

# Merged Characteristic Basis Function Method for Analysis of Electromagnetic Scattering Characteristics from Conducting Targets

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**Abstract**—In this paper, a merged characteristic basis function method (MCBFM) is proposed to analyze the electromagnetic scattering characteristics from conducting targets. A merged characteristic basis function (M-CBF) is newly defined in the MCBFM. Considering the mutual interaction of surrounding blocks, the M-CBF is generated by merging the conventional secondary characteristic basis functions (SCBFs) and the high order characteristic basis functions (HO-CBFs) of each block in the conventional primary characteristic basis function (PCBF). Thus, the true current distribution of the targets is approached by using a single M-CBF reducing the number of CBFs when the incident plane waves (PWs) are certain. The numerical results of a PEC hexahedron demonstrate that the proposed MCBFM improves the accuracy without increasing the number of PWs and the CBFs compared to the improved primary CBFM (IP-CBFM). The results also demonstrate that the MCBFM is capable of effectively reducing the CPU time by 63.38% without losing any accuracy compared to the conventional characteristic basis function method (CBFM). Other results of a PEC cylinder demonstrate that when a considerable computational accuracy is required, the efficiency of the proposed MCBFM is the highest among these three methods.

## 1. INTRODUCTION

The method of moments (MoM) has been widely applied to accurate solution of electromagnetic problems. However, it places a considerable burden on the computational time and memory requirements when electrically large problems are analyzed. The iterative techniques, such as the fast multipole method (FMM) [1] and multilevel fast multipole method (MLFMM) [2, 3], are known as effective analysis techniques to improve the efficiency of the MoM. But the involvement of Bessel and Legendre functions in these methods makes them complicated and difficult to implement. Moreover, the dependency of incident field makes the FMM and MLFMM unsuitable for the analysis of monostatic problems.

In recent years, the characteristic basis function method (CBFM) has been proposed [4, 5] as a fast, stable, and simple algorithm for the large scale and ill-conditioned problem. This method can be performed using the direct method for matrix calculation. In the CBFM, the scatterer is divided into multiple blocks, and characteristic basis functions (CBFs) are generated in each block. The number and the size of blocks are known to be the key parameters. The dimension of reduced matrix is determined by the number of blocks. While dealing with electrically large scattering problems, the number of unknowns in each block should be increased in order to keep low number of blocks. Although the small dimension of reduced matrix reduces the solving time of reduced equation, it also rapidly reduces the computational efficiency of the generation of impedance matrix and the construction of the reduced matrix. Meanwhile, it is known that there is a tradeoff relationship between the precision

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*Received 15 March 2017, Accepted 27 April 2017, Scheduled 30 June 2017*

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and calculation time [6]. Several studies have been conducted for the accuracy, calculation time and memory improvement of the CBFM. Singular value decomposition (SVD) has been introduced to reduce the lengthy CBFs [7, 8]. The adaptive cross approximation (ACA) algorithm has been introduced to accelerate the generation of the reduced matrix [9]. Multiple plane waves (PWs) with two orthogonal polarizations are used as the incident electromagnetic fields [10, 11]; however, in large scale scattering problems, the incident electromagnetic fields in multiple directions generate an extremely large number of CBFs, and it takes a long time to perform SVD procedure. Limiting the range of incidence angles can reduce the number of generated CBFs. However, decreasing the CPU time by this approach has a negative impact on accuracy. The secondary CBFs (SCBFs) are proposed in [12] to decrease the number of the incident PWs without reducing the accuracy of the results. The high order CBFs is proposed to enhance the accuracy of the CBFM, and a connected patch array is analyzed using the CBFM along with the HO-CBFs [13]. Besides, the HO-CBFs combined with a volume integral equation is used in the analysis of various antennas in the vicinity of a dielectric object. The HO-CBFs provide results accurately even if a block division is arbitrary [14]. However, the uses of SCBFs and HO-CBFs result in problematic number of CBFs. An improved primary CBFM (IP-CBFM) is proposed in [15], which reduces the amount of memory used for reduced matrix by combining the SCBFs with the PCBF. However, this approach also reduces the precision compared to the conventional CBFM.

