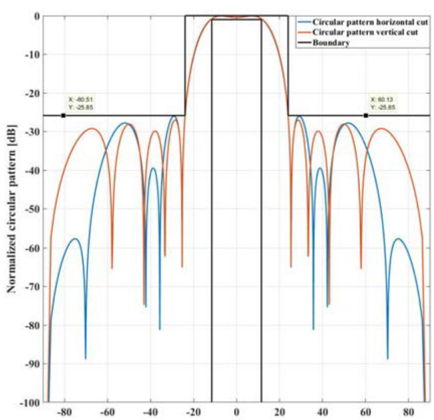

Figure 6. 2D circular pattern cuts. The vertical ($u = 0$) and horizontal ($v = 0$) cuts are reported in the figure together with the boundary required by the benchmark. Datatips have been inserted to show precisely the SLL.

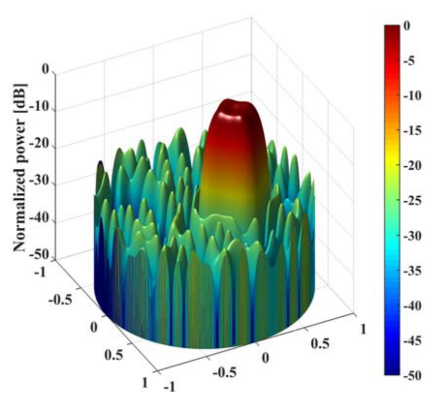

Figure 7. Diamond-shape 3D pattern.

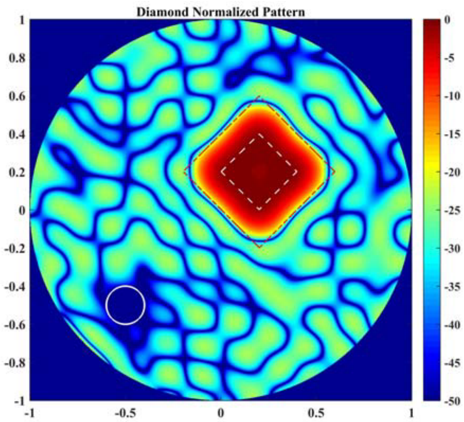

Figure 8. 2D diamond-shape pattern. The white circle and dashed square, together with the red dash-dotted square represent the null region, the mainlobe and sidelobe regions boundaries respectively.

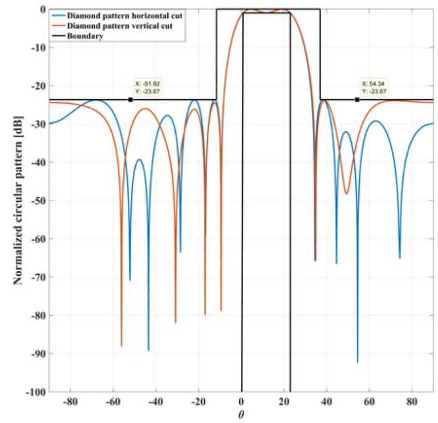

Figure 9. 2D diamond-shape cuts. The vertical ($u = 0$) and horizontal ($v = 0$) cuts are reported in the figure together with the boundary required by the benchmark. Datatips have been inserted to show precisely the SLL.

of the Element Number Reduction Ratio [2] as $\text{ENRR} = 1 - \frac{N_e}{F}$, where $N_e = 119$ and F =196, in this case we have ENRR = 39.29% *Vs* ENRR = 23.47% obtained in [5]. Fig. 2 and Fig. 3 show the layouts obtained in [5] and in the work under discussion, respectively.

Figures 4, 5 and 6 show the circular 3D/2D patterns and the main axes cuts, respectively, whereas Fig. 7, Fig. 8 and Fig. 9 show the diamond-shaped 3D/2D patterns and the main axes cuts, respectively. In particular, in Fig. 9 the cuts are apparently centered at $\theta = \text{asin}(0.2)$.

4. CONCLUSIONS

In this paper, a new synthesis method for achieving *maximally thinned*, *planar*, and *steerable*, array has been proposed. Starting from [1], where maximally sparse, linear, broadside arrays have been considered, a new formulation based on the introduction of a binary vector, whose distribution of ones represents the array layout, is proposed [2]. Such a formulation, in its general form, allows to easily impose a sufficient condition to obtain non-superdirective arrays and at the same time to minimize their cardinality, obtained through a constraint and a cost function being both affine. Nevertheless, the binary problem that derives from this general formulation falls in the context of MIP. In order to make the problem tractable, two approaches, one based on reweighted ℓ_1-norm and the other one on a fast-B&B technique, have been introduced. For the considered benchmark problem, a considerable improvement is obtained in terms of the minimum number of required radiating elements: ENRR = 39.29%.

As regards to the performances of the two proposed approaches, there is a tradeoff: sparsity *versus* time consumption. In fact, the synthesis being equal, the method based on the fast-B&B algorithm can generally return smaller numbers of elements than the ones obtained from the reweighted ℓ_1-norm technique in spite of longer time required for convergence. Whether to use the former or the latter method depends on the requirements.

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