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A Corrected Method to Extract Dielectric Parameters from Transmission Lines with Conductor Surface Roughness at Terahertz Frequencies

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Abstract—“Curve-fitting” method is an important method to extract dielectric parameters of substrate materials from planar transmission lines. At gigahertz frequencies, effective conductivity concept is adopted to model the conductor’s surface roughness effects in planar transmission lines, and differential extrapolation method is used to remove surface roughness effects. However, such a concept and method lose their accuracy at extremely high frequency such as terahertz waves. This paper details some new limitations in the terahertz regime and proposes corrections in calculating effective conductivity with rough conductor and curve-fitting method for transmission performance characterization in eliminating the effects of surface roughness. The proposed method is validated by simulation data for conductivity with parallel plate waveguide model, and the corrected method presented here can effectively extract dielectric parameters with an error less than 7%.

1. INTRODUCTION

In radar and communication field, full knowledge about dielectric parameters is important for design and analysis of microwave components. Methods for measuring dielectric parameters can mainly be categorized in two groups: transmission methods and resonator methods [1]. For a given transmission line, the propagation parameters and resonant frequencies are determined by dielectric parameters of material. Therefore, dielectric parameters can be derived from measured propagation constant in transmission methods and from measured resonant frequencies in resonator methods. Compared with transmission methods, resonator methods are usually more accurate but can only extract dielectric parameters at a few discrete resonant frequencies [2]. In order to obtain dielectric parameters over a wideband frequency, transmission methods are most preferred.

There is a variety of existing transmission methods to extract dielectric parameters. The transmission structures range from rectangular waveguide [3] to coaxial line [4] or stripline [5], among which transmitting parameters like propagation constants or S parameters are measured. All of these methods have in common that dielectric parameters are calculated explicitly from given formulas at microwave frequencies.

Different from the aforementioned traditional transmission methods, Zhang proposed a “curve-fitting” method which calculates dielectric parameters by estimating coefficients of dispersive dielectric model [6]. The curve-fitting method loses its accuracy if the influences of rough conductor surface aren’t considered in transmission line. To solve this problem, Huang and Jia use differential extrapolation method to remove effects of conductor’s surface roughness [7], and the extraction error is less than 5% on the GHz frequency region. However, because modeling method of conductor’s surface roughness and removal of its effects are no longer effective at extremely high frequencies, accuracy of parameters

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extraction decreases significantly. To solve this problem, herein, the treatment of conductor's surface roughness is reconsidered in terahertz band, and two modifications of the original method are proposed in the present paper. Section 2 gives a brief introduction to Huang's original method, in which the problems appearing in modeling of conductor's surface roughness and removal of surface roughness' effects in terahertz band are highlighted. To address these issues encountered at extremely high frequencies, methods and implementations are given and verified in Section 3. In Section 4, a simple example of dielectric parameters extraction in terahertz band is given with a parallel plate waveguide model. Section 5 summarizes this paper.

2. PROPOSED APPROACH OF DIELECTRIC PARAMETERS EXTRACTION

2.1. General Procedure for Dielectric Parameters Extraction

A flowchart of dielectric parameters extraction procedure is described in Figure 1. On one hand, attenuation constant α_T and phase constant β are firstly obtained from measured or simulated data for transmission line containing conductor surface roughness. Then, differential extrapolation method [5] is adopted to remove the influence of surface roughness, thus the experimental or simulated solutions of α_T and β for smooth surfaces can be obtained. On the other hand, the Debye dispersive model for substrate material is assumed, and with the help of theoretical formulas, analytical solutions of α_T and β are calculated for the same transmission line model as that of experimental or simulated solutions. To determine the unknown Debye model's coefficients in analytical model, a genetic algorithm (GA) based curve-fitting method is used. By minimizing objective function which describes the gap between analytical solutions and experimental or simulated solutions, Debye model's coefficients are estimated, and dielectric parameters including permittivity and loss tangent can be obtained immediately.