In order to resolve the above mentioned problems, this paper proposes a merged characteristic basis function method (MCBFM) that realizes the efficiency and accuracy synchronously. Taking the mutual interaction of surrounding blocks into account in the primary CBFs, the SCBFs and HO-CBFs of each block are merged in the PCBF to generate a merged characteristic basis function (M-CBF). The proposed MCBFM achieves better efficiency than the conventional CBFM without losing any accuracy. Moreover, the MCBFM is more accurate than the IP-CBFM.

The remainder of the paper is organized as follows. In Section 2, the conventional CBFM and IP-CBFM are briefly reviewed first. Then the formulation of the MCBFM is described in detail. Section 3 provides some numerical results about the monostatic radar cross section (RCS) to demonstrate the accuracy and efficiency of the MCBFM. Finally, the conclusions are drawn in Section 4.

## 2. FORMULATION

### 2.1. Review of the Conventional CBFM

In this section, the computation procedure of the conventional characteristic basis function method (CBFM) is briefly described. Further details about the CBFM can be found in [4]. In the CBFM, the target is divided into  $M$  blocks. The CBFs of each block consist of two parts: the primary CBF (PCBF) and secondary CBFs (SCBFs).

The PCBF refers to the self-interaction component in the block. Suppose the number of incident PWs is  $N_{\text{PWs}}$  ( $N_{\text{PWs}} = 2N_\phi N_\theta$ ), where  $N_\phi$  and  $N_\theta$  represent the numbers of PWs in directions of  $\phi$  and  $\theta$ , respectively. Two polarization modes are considered, and the incident field is denoted as  $\mathbf{E}_{ii}^{e, N_{\text{PWs}}}$ . The PCBF of the block  $i$  is generated as follow:

$$\mathbf{Z}_{ii}^e \mathbf{J}_{ii}^e = \mathbf{E}_{ii}^{e, N_{\text{PWs}}} \quad (1)$$

where  $\mathbf{Z}_{ii}^e$  represents the self-impedance matrix for the extended block  $i$ , with  $i = 1, 2, \dots, M$ ;  $\mathbf{E}_{ii}^{e, N_{\text{PWs}}}$  is an excitation matrix, and  $\mathbf{J}_{ii}^e$  is the response to the corresponding excitation. Eq. (1) can be solved directly by the LU decomposition, and then the PCBF  $\mathbf{J}_{ii}^b$  of block  $i$  is obtained by removing the components of  $\mathbf{J}_{ii}^e$  corresponding to the overlapping segments. Superscripts  $e$  and  $b$  stand for the extended and un-extended blocks, respectively.

It is known that a higher precision solution can be obtained by considering the SCBFs [4]. The SCBF is the mutual interaction component between blocks  $i$  and  $j$ . The SCBF is generated as:

$$\mathbf{Z}_{ij}^e \mathbf{J}_{ij}^e = -\mathbf{Z}_{ij}^{e'} \mathbf{J}_{jj}^{b'} \quad (2)$$

where  $\mathbf{Z}_{ij}^{e'}$  is a part of the mutual impedance matrix  $\mathbf{Z}_{ij}^e$ , and  $\mathbf{Z}_{ij}^e$  represents the mutual interaction of extended block  $i$  and un-extended block  $j$ . The dimensions of  $\mathbf{Z}_{ij}^{e'}$  are  $N_i^e \times N_j^{e'}$ , where  $N_i^e$  is the

number of unknowns in the extended block  $i$ ;  $N_j^{e'} = N_j - N_{ij}^{\text{overlapping}}$ ;  $N_j$  is the number of unknowns in the un-extended block  $j$  and  $N_{ij}^{\text{overlapping}}$  the number of unknowns corresponding to the overlapping segments between extended block  $i$  and un-extended block  $j$ .  $\mathbf{J}_{jj}^{b'}$  is a part of the PCBF  $\mathbf{J}_{jj}^b$ , and its dimensions are  $N_j^{e'} \times 1$ . The SCBF  $\mathbf{J}_{ij}^e$  is obtained by removing the extended components of  $\mathbf{J}_{ij}^e$ .

## 2.2. Review of the IP-CBFM

It is easy to know from Subsection 2.1 that the number of CBFs in each block is  $M$  under each incident PW, one PCBF and  $M - 1$  SCBFs. When  $N_{\text{PWs}}$  PWs are set to irradiate to each block, the total number of the CBFs is  $M^2 \times N_{\text{PWs}}$ . That is an enormous number of CBFs which significantly increases the CPU time and memory requirement. The number of CBFs can be reduced by limiting the number of incident PWs. However, decreasing the CPU time by this approach has a negative impact on the accuracy.

