

An Efficient Localization Method Using Signal Reconstruction

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Abstract—This paper considers the localization of an emitter where the transmitted signal is unknown for receivers. To improve the localization accuracy, we propose an efficient method to estimate the emitter position by reconstructing the transmitted signal jointly. Simulation results show that the localization performance of the proposed method is much better than the existing algorithms.

1. INTRODUCTION

Emitter localization based on several widely distributed stations attracts more and more interest [1–7]. The traditional localization algorithms for emitters are generally based on time difference of arrival (TDOA), angle of arrival (AOA) [8–11], etc. These types of algorithms are seen as a kind of two-step localization approaches, which have been proved suboptimal because much information of the emitter is discarded when the TDOA or AOA is extracted separately in the different stations [12].

An alternative approach is commonly referred to as the direct position determination (DPD) methods. These approaches reserve as much information as possible and have been shown to outperform the two-step localization methods at the cost of the computation complexity and increased transmission demands [13–17]. As for this approaches, there are two cases encountered in general. The first case is that the transmitted signal is completely known to the stations, e.g., a training signal or synchronization signal. The DPD algorithm of this case can acquire the optimal localization performance by joint processing of the observed signals from all stations and the waveform information. This case is rarely encountered in actual applications and is referred to as DPD-known in this paper [13]. In noncooperative emitter localization, the second case that the waveform of the transmitted signal is unknown to the receivers, is encountered generally. In [13], the DPD algorithm is proposed to estimate the position of the emitter without the need of any waveform information. Here we refer to this method as DPD-unknown so as to distinguish from the DPD-known algorithm. Although the transmitted signal is ignored in the localization, the DPD-unknown method is still shown to considerably outperform the conventional two-step localization methods, especially in low signal-to-noise ratio (SNR) scenarios. However, due to the unknown waveform information, it is obvious that the DPD-unknown approach is not optimal and suffers considerable performance loss compared to the DPD-known approach, in the sense that the waveform information is not taken into account during the localization process.

In practice, motivated by an attempt to approximate the DPD-known algorithm, the transmitted signal can be estimated employing some techniques. This paper proposed an approach to solve this problem, which estimates the emitter position by reconstructing the transmitted signal jointly. Simulation results show the superiority and robustness of the proposed algorithm.

The paper is organized as follows. Section 2 briefly reviews the signal model for widely separated stations. Section 3 states the localization problem formulated in a maximum likelihood estimation. The proposed new algorithm is presented in detail in Section 4, followed by numerical simulations in Section 5. Finally, conclusions are drawn in Section 6.

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2. SIGNAL MODEL

Considering a noncooperative stationary emitter and D widely distributed stations intercepting the transmitted signal, the stations are located at (x_d, y_d) , $d = 1, 2, \dots, D$, which employ synchronized oscillators. The emitter, located at $\mathbf{v} = (x, y)$, transmitted the signal $g(t)$ at time t_0 with the length, T_l . The intercepted signal by the different stations are transmitted to a central processor to locate the emitter. The observed signal for the d -th station can be expressed as

$$\begin{aligned} w_d(t) &= \begin{cases} c_d g(t - t_{d,b}) + n_d(t), & t_{d,b} \leq t < t_{d,f} \\ n_d(t), & \text{else} \end{cases} \\ &= c_d g_d(t) + n_d(t) \end{aligned} \quad (1)$$

where c_d is an unknown complex scalar representing the attenuation coefficient related to the path from the emitter to the different receivers, and $n_d(t)$ is assumed to be the independent identically distributed zero-mean complex Gaussian white noise. In addition,

$$g_d(t) \triangleq \begin{cases} g(t - t_{d,b}), & t_{d,b} \leq t < t_{d,f} \\ 0 & \text{else} \end{cases} \quad (2)$$

with the starting time $t_{d,b}$ and terminal time $t_{d,f}$ of the transmitted signal during the observation interval $[0, T]$ which is assumed to be long enough for the complete signal waveform to be captured by all receivers. The starting and terminal times are the dependence between the observations and source location,