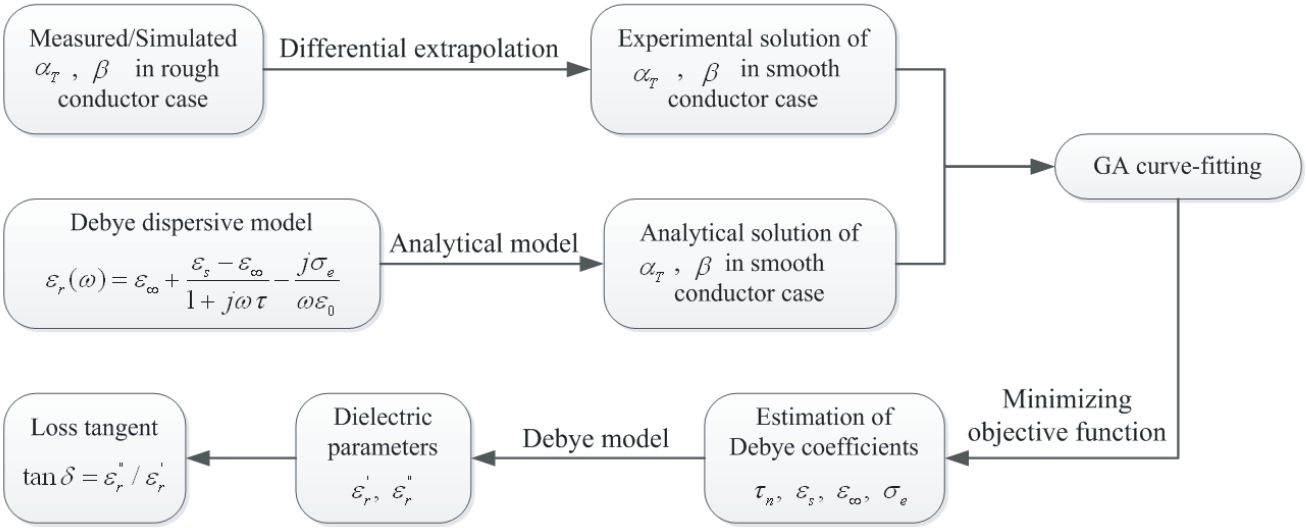


Figure 1. Flowing chart of dielectric parameters extraction procedure.

2.2. Characterization of Conductor Surface Roughness

To obtain the experimental or simulated solutions of α_T and β with smooth surface conductor, modeling method of conductor's surface roughness and removal of its influence on propagation constants need to be considered. This subsection mainly discusses modeling method of conductor's surface roughness and its limitations in terahertz band.

The interactions between the surface roughness and the passing waves depend on the operating frequency and the corresponding skin penetration depth. Actually the passing waves do not "see" individual peaks and pits of the rough surface, but rather the "mean" plane surface. Hence, it is sufficient to model the surface roughness as long as the root-mean-square (RMS) value of surface profile is on or larger than the order of the skin depth. As illustrated in Eq. (1), conductivity gradient model which describes variation of conductivity for a rough surface conductor is firstly introduced to model conductor's surface roughness, where σ_b is the bulk conductivity, R_q the RMS of peak-to-valley roughness amplitude, and CDF$(\cdot)$ the cumulative distribution function of the probability density function (PDF) of surface roughness [8].

$$\sigma_{rough}(x) = \sigma_b \mathrm{CDF}(x) = \sigma_b \frac{1}{\sqrt{2\pi} R_q} \int_{-\infty}^{x} e^{-\frac{u^2}{2R_q^2}} du \tag{1}$$

Concept of effective conductivity is then brought in to facilitate the calculation of rough surface conductor loss. This concept assumes a virtual smooth conductor which has the same conductor loss as the actual rough conductor, shown in Eq. (2), where $\sigma_{rough}(x)$ is the gradient conductivity, σ_{eff} the equivalent conductivity of virtual smooth conductor, and $\bar{\mathbf{J}}_{rough}$ and $\bar{\mathbf{J}}_{smooth}$ are current density in rough conductor and virtual smooth conductor respectively. Equivalent conductivity of such virtual smooth conductor can be obtained with the following Equation (2).