To resolve this problem, an improved primary characteristic basis function method (IP-CBFM) is proposed in [14]. The IP-CBFM reduces the number of CBFs by combining the SCBFs with PCBF. The improved primary CBF (IP-CBF) of each block is generated using Eqs. (1) and (2) as follows:

$$\mathbf{Z}_{ii}^e \mathbf{J}_{ii}^e + \sum_{\substack{j=1 \\ j \neq i}}^M \mathbf{Z}_{ii}^e \mathbf{J}_{ij}^e = \mathbf{Z}_{ii}^e \left( \mathbf{J}_{ii}^e + \sum_{\substack{j=1 \\ j \neq i}}^M \mathbf{J}_{ij}^e \right) = \mathbf{E}_{ii}^{e, N_{\text{PWs}}} - \sum_{\substack{j=1 \\ j \neq i}}^M \mathbf{Z}_{ij}^{e'} \mathbf{J}_{jj}^{b'} \quad (3)$$

$$\mathbf{Z}_{ii}^e \mathbf{J}_{ii}^{e, \text{IP}} = \mathbf{E}_{ii}^{e, N_{\text{PWs}}} - \sum_{\substack{j=1 \\ j \neq i}}^M \mathbf{Z}_{ij}^{e'} \mathbf{J}_{jj}^{b'} \quad (4)$$

The IP-CBF  $\mathbf{J}_{ii}^{b, \text{IP}}$  of block  $i$  can be obtained by removing the extended components of  $\mathbf{J}_{ii}^{e, \text{IP}}$ . The total number of CBFs is reduced to  $M \times N_{\text{PWs}}$ . Thus, the IP-CBFM realizes the reduction of CBFs. However, the precision is also reduced compared to that of the conventional CBFM.

## 2.3. The Formulation of the MCBFM

This subsection presents the merged characteristic basis function (MCBFM) to enhance the accuracy of the IP-CBFM without increasing the number of PWs and CBFs. A merged CBF (M-CBF) is newly defined. Considering the high-order mutual interaction of surrounding blocks, the M-CBF is generated by merging the conventional secondary characteristic basis function (SCBFs) and the high-order characteristic basis functions (HO-CBFs) of each block in the conventional primary characteristic basis function (PCBF).

First, the HO-CBFs are generated from the sum of corresponding SCBFs as follows:

$$\mathbf{Z}_{ii}^e \mathbf{J}_{ij}^{e, \text{HO}} = -\mathbf{Z}_{ij}^{e'} \sum_{\substack{j'=1 \\ j' \neq j}}^M \mathbf{J}_{jj'}^{b'} \quad (5)$$

where  $i = 1, 2, \dots, M$ ;  $j = 1, 2, \dots, (i-1), (i+1), \dots, M$ , and  $j' = 1, 2, \dots, (j-1), (j+1), \dots, M$ . Let's take  $i = 1$  as an example to clearly describe the construction of the HO-CBFs. In Eq. (5), when  $i = 1$ ,  $j$  should be  $j = 2, 3, \dots, M$ . The HO-CBFs of extended block  $i$  are  $\mathbf{J}_{12}^{e, \text{HO}}, \mathbf{J}_{13}^{e, \text{HO}}, \dots, \mathbf{J}_{1M}^{e, \text{HO}}$ , respectively. While solving  $\mathbf{J}_{12}^{e, \text{HO}}$  ( $i = 1, j = 2$ ),  $\mathbf{J}_{21}^{b'}, \mathbf{J}_{23}^{b'}, \dots, \mathbf{J}_{2M}^{b'}$  are used in the term  $\mathbf{J}_{jj'}^{b'}$  at the right side of Eq. (5), at this moment  $j' = 1, 3, \dots, M$ .