$$t_{d,b} \triangleq t_{d,b}(\mathbf{v}, t_0) = \tau_d(\mathbf{v}) + t_0, \quad (3)$$

$$t_{d,f} \triangleq t_{d,f}(\mathbf{v}, t_0, T_l) = \tau_d(\mathbf{v}) + t_0 + T_l, \quad (4)$$

where t_0 is the time when the signal is transmitted by the emitter and T_l the length of the transmitted signal. $\tau_d(\mathbf{v})$ denotes the propagation time between the emitter and the d -th receiver, which is given by

$$\tau_d(\mathbf{v}) = \frac{\sqrt{(x - x_d)^2 + (y - y_d)^2}}{c} \quad (5)$$

with c denoting the speed of light. Note that the starting time and terminal time are written as $t_{d,b}(\mathbf{v}, t_0)$ and $t_{d,f}(\mathbf{v}, t_0, T_l)$ sometimes in this paper, to emphasize that they are the functions of unknown parameters $\mathbf{v}$, t_0 and T_l .

After sampling, the continuous signal of Eq. (1) can be written in a vector form as

$$\mathbf{w}_d = c_d \mathbf{g}_d + \mathbf{n}_d \quad (6)$$

with

$$\begin{aligned} \mathbf{w}_d &\triangleq [w_d[0] \ w_d[1] \ \dots \ w_d[N_s - 1]]^\top \\ \mathbf{g}_d &\triangleq [g_d[0] \ g_d[1] \ \dots \ g_d[N_s - 1]]^\top \\ \mathbf{n}_d &\triangleq [n_d[0] \ n_d[1] \ \dots \ n_d[N_s - 1]]^\top \end{aligned} \quad (7)$$

where N_s is the number of samples under the sampling interval T_s , and the superscript $[\cdot]^\top$ represents the transpose operator. The receiver stations are assumed to be spaced widely enough to warrant mutual independence of noise vectors,

$$\mathbf{n}_d \perp \mathbf{n}_{d'}, \quad d \neq d' \quad (8)$$

3. PROBLEM FORMULATION

In order to deduce the estimator of localization, the transmitted signal is assumed first to be known for receivers. The unknown parameters are the transmitted time t_0 , the attenuation coefficient c_d and the

emitter location $\mathbf{v}$. Following [18], the joint maximum likelihood (ML) estimate of unknown parameters can be found by maximising the likelihood ratio,

$$\begin{aligned} D(\mathbf{w}_d; \mathbf{v}, t_0, c_d) &\propto \frac{\exp \left\{ -\frac{1}{2} (\mathbf{w}_d - c_d \mathbf{g}_d)^H \mathbf{R}_d^{-1} (\mathbf{w}_d - c_d \mathbf{g}_d) \right\}}{\exp \left\{ -\frac{1}{2} \mathbf{w}_d^H \mathbf{R}_l^{-1} \mathbf{w}_d \right\}} \\ &= \exp \left\{ \frac{1}{2} c_d \mathbf{w}_d^H \mathbf{R}_d^{-1} \mathbf{g}_d + \frac{1}{2} (c_d \mathbf{g}_d)^H \mathbf{R}_d^{-1} \mathbf{w}_d - \frac{1}{2} (c_d \mathbf{g}_d)^H \mathbf{R}_d^{-1} (c_d \mathbf{g}_d) \right\} \end{aligned} \quad (9)$$

where the superscript $[\cdot]^H$ represents the conjugate transpose operator and c_d can be estimated based on the general likelihood ratio by maximizing the likelihood in Eq. (9)

$$\frac{\partial}{\partial c_d} D(\mathbf{w}_d; \mathbf{v}, t_0, c_d) |_{c_d = \hat{c}_{d, \text{ML}}} = 0 \quad (10)$$

and the ML estimation for c_d is

$$\hat{c}_{d, \text{ML}} = \frac{\mathbf{g}_d^H \mathbf{R}_d^{-1} \mathbf{w}_d}{\mathbf{g}_d^H \mathbf{R}_d^{-1} \mathbf{g}_d}. \quad (11)$$