$$\int \frac{\left|\bar{\mathbf{J}}_{rough}(x)\right|^2}{\sigma_{rough}(x)} dx = \int \frac{\left|\bar{\mathbf{J}}_{smooth}(x)\right|^2}{\sigma_{\mathit{eff}}} dx \tag{2}$$

Conductivity gradient model relies on the hypothesis that conductor's bulk conductivity is constant, which is no longer correct at extremely high frequencies. In terahertz band, conductor shows dispersive behavior with frequency, and its conductivity can be characterized by Drude model:

$$\sigma(\omega) = \frac{\sigma_0}{1 + j\omega\tau} \tag{3}$$

The dispersive conductivity model indicates that calculation formula of conductor loss needs to be revised again. Since effective conductivity is based on the sense of "equivalent conductor loss", results of effective conductivity must be corrected when the dispersive conductivity model is introduced in terahertz band.

2.3. Removal of Conductor Surface Roughness Effects

Conductor surface roughness mainly increases attenuation constant α and has little influence on phase constant β [5]. Differential extrapolation method [9] can effectively remove this additional increase on attenuation constant. This method supposes that both conductor and dielectric contribute to the total attenuation constant α_T. The conductor part α_c is proportional to $\sqrt{\omega}$ term, and the dielectric part α_D is proportional to a sum of ω and ω^2 terms at microwave frequencies, as shown in Eqs. (4) and (5). Detailed derivations of these frequency-dependent properties can be referenced in [5].

$$\alpha_c = A_1\sqrt{\omega} \tag{4}$$

$$\alpha_D = A_2\omega + A_3\omega^2 \tag{5}$$

Therefore, total attenuation constant α_T can be curve-fitted by Eq. (6).

$$\alpha_T = A_1\sqrt{\omega} + A_2\omega + A_3\omega^2 \tag{6}$$

When surface roughness is considered, the curve-fitting form does not change, but the fitting coefficients A_i $(i = 1, 2, 3)$ vary with R_q, i.e., the RMS of peak-to-valley roughness amplitude. Each A_i can be plotted as a function of R_q, and such a curve can be extrapolated to the smooth case where $R_q = 0$. Using the extrapolated smooth coefficients, attenuation constants corresponding to the smooth case can be acquired.

It should be noticed that the frequency-dependent law in Eq. (4) only holds if the conductor conductivity is constant. When frequency increases significantly in terahertz band, conductivity becomes dispersive, and Eq. (4) is no longer correct. Besides, to derive Eq. (5), some approximations which are reasonable only in gigahertz band are made, and in terahertz band, frequency-dependent law of Eq. (5) will not hold either.

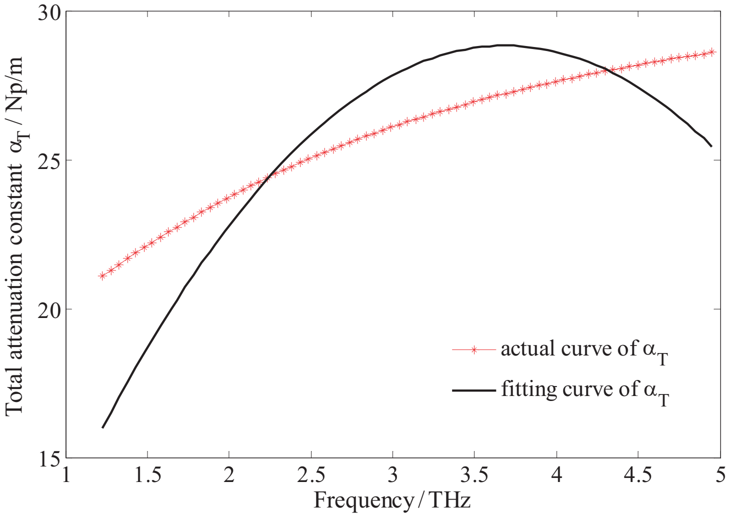


Figure 2. Actual and fitting curves of α_T in terahertz band.