Then the M-CBF of each block is generated based on Eqs. (1), (2) and (5) as follows:

$$\mathbf{Z}_{ii}^e \mathbf{J}_{ii}^e + \sum_{\substack{j=1 \\ j \neq i}}^M \mathbf{Z}_{ii}^e \mathbf{J}_{ij}^e + \sum_{\substack{j=1 \\ j \neq i}}^M \mathbf{Z}_{ii}^e \mathbf{J}_{ij}^{e,\text{HO}} = \mathbf{E}_{ii}^{e, N_{\text{pws}}} - \sum_{\substack{j=1 \\ j \neq i}}^M \mathbf{Z}_{ij}^{e'} \mathbf{J}_{jj}^{b'} - \sum_{\substack{j=1 \\ j \neq i}}^M \mathbf{Z}_{ij}^{e'} \sum_{\substack{j'=1 \\ j' \neq j}}^M \mathbf{J}_{jj'}^{b'} \quad (6)$$

$$\mathbf{Z}_{ii}^e \left( \mathbf{J}_{ii}^e + \sum_{\substack{j=1 \\ j \neq i}}^M \mathbf{J}_{ij}^e + \sum_{\substack{j=1 \\ j \neq i}}^M \mathbf{J}_{ij}^{e,\text{HO}} \right) = \mathbf{E}_{ii}^{e, N_{\text{pws}}} - \sum_{\substack{j=1 \\ j \neq i}}^M \mathbf{Z}_{ij}^{e'} \mathbf{J}_{jj}^{b'} - \sum_{\substack{j=1 \\ j \neq i}}^M \mathbf{Z}_{ij}^{e'} \sum_{\substack{j'=1 \\ j' \neq j}}^M \mathbf{J}_{jj'}^{b'} \quad (7)$$

$$\mathbf{Z}_{ii}^e \mathbf{J}_{ii}^{e, \text{M-CBF}} = \mathbf{E}_{ii}^{e, N_{\text{pws}}} - \sum_{\substack{j=1 \\ j \neq i}}^M \mathbf{Z}_{ij}^{e'} \mathbf{J}_{jj}^{b'} - \sum_{\substack{j=1 \\ j \neq i}}^M \mathbf{Z}_{ij}^{e'} \sum_{\substack{j'=1 \\ j' \neq j}}^M \mathbf{J}_{jj'}^{b'} \quad (8)$$

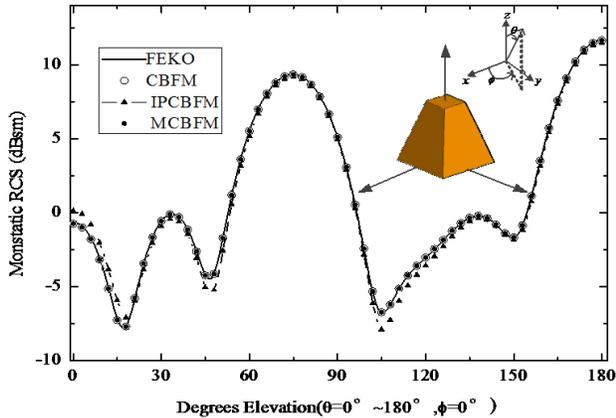
The M-CBF  $\mathbf{J}_{ii}^{e, \text{M-CBF}}$  of block  $i$  can be obtained by removing the extended components of  $\mathbf{J}_{ii}^{e, \text{M-CBF}}$ . The total number of M-CBFs is still  $M \times N_{\text{pws}}$ . It can be observed from Eq. (8) that the HO-CBFs do not need to be solved. Since the mutual interaction of surrounding blocks is properly considered in the M-CBF, the high precision can also be obtained when the same number of PWs is set to irradiate each block. Moreover, lower number of PWs is required in the MCBFM than the IP-CBFM when a considerable computational accuracy is required. It should be explained that the generation time of the M-CBFs will increase slightly, but it is negligible relative to the overall time.

Lastly, it is desirable to use the SVD procedure to remove the redundancy in the obtained M-CBFs before constructing the reduced matrix. Since the number of M-CBFs is much lower than the CBFs generated by the CBFM, the time of SVD procedure is also saved in the MCBFM.