By substituting Eq. (11) into Eq. (9) and employing the assumption that $\mathbf{n}_d$ is independent across different paths, the logarithmic form of the likelihood ratio function for all the paths is expressed as

$$\ell(\mathbf{w}; \mathbf{v}, t_0) \propto \sum_{d=1}^D \frac{1}{\mathbf{g}_d^H \mathbf{R}_d^{-1} \mathbf{g}_d} |\mathbf{g}_d^H \mathbf{R}_d^{-1} \mathbf{w}_d|^2. \quad (12)$$

Therefore, the position of the emitter $\mathbf{v}$ can be estimated using the following ML estimator,

$$\begin{aligned} [\hat{\mathbf{v}}, \hat{t}_0] &= \arg \max_{\mathbf{v}, t_0} \ell(\mathbf{w}; \mathbf{v}, t_0) \\ &= \arg \max_{\mathbf{v}, t_0} \sum_{d=1}^D \frac{1}{\mathbf{g}_d^H \mathbf{R}_d^{-1} \mathbf{g}_d} |\mathbf{g}_d^H \mathbf{R}_d^{-1} \mathbf{w}_d|^2. \end{aligned} \quad (13)$$

Note that $\mathbf{g}_d$ is related to $\mathbf{v}$ and t_0 according to Eq. (2). When the transmitted signal is known, the emitter location can be determined by searching for all the $\mathbf{v}$ and t_0 of interest.

As for the unknown transmitted signal, the predominant challenge of Eq. (13) is to generate the function $\mathbf{g}_d$ related to position. DPD-unknown avoids this challenge and employs the mathematical tool to locate the emitter by maximising the eigenvalue for all the $\mathbf{v}$ of interest [13]. However, it ignores the waveform information about the transmitted signal, resulting in not good enough performance comparing with the DPD-known algorithm. This paper introduces another approach to acquire better localization performance as for the case that the transmitted signal is unknown.

4. THE LOCALIZATION ALGORITHM BASED ON THE ESTIMATION OF SIGNAL

In this section, we propose a localization algorithm that estimates emitter position by reconstructing the transmitted signal jointly. Different from the DPD-unknown algorithm, this method requires an additional estimation for the t_0 and T_l to extract the transmitted signal from receivers according to Eq. (1). The benefit is that it exploits waveform information as much as possible and obtains better localization performance.

For notational convenience, all the parameters estimated are embodied in $\boldsymbol{\eta} \triangleq (\mathbf{v}, t_0, T_l)$. This approach firstly extracts the samples of transmitted signal from receivers according to a hypothetical $\boldsymbol{\eta}$. Then the transmitted signal will be estimated based on the extracted samples of the hypothetical $\boldsymbol{\eta}$. At last, a cost function is constructed to determine the $\boldsymbol{\eta}$ and locate the emitter as well.

4.1. Extraction of the Samples of the Transmitted Signal

As indicated in Eq. (1), the transmitted signal occurs in an interval from $t_{d,b}(\boldsymbol{\eta})$ to $t_{d,f}(\boldsymbol{\eta})$, which is determined by a hypothetical $\boldsymbol{\eta}$. As for a hypothetical $\boldsymbol{\eta}$, the indexes of the interval in $\mathbf{w}_d$ can be computed as

$$n_{d,b}(\boldsymbol{\eta}) = \lfloor \frac{t_{d,b}(\boldsymbol{\eta})}{T_s} \rfloor \quad (14)$$

$$n_{d,f}(\boldsymbol{\eta}) = \lfloor \frac{t_{d,f}(\boldsymbol{\eta})}{T_s} \rfloor \quad (15)$$

where $n_{d,b}(\boldsymbol{\eta})$ and $n_{d,f}(\boldsymbol{\eta})$ denote the starting index and terminal index respectively and $\lfloor \cdot \rfloor$ represents the operation of rounding down. Hence the samples of the transmitted signal can be extracted from the $n_{d,b}(\boldsymbol{\eta})$ -th to $n_{d,f}(\boldsymbol{\eta})$ -th elements in the observed signal $\mathbf{w}_d$ as,