Therefore, when the frequency is extremely high to terahertz band, total attenuation constant α_T cannot be curve-fitted by the form in Eq. (6), and differential extrapolation method will also become invalid. Taking a smooth parallel plate waveguide for example, Figure 2 gives its fitting results of total attenuation constant α_T for TM_1 mode with $A_1\sqrt{\omega} + A_2\omega + A_3\omega^2$ form. It can be observed that in terahertz band, α_T cannot be curve-fitted by the form in Eq. (6). To remove effects of conductor's surface roughness in terahertz band, a new curve-fitting form needs thus to be considered.

3. CORRECTIONS OF DIELECTRIC PARAMETERS EXTRACTION METHOD IN TERAHERTZ BAND

It has been presented in Section 2 that when frequency becomes extremely high, calculations of effective conductivity and differential extrapolation method at microwave frequencies are no longer valid. This section mainly gives their corrections in terahertz band, and verifications of these corrections are also provided.

3.1. Corrections of Surface Roughness Modeling Method in Terahertz Band

As indicated in Eq. (3), conductor's conductivity becomes frequency-dependent in terahertz band. Because of the dispersion characteristic of conductor, two modifications need to be made during the calculation of effective conductivity for rough surface conductor. The first one is the correction of conductivity gradient model. Conductivity in Eq. (1) needs to be replaced by Eq. (3), and the new conductivity gradient model becomes:

$$\sigma_{rough}(x) = \frac{\sigma_0}{1+j\omega\tau}\frac{1}{\sqrt{2\pi}R_q}\int_{-\infty}^{x} e^{-\frac{u^2}{2R_q^2}}du \tag{7}$$

In terahertz band, Eq. (7) should be used to solve the current density $\bar{\mathbf{J}}$ inside the conductor.

The second correction is the calculation of conductor loss. In Eq. (2), conductor loss P is calculated by:

$$P = \int \frac{|\bar{\mathbf{J}}|^2}{\sigma}dx \tag{8}$$

When conductivity becomes complex, according to Joule's law and in reference to the relationship of $\bar{\mathbf{J}} = \sigma\bar{\mathbf{E}}$, conductor loss becomes:

$$P = \mathrm{Re}\left\{\int \bar{\mathbf{E}}\cdot\bar{\mathbf{J}}^* dx\right\} = \mathrm{Re}\left(\frac{1}{\sigma}\right)\int |\bar{\mathbf{J}}|^2 dx \tag{9}$$

In terahertz band, Eq. (9) should be used to calculate the effective conductivity.

In order to verify the revised surface roughness modeling method in terahertz band, the effective conductivity calculated is compared with actual measured conductivity. Taking gold for example, Yang et al. give its measured conductivity in 400 GHz and 650 GHz when R_q equals 160 nm, 190 nm and 210 nm, respectively [10]. Besides, Drude coefficients τ and σ_0 are also provided in [10]. With these coefficients, effective conductivity is calculated, and comparisons are shown in Table 1 and Table 2. All the conductivities are normalized to smooth gold conductivity.

Table 1. Comparison of normalized measured conductivity and effective conductivity in 400 GHz.

	Smooth	$R_q = 160\,\text{nm}$	$R_q = 190\,\text{nm}$	$R_q = 210\,\text{nm}$
Measured conductivity (normalized)	1	0.48	0.32	0.28
Effective conductivity (normalized)	1	0.46	0.41	0.35

Table 2. Comparison of normalized measured conductivity and effective conductivity in 650 GHz.

	Smooth	$R_q = 160\,\text{nm}$	$R_q = 190\,\text{nm}$	$R_q = 210\,\text{nm}$
Measured conductivity (normalized)	1	0.43	0.35	0.28
Effective conductivity (normalized)	1	0.39	0.35	0.29

Comparisons in Table 1 and Table 2 show that actual measured conductivity and effective conductivity are very close to each other, especially in 650 GHz. One source of the difference between these two conductivities is that the effective conductivity requires the PDF of the profile of surface roughness to be Gaussian distributed, and such a hypothesis may not exactly hold in actual measured case. However, the close agreement between these two conductivities still validates the corrections of effective conductivity for rough surface conductor in terahertz band.