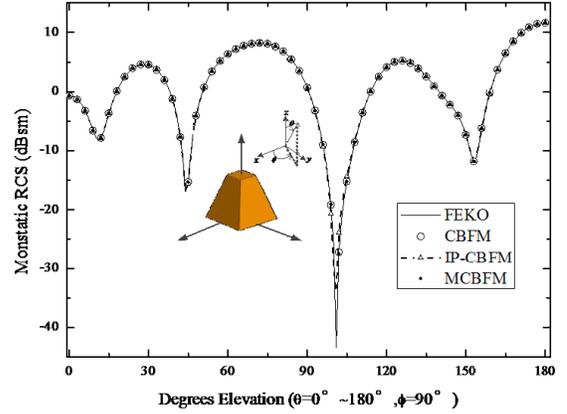
### 3. NUMERICAL RESULTS

In order to verify the validity of the proposed method, the MCBFM is applied to different samples to calculate the monostatic RCS. The obtained results are compared with the those obtained by the CBFM, IP-CBFM and the simulation of FEKO. The simulations are performed on a personal computer equipped with Intel<sup>®</sup> Core<sup>™</sup> i7-3820 at 3.60 GHz and 64 GB RAM. The full impedance matrix is stored in the computer memory, and the threshold of the SVD is 1.0E-3.

First, the scattering problem of a PEC hexahedron at a frequency of 300 MHz is presented. The electrical lengths of bottom and top sides are  $1\lambda$  and  $0.4\lambda$ , respectively, and  $1\lambda$  high. The hexahedron is divided into 1598 triangular patches with total 4491 unknowns. All the unknowns are assigned to



**Figure 1.** Monostatic RCS of a PEC hexahedron in  $HH$  polarization.



**Figure 2.** Monostatic RCS of a PEC hexahedron in  $VV$  polarization.

8 blocks. The results of monostatic RCS in  $HH$  and  $VV$  polarization calculated by the CBFM, IP-CBFM, MCBFM and the simulation results of FEKO are presented in Fig. 1 and Fig. 2, respectively. The conditions required for the three methods are shown in Table 1.  $N_\theta, N_\phi$  indicate the number of incident PWs in each direction.  $\Delta\theta, \Delta\phi$  indicate the angle intervals of the incident PWs. The calculation time for different procedures and relative errors are summarized in the Table 2.

**Table 1.** The condition of incident PWs for the three methods.

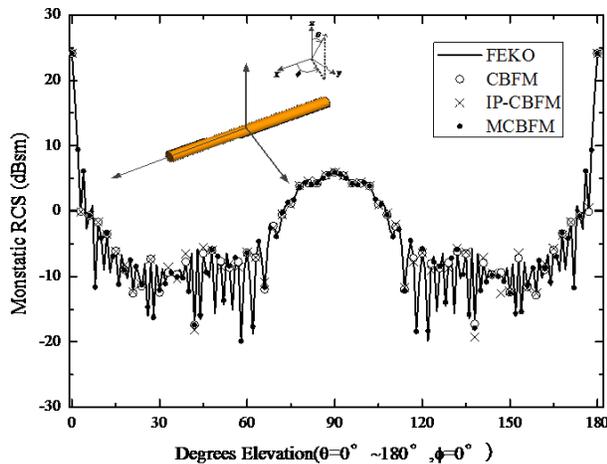
Method	$(N_\theta, N_\phi)$	$(\Delta\theta, \Delta\phi)$	Number of PWs
CBFM			
IP-CBFM	(9, 4)	$(36^\circ, 90^\circ)$	72
MCBFM			

**Table 2.** The calculation time of different procedures and the relative errors.

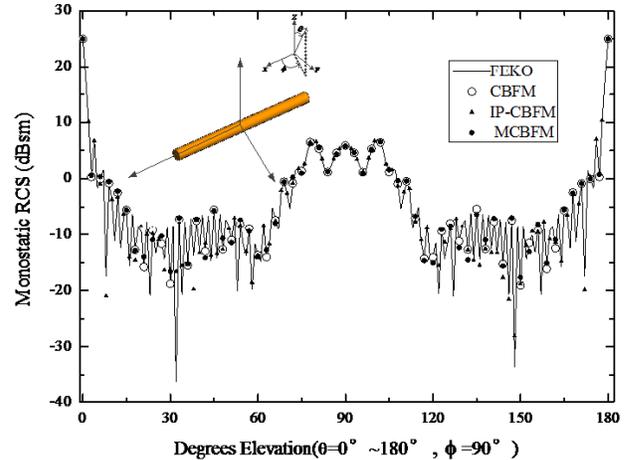
Methods	CBFs generation time (s)	SVD time (s)	Reduced matrix construction time (s)	Solving time (s)	Total Time (s)	Number of CBFs (After SVD)	Relative errors (%)
FEKO	-	-	-	-	131.37	-	-
CBFM	21.53	42.21	255.25	7.72	329.71	778	0.701%
IP-CBFM	14.79	10.67	69.84	5.49	100.79	416	3.043%
MCBFM	27.55	5.58	82.04	5.58	120.75	448	0.704%