$$\mathbf{g}^d(\boldsymbol{\eta}) = \mathbf{w}_d [n_{d,b}(\boldsymbol{\eta}) : n_{d,f}(\boldsymbol{\eta})] \quad (16)$$

where $[n_{d,b}(\boldsymbol{\eta}) : n_{d,f}(\boldsymbol{\eta})]$ denotes the operation that extracts the samples from the $n_{d,b}(\boldsymbol{\eta})$ -th to $n_{d,f}(\boldsymbol{\eta})$ -th indexes in the column vector $\mathbf{w}_d$. Note that the indexes are related to $\boldsymbol{\eta}$, hence the size of $\mathbf{g}^d(\boldsymbol{\eta})$ is unfixed. The extracted samples contain the complete signal waveform if the hypothetical $\boldsymbol{\eta}$ is the correct, namely the emitter position $\mathbf{v}$ transmitted time t_0 and the signal length T_l are the correct. In contrast, with regard to an incorrect $\boldsymbol{\eta}$, the extracted samples consist of noise segments and maybe part of the signal segments. As for D receivers, we combine all the extracted samples in a signal matrix as,

$$\mathbf{G}(\boldsymbol{\eta}) \triangleq [\mathbf{g}^1(\boldsymbol{\eta}), \dots, \mathbf{g}^d(\boldsymbol{\eta}), \dots, \mathbf{g}^D(\boldsymbol{\eta})] \quad (17)$$

4.2. Reconstruct the Transmitted Signal

This section introduces how to reconstruct the transmitted signal from the signal matrix in Eq. (17). The principal component analysis is utilized generally in signal processing [19–26]. As for the correct $\boldsymbol{\eta}$, the signal matrix includes waveform information of the transmitted signal. In order to reserve waveform information as much as possible, we choose the first principal component as an estimation of transmitted signal. The first principal component can give a weight vector $\boldsymbol{\omega}^*$ to yield the reconstructed signal as

$$\mathbf{g}_{\text{esti}}(\boldsymbol{\eta}) = \mathbf{G}(\boldsymbol{\eta})\boldsymbol{\omega}^* \quad (18)$$

In order to obtain the weight vector, we first construct an $D \times D$ variance matrix as

$$\boldsymbol{\Sigma}(\boldsymbol{\eta}) \triangleq [\mathbf{G}(\boldsymbol{\eta}) - \bar{\mathbf{G}}(\boldsymbol{\eta})]^H [\mathbf{G}(\boldsymbol{\eta}) - \bar{\mathbf{G}}(\boldsymbol{\eta})] \quad (19)$$

where

$$\bar{\mathbf{G}}(\boldsymbol{\eta}) \triangleq \mathbf{E}_{N(\boldsymbol{\eta}) \times 1} \bar{\mathbf{g}}(\boldsymbol{\eta}) \quad (20)$$

with

$$\mathbf{E}_{N(\boldsymbol{\eta}) \times 1} \triangleq [1, 1, 1, \dots, 1]^T \quad (21)$$

$$N(\boldsymbol{\eta}) \triangleq n_{d,b}(\boldsymbol{\eta}) - n_{d,f}(\boldsymbol{\eta}) + 1 \quad (22)$$

note that $\bar{\mathbf{g}}(\boldsymbol{\eta})$ is a row vector that each element is the mean of each column in signal matrix of Eq. (17).

The eigenvalues of the variance matrix are as follows

$$\lambda_1(\boldsymbol{\eta}) \geq \lambda_2(\boldsymbol{\eta}) \geq \dots \geq \lambda_D(\boldsymbol{\eta}). \quad (23)$$

The different principal components are obtained by computing the eigenvectors corresponding to different eigenvalues. The size of eigenvalues reveals the amount of information in the principal component about the transmitted signal. Thus, we choose the first principal component, whose eigenvector is corresponding to the largest eigenvalue, as the estimated signal to reserve as much information as possible. Namely, the weight vector $\boldsymbol{\omega}^*$ in Eq. (18) is obtained by computing the eigenvector corresponding to the largest eigenvalue $\lambda_1(\boldsymbol{\eta})$.