3.2. Corrections of Differential Extrapolation Method in Terahertz Band

First of all, frequency-dependent laws of conductor attenuation constant α_c and dielectric attenuation constant α_D need to be modified. Propagation constant γ of imperfect conductors can be written as [11]:

$$\gamma = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}} \tag{10}$$

To derive the frequency-dependent law of α_c, we substitute the dispersion conductivity model σ in Eq. (3) into Eq. (10), and α_c can finally be formulated as:

$$\alpha_c = \mathrm{Re}(\gamma) = \sqrt{\frac{\sigma_0\mu}{2}}\sqrt{\frac{\omega}{1+(\omega\tau)^2}} \cdot \left[\sqrt{\frac{\sqrt{(\omega\tau)^2+1}+1}{2}} + \sqrt{\frac{\sqrt{(\omega\tau)^2+1}-1}{2}}\right] \tag{11}$$

For dielectric material, its propagation constant γ can be written as [11]:

$$\gamma = j\omega\sqrt{\mu\varepsilon_0\varepsilon_r} \tag{12}$$

At high frequency, Debye model is used to describe dielectric material's dispersive behavior [12]:

$$\varepsilon_r(\omega) = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1+j\omega\tau} - \frac{j\sigma_e}{\omega\varepsilon_0} \tag{13}$$

where ε_s, ε_∞ are static dielectric constant and relative permittivity at high-frequency respectively, and σ_e is the effective conductivity of dielectric materials, τ the characteristic relaxation time of the medium, ε_0 the permittivity of free-space.

Replace the relative permittivity ε_r in Eq. (12) with Eq. (13), and after mathematical approximations in terahertz band, dielectric attenuation constant α_d can finally be expressed by:

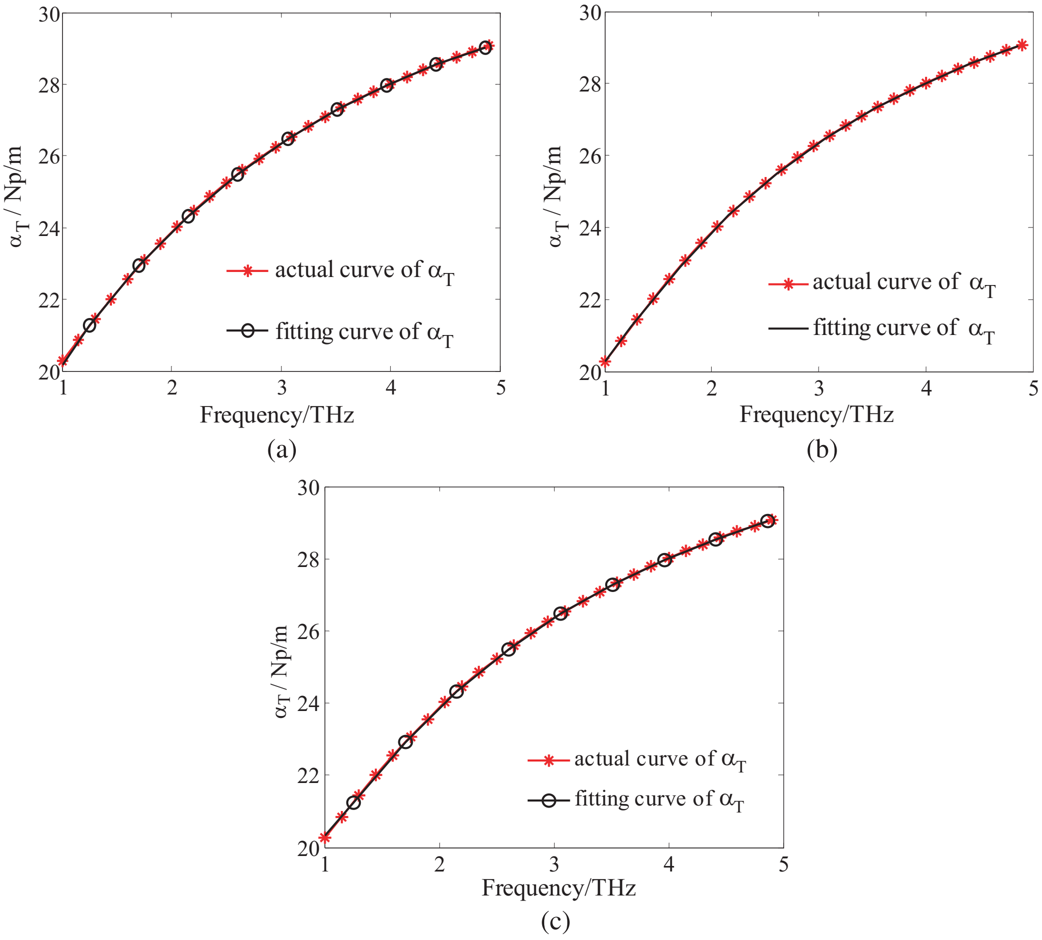
$$\alpha_d \approx \frac{\sqrt{\mu\varepsilon_\infty\varepsilon_0}}{2}\frac{(\varepsilon_s-\varepsilon_\infty)\tau}{\varepsilon_\infty}\frac{\omega^2}{1+(\omega\tau)^2} \tag{14}$$