The relative error is defined as  $(|RCS_x - RCS_{FEKO}| / |RCS_{FEKO}|) \times 100\%$ , where  $RCS_{FEKO}$  are the simulation results from the software FEKO, and  $RCS_x$  are the results of the three methods. It can be easily observed from Fig. 1 and Table 2 that the proposed MCBFM is more accurate than the IP-CBFM, and the relative error is reduced by 2.3%. The generation time of the CBFs is slightly increased by the MCBFM, but the overall total time is reduced by 63.38% compared to the CBFM.

In order to further demonstrate the accuracy and the efficiency of the MCBFM, the results for the scattering problem from one PEC cylinder at a frequency of 600 MHz are presented. The electrical size of the cylinder is  $18\lambda$  length and  $0.6\lambda$  radius. The cylinder is divided into 12802 triangular patches, and



**Figure 3.** Monostatic RCS of a PEC Cylinder in  $HH$  polarization.



**Figure 4.** Monostatic RCS of a PEC Cylinder in  $VV$  polarization.

the total number of unknowns is 26686. All the unknowns are assigned to 25 blocks. The monostatic RCS in  $HH$  and  $VV$  polarizations calculated by the CBFM, the IP-CBFM and the MCBFM with considering different numbers of PWs are presented in Fig. 3 and Fig. 4, respectively. The conditions required for these three methods are shown in Table 3.

**Table 3.** The condition for the three methods.

Method	$(N_\theta, N_\phi)$	$(\Delta\phi, \Delta\theta)$	Number of PWs	Relative errors (%)
CBFM	(11, 5)	$(30^\circ, 72^\circ)$	110	< 1%
IP-CBFM	(15, 6)	$(22.5^\circ, 60^\circ)$	180	
MCBFM	(11, 5)	$(30^\circ, 72^\circ)$	110	

It can be seen from Fig. 3 and Fig. 4 that the results obtained by the three methods are in good agreement with the simulation results of FEKO. When the relative error is less than 1%, the number of PWs required in the IP-CBFM is more than that in the CBFM and MCBFM. The CPU time and number of CBFs for the three methods are summarized in the Table 4.

**Table 4.** The calculation time of different procedures for the three methods.

Methods	CBFs generation time (s)	SVD time (s)	Reduced matrix construction time (s)	Solving time (s)	Total Time (s)	Number of CBFs (After SVD)
FEKO	-	-	-	-	9612.17	-
CBFM	578.78	10803.8	11743.5	231.10	23357	4263
IP-CBFM	301.19	315.28	5785.61	146.33	6548.41	2964
MCBFM	855.31	116.76	4968.13	85.09	6025.18	2731

It can be seen easily seen from Table 3 that when a considerable computational accuracy is required, the efficiency of the proposed MCBFM is the highest among these three methods.

#### 4. CONCLUSION

In this paper, a merged characteristic basis function method (MCBFM) is proposed to analyze the electromagnetic scattering characteristics from conducting targets. First, a merged characteristic basis functions (M-CBF) is defined and generated by merging the conventional secondary characteristic basis functions (SCBFs) and the high order characteristic basis functions (HO-CBFs) of each block in the conventional primary characteristic basis function (PCBF). Then the proposed MCBFM is used to calculate the monostatic RCS in different polarizations of a PEC hexahedron and a PEC cylinder. The numerical results validate and demonstrate that the proposed MCBFM significantly reduces the CPU time without losing the accuracy compared to the conventional CBFM. Moreover, the proposed method improves the accuracy without increasing the number of PWs and CBFs compared to the IP-CBFM. Hence, the proposed MCBFM is much more efficient than the other two methods when a considerable computational accuracy is required.

#### ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China under Grant No. 61172020 and No. 61401003; the Specialized Research Fund for the Doctoral Program of Higher Education of China under Grant No. 20123401110006; the Natural Science Research Project of Anhui Education under Grant No. KJ2015A436.

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