4.3. Design of the Enhanced Estimator

In principal component analysis, although we choose the largest eigenvalue, some information about the transmitted signal is still abandoned. In order to evaluate the amount of information remained, a first principal component coefficient is defined as,

$$\varepsilon(\boldsymbol{\eta}) \triangleq \frac{\lambda_1(\boldsymbol{\eta})}{\sum_{d=1}^D \lambda_d(\boldsymbol{\eta})} \quad (24)$$

which is the function of $\boldsymbol{\eta}$. A larger value denotes more accurate estimation for the transmitted signal. According to Eq. (18), the estimated signal also is the function of $\boldsymbol{\eta}$. Only correct $\boldsymbol{\eta}$ can we obtain the accurate estimate for the transmitted signal. Hence, the first principal component coefficient will be maximised on the correct $\boldsymbol{\eta}$.

Moreover, considering that the transmitted signal has more power than noise, an correct estimation for the transmitted signal in Eq. (18) will generate a large power, which is defined as

$$E(\boldsymbol{\eta}) \triangleq \frac{\sum_j |g_{\text{esti}}(j; \boldsymbol{\eta})|^2}{T_l} \quad (25)$$

where $g_{\text{esti}}(j; \boldsymbol{\eta})$ denotes the j th element in the estimated signal $\mathbf{g}_{\text{esti}}(\boldsymbol{\eta})$. The power is also a function of $\boldsymbol{\eta}$ and the correct $\boldsymbol{\eta}$ will yield a large value.

Both the first principal component coefficient in Eq. (24) and power in Eq. (25) can be utilized to estimate $\boldsymbol{\eta}$. Yet it is rough and we construct a cost function to estimate them accurately.

Similarly with the DPD-known algorithm, the logarithmic likelihood ratio function can be obtained based on the transmitted signal,

$$\ell(\mathbf{w}; \boldsymbol{\eta}) = \sum_{d=1}^D \frac{1}{\mathbf{g}_d(\boldsymbol{\eta})^H \mathbf{R}_d^{-1} \mathbf{g}_d(\boldsymbol{\eta})} \left| \mathbf{g}_d(\boldsymbol{\eta})^H \mathbf{R}_d^{-1} \mathbf{w}_d \right|^2 \quad (26)$$

where $\mathbf{g}_d(\boldsymbol{\eta})$ is a function of $\boldsymbol{\eta}$ and is expressed as

$$\mathbf{g}_d(\boldsymbol{\eta}) \triangleq [\mathbf{0}_{d,1}; \mathbf{g}_{\text{esti}}(\boldsymbol{\eta}); \mathbf{0}_{d,2}] \quad (27)$$

which consists of the reconstructed signal and two zero vectors. $\mathbf{0}_{d,1}$ is an $n_{d,b}(\boldsymbol{\eta}) \times 1$ column vector, and $\mathbf{0}_{d,2}$ is an $[N_s - n_{d,f}(\boldsymbol{\eta}) - 1] \times 1$ column vector.

Combining the logarithmic likelihood ratio function with the first principal component coefficient and power, the cost function is defined as,

$$\ell_f(\boldsymbol{\eta}) \triangleq \varepsilon(\boldsymbol{\eta}) \times E(\boldsymbol{\eta}) \times \ell(\mathbf{w}; \boldsymbol{\eta}). \quad (28)$$

The cost function is maximised for the correct $\boldsymbol{\eta}$, and a high dimensional search for the region of interest is carried out to obtain the estimation of $\boldsymbol{\eta}$. Hence, the estimator is,

$$\hat{\boldsymbol{\eta}} = \arg \max_{\boldsymbol{\eta}} \ell_f(\boldsymbol{\eta}). \quad (29)$$

Compared with the DPD-unknown algorithm, this estimator exploits more waveform information of the transmitted signal and therefore is able to produce better localization performance. In essence, it can also be viewed as an approximation of the DPD-known algorithm. Different from DPD-known, the function related to emitter position is constructed by the reconstructed signal rather than the transmitted signal which is unknown to receivers.