Notice that in terahertz band, Eqs. (11) and (14) have already been the simplified forms. Therefore, it is difficult to find one physically reasonable curve-fitting type like the one in Eq. (6). Differential extrapolation method requires that the total attenuation constant α_T to be curve-fitted by a frequency-dependent formula. In order to find such a curve-fitting formula, three mathematical forms are considered: the polynomial form in Eq. (15), the Fourier form in (16) and the exponential form in Eq. (17).

$$\alpha_T(\omega) = \sum_{k=0}^{n} b_k\omega^k \tag{15}$$

$$\alpha_T(\omega) = a_0 + \sum_{k=1}^{n}\left[a_k\cos(kf_0\omega) + b_k\sin(kf_0\omega)\right] \tag{16}$$

$$\alpha_T(\omega) = \sum_{k=1}^{n} a_k e^{b_k\omega} \tag{17}$$



(a)

(b)

(c)

Figure 3. Fitting curve of α_T with different mathematical form (a) of exponential form, (b) of Fourier form, (c) of polynomial form.

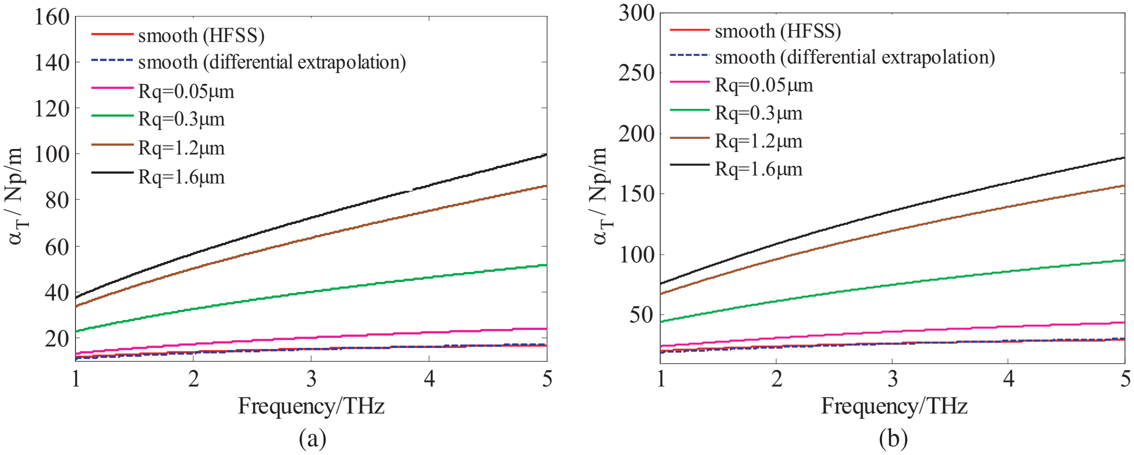
where $a_k(k \in [0, n])$ in Eq. (15), $a_0, a_k, b_k(k \in [1, n])$ and f_0 in Eq. (16), $a_k, b_k(k \in [1, n])$ are all fitting parameters, which will be determined through the experimental or simulation data α_T.