5. SIMULATION RESULTS

In this section, we examine the performance and verify the superiority for the proposed method (DPD-enhanced), which is compared with the DPD-unknown and the DPD-known methods in [13]. The

DPD-known algorithm assumes that the transmitted signal is known perfectly for receivers. Therefore, the performance of the DPD-known algorithm can be viewed as an upper bound for all DPD algorithms, since it utilizes the waveform information of signal but suffers no signal estimation error.

The distributed placement simulated for four static receivers is shown on Fig. 1. The receivers are located at $(8, -6)$ km, $(-4, 2)$ km, $(5, 4)$ km, $(-5, -10)$ km respectively. The position of emitter is selected at random and it is placed at $(0, -5)$ km. We focus on the position root mean error (ME) defined respectively by

$$\text{ME} = \frac{1}{N} \sum_j^N \|\hat{\mathbf{v}}_j - \mathbf{v}\| \quad (30)$$

where N is the number of Monte Carlo experiments, and $\hat{\mathbf{v}}_j$ denotes emitter position estimated at the j th trial.

We choose three different signals to verify the localization performance for the proposed algorithm.

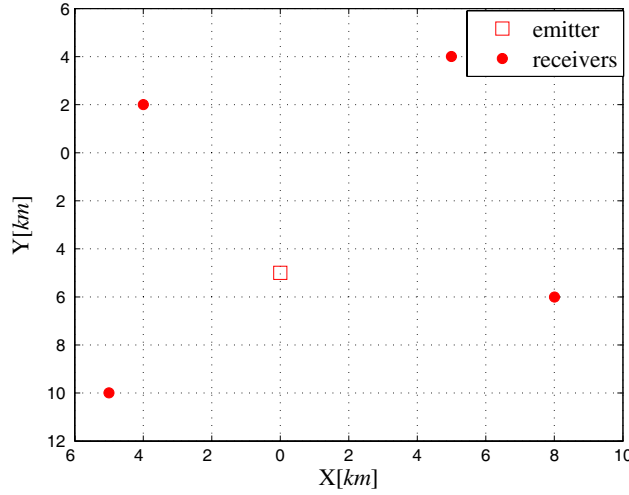


Figure 1. The placement of simulation scenario: the dots indicate the position of the receivers, and the square denotes the position of emitter.

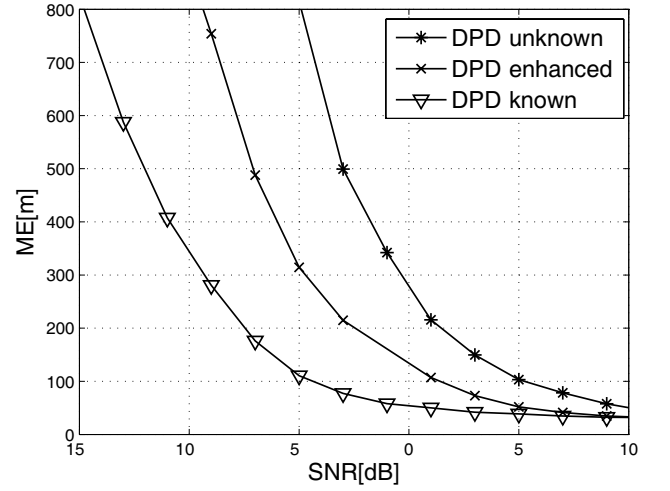


Figure 2. The comparison of three different algorithms for signal 1.