Taking the same parallel plate waveguide as the one corresponding to Figure 2 for example, Figure 3 gives its fitting results of α_T for TM_1 mode with 8-order polynomial form, 3-order Fourier form and 2-order exponential form, respectively. The number of points used in the fit is 10 for the three forms.

The fitting results in Figure 3 show that α_T can be well curve-fitted by all the three proposed mathematical forms. Although these forms do not have clear physical meanings, they can still be used to remove the surface roughness effects on attenuation. This is because differential extrapolation method relies on the hypothesis that the decrease process of α_T is continuous if R_q decreases continuously. Coefficients of the fitting form should also vary continuously.

To verify the hypothesis mentioned above, a parallel plate waveguide model is used. The upper and lower conductor plates are made of gold, whose R_q for conductor surface equals 0.05 μm, 0.3 μm, 1.2 μm and 1.6 μm, respectively. The dielectric material filled in the parallel plate waveguide is FR4, and the simulated frequencies range from 1 THz to 5 THz. Since the fit process with 8-order polynomial form has better fitting accuracy than that of the other two forms, it is finally used to fit the total attenuation constant α_T, and herein only TEM mode and TM_1 mode are analyzed.

Figure 4 gives α_T for both the rough and smooth conductor surfaces as well as the extrapolated smooth surface α_T for TEM mode and TM_1 mode. It can be observed that the smooth surface α_T obtained from software simulation and the smooth surface α_T obtained from differential extrapolation method almost coincide for both modes. Actually, the maximal relative errors of α_T for smooth surface between the software simulation and the differential extrapolation method are 4.85% and 6.8% for TEM mode and TM_1 mode, respectively, which indicates that in terahertz band, the differential extrapolation method can still remove conductor's surface roughness effects effectively through a change of curve-fitting form.



(a) (b)

Figure 4. Comparison of α_T obtained from different methods. (a) TEM mode. (b) TM_1 mode.

4. EXAMPLE OF DIELECTRIC PARAMETERS EXTRACTION IN TERAHERTZ BAND

As the parallel plate waveguide model is simple and its propagation constant can be predicted by analytical solution, it is used in this section to give an example of dielectric parameters extraction in terahertz band. Dielectric filled in the parallel plate waveguide is FR4, and the conductor plates are gold. Distance between the two conductor plates is 0.2 mm.

The extraction process in gigahertz frequency has been described in Section 2. At terahertz band, to obtain experimental solutions, the Debye-type FR4 and the Drude-type gold are imported in HFSS software, where coefficients of Debye model and Drude model are in reference to Zhang's measured data [6] and Yang's measured data [10]. These materials are used to define the parameters

Table 3. Parameters for Debye dielectric model and Drude conductor model.

Parameters	τ_{Debye}	ε_s	ε_∞	σ_e	τ_{Drude}	σ_0
Specified value	3.74×10^{-11}	4.41	4.24	3.65×10^{-3}	2.713×10^{-14}	4.517×10^7
Predicted value	3.884×10^{-11}	4.310	4.243	3.258×10^{-3}	2.420×10^{-14}	4.206×10^7
Relative error	3.85 %	2.26%	0.07%	10.7%	12.1%	6.88%

of parallel plate waveguide model. Besides, the corrected effective conductivity is also used to model rough conductors. By simulation and with the help of corrected differential extrapolation method, extrapolated propagation constant for smooth surface conductor can be obtained. Methods of acquiring analytical solution of α_T and estimating Debye and Drude coefficients keep unchanging as those at microwave frequencies [7]. The comparison of the specified and predicted value for Debye and Drude parameters are shown in Table 3. In GA optimization, the number of individuals in each generation is 300; the ratio of variation for the adjacent generations is 20%; the maximum iteration times are 100. When the value of optimization objective function is less than 10^{-6}, or when the five consecutive generations get the same "elite" solution, or when the maximum iteration times is reached, the search is stopped.