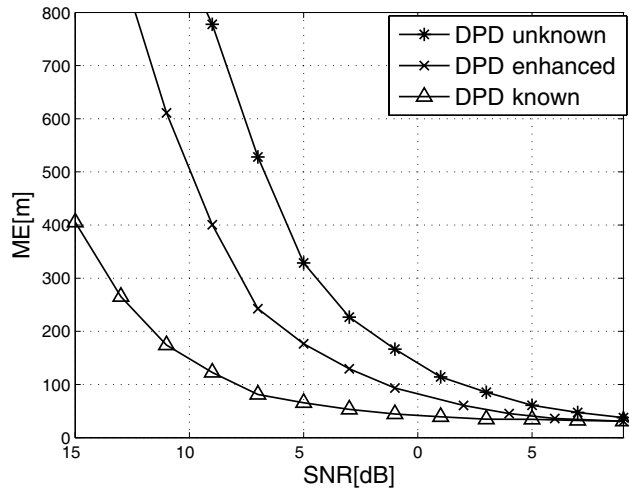


Figure 3. The comparison of three different algorithms for signal 2.

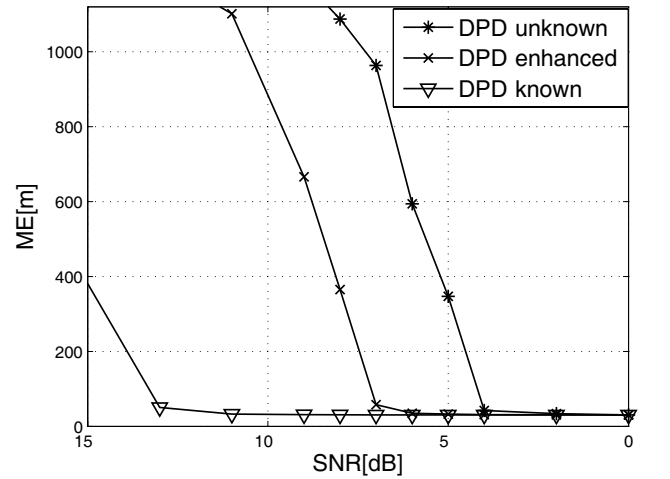


Figure 4. The comparison of three different algorithms for signal 3.

Signal 1 is a single carrier frequency pulse with the frequency 2 MHz and pulse width 10 μ s. Signal 2 is a gaussian pulse with the pulse width T_l is 20 μ s and frequency f_0 is 2 MHz. Signal 3 is a linear frequency modulated signal with the initial frequency 3 MHz, pulse width 10 μ s, and chirp rate 0.2 MHz/ μ s.

Figures 2–4 show the performance comparison for the DPD-known, DPD-unknown and DPD-enhanced algorithms. It can be seen clearly that in high SNR condition, all the algorithms achieve superior localization accuracy and perform nearly equally. As SNR decreases, the performance of the DPD-unknown algorithm deteriorates rapidly. In contrast, the DPD-enhanced algorithm can hold a good localization accuracy and perform similarly as the DPD-known algorithm. Besides, as for different signals, the DPD-enhanced algorithm holds a good robustness. Rapid performance loss for the DPD-enhanced algorithm for the extremely low SNR is observed as well.

6. CONCLUSIONS

In this paper, we consider the localization of a noncooperative emitter with the unknown signal waveform. To improve the localization performance, we propose an approach to determine the emitter position by reconstructing the transmitted signal based on the principal component analysis. In the maximum likelihood context, the DPD-known algorithm is an optimization search for the position of the emitter $\mathbf{v}$ and the transmitted time t_0 , which is, however, required to acquire the completely transmitted signal for receivers. Aiming at the scene of the transmitted unknown signal waveform, motivated by an attempt to approximate the DPD-known algorithm, a new DPD algorithm, i.e., DPD-enhanced, is deduced by the signal reconstruction using the principal component analysis. Numerical simulations are carried out to verify the localization performance of the proposed algorithm.

The simulation results show that the proposed DPD-enhanced algorithm significantly outperforms the DPD-unknown algorithm and can achieve the nearly same localization performance as the DPD-known algorithm when the SNR is quite high. Moreover, as for the three different signals, the DPD-enhanced algorithm has obvious advantages compared to the DPD-unknown and hence holds a good robustness. However, the DPD-enhanced algorithm suffered performance loss compared to the DPD-known algorithm if the SNR is extremely low. Further work is currently underway to consider the scenario with multiple emitters transmitting the different unknown signals.

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