The final extracted dielectric parameters are given in Figure 5, where the real part of relative permittivity ε_r' and loss tangent $\tan\delta$ are considered. The extracted dielectric parameters are also compared with the actual ones defined in HFSS software.

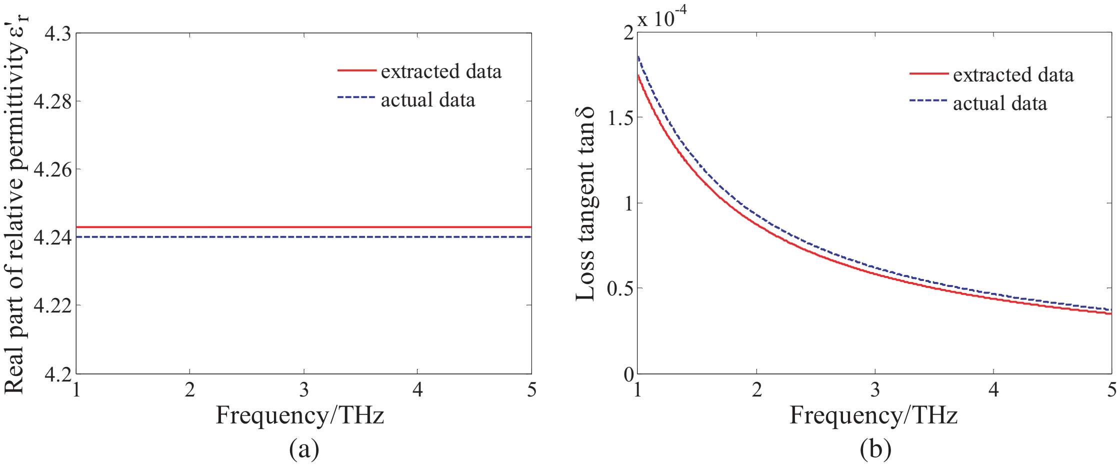


Figure 5. Extracted dielectric parameters at terahertz frequencies (a) for ε_r', (b) for $\tan\delta$.

It can be observed in Figure 5 that discrepancies between the extracted parameters and the actual data are small for both ε_r' and $\tan\delta$. To analyze quantitatively, in terahertz band, the relative extraction error of ε_r' is less than 0.1%, and that of $\tan\delta$ is less than 7%. Even though the errors in extracting dielectric parameter are a little greater than the ones in gigahertz frequencies [7], the results can still be accepted.

5. DISCUSSION AND CONCLUSION

In this paper, a "curve-fitting" dielectric parameters extraction method is reconsidered in terahertz band. Compared with the original method at microwave frequencies [7], two corrections are made. The first one is modeling method of conductor's surface roughness. In this correction, a dispersive conductor model is introduced and applied in calculation of effective conductivity with conductivity gradient model for roughness surface. This correction is verified by comparison with actual measured conductivity of rough conductors. The second corrected point is the elimination of surface roughness' effects. At terahertz frequencies, as conductor becomes dispersive, the frequency-dependent curve-fitting form for attenuation parameter needs to be changed. An 8-order polynomial curve-fitting form is finally used in differential extrapolation method and proved to be able to remove surface roughness' effects effectively. Considering these two corrections, the extracted dielectric parameters are less than 7% in terahertz band for a given parallel plate structure filled with FR4 material.

Due to limitations of materials and measurement equipment, experimental solutions of total attenuation constant α_T and phase constant β are obtained from software simulation. However, because the corrected effective conductivity is proved to have a close agreement with actual measured conductivity, the proposed corrected method is practical to real materials and can be applied to various types of transmission lines as long as the measured random errors are eliminated in advance at such high frequencies. More researches and experiments need to be conducted to give further support of this proposed method in practical applications.

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