

High Resolution Radar Focusing Using Spectral Estimation Methods in Wide-Band and Near-Field Configurations: Application to Millimeter-Wave Near-Range Imaging

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Abstract—The spatial resolution of an imaging system is a key factor, which steers its performance for complex target detection, characterization and recognition. Active electromagnetic imaging systems with limited frequency bandwidth and synthetic aperture may fail to discriminate important details during the imaging process, due to their insufficient resolution properties. Spectral estimation methods may be used to overcome such limitations through dedicated signal processing techniques. This study proposes a new signal processing chain, which is able to cope with near-field and wide-band configurations, to significantly improve 2-D resolution, using classical spectral estimation methods. This work is based on the efficient handling and compensation of critical signal properties, such as near-field and large bandwidths, which make the proposed technique able to deal with very general imaging configurations, such as near/far-range, narrow/wide-beamwidths and -bandwidths, very short aperture... Experimental results obtained at millimeter-wave are shown to demonstrate the performance and versatility of the proposed approach.

1. INTRODUCTION

Synthetic Aperture Radar (SAR) combines 1-D or 2-D spatial diversity with spectral diversity to produce 2-D or 3-D maps of the electromagnetic reflectivity of environments. Exact focusing methods in time or frequency domain such as the Back-Projection [1,2] or the Range migration (ω - k) inversion [3–5] algorithms may be used to transform the coherent acquired raw data into images. The achieved spatial resolution is inherently limited by the processed frequency band and by the synthetic aperture dimensions. To overcome this limitation, imaging techniques based on inverse scattering methodology [6] or spectral estimation methods are used to improve spatial resolution, that is a key factor for target detection and recognition. Spectral estimation methods consider the problem of determining the spectral content of a finite noisy set of measurements, by means of either parametric or nonparametric techniques [7,8]. The spectral estimation methods considered here are covariance matrix-based algorithms, which have been originally adapted to array processing, according to a data model based on several assumptions, corresponding to the plane-wave (far-field region) and the narrow-frequency bandwidth configurations [9,10]. The data model considers that the region of interest is located relatively far from the radar system so that the spherical wave-front arriving upon the synthetic aperture is considered as a plane wave. It also assumes that the transmitted signals have a narrow frequency bandwidth. In such cases, the signal history of a point-scatterer is considered as a 2D-sinusoid where the phase variation is considered as a linear function of the scatterer's position.

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However, when the model is compromised, compensations are required to give full capability to spectral estimation methods. In the literature, specific spectral estimation methods are modified to consider the wide-band or near-field configurations as in [11–15]. Firstly, for a wide-band far-field configuration, the distance variation between one target and the full aperture may exceed the range resolution of the system. The target response is going to spread over multiple range cells leading to a reduction of the capability of spectral estimation methods. Because the spectral estimation methods considered uses the covariance matrix from one particular range of the SAR data, the selected vector provides only a portion of the phase history of the target response. This therefore reduces the capability of spectral estimation methods to discriminate closely spaced targets. The wide-band compensation algorithm in [16, 17] has the ability to cancel out the range cell migrations that occur by means of a spatial re-sampling of the SAR data. Then the spectral estimation methods are applied on the compensated SAR data.

Secondly, for a narrow-band near-field configuration, the wave phase-front cannot be considered as a plane phase-front without producing severe errors and distortions, since phase history is no longer a linear function. Near-field compensation algorithms [18–20] locally compensate for the spherical phase-front over the aperture. The estimation problem consists in solving a set of two non-linear functions (the range and the Direction of Arrival (DoA)). The compensation is done using the exact geometry of the problem. Once the compensation is performed, spectral estimation methods are applied on the resulting SAR data.

However, where the SAR data acquired in a wide-band and near-field configuration, the spherical wave-front curvature generates nonlinear range cell migrations, whose imperfect compensation, using the plane wave assumption, generates residual range shifts whose magnitude might be comparable to the range resolution, and whose distribution over the aperture highly depends on the location of a scatterer and on the acquisition geometry. This variability prevents a generic correction procedure. Similarly, near-field phase patterns cannot be written under a convenient and generic formalism, i.e., the phase distribution over the aperture not only depends on a target azimuth, but on its range position too.

To avoid this, a solution is to perform spectral estimation methods from SAR images focused using an exact technique as in [21–24]. The near-field and wide-band compensations that were previously needed are no longer necessary. Then spectral estimation methods are directly applied on the resulting compensated complex SAR image.

Most of the SAR configurations encountered in the litterature [25] are data processed at zero doppler, i.e., all the scatterers are seen by the radar over a symmetric angular domain with a similar range of the observation angles and focused SAR image are then well represented over a Cartesian (range-azimuth) grid.

However, in a fan-beam configuration [26], where the dimension of the synthetic aperture is smaller than the area that is imaged, multiple targets may be measured over different ranges of observation angles, i.e., each scatterer is seen over a non-symmetric angular domain, where the median value depends on the acquisition geometry. As a result, such an information is much better represented using polar coordinates, rather than a Cartesian coordinates. This effect can be well observed on focused images where sidelobe target responses spread along a quasi-circular trajectory.

This property whose validity is based on the fact that the synthetic aperture is lower than the scene is used in this paper to significantly improve high-resolution focusing results thanks to spectral estimation methods.

First, the SAR geometry configuration and theory to build SAR raw data are presented in Section 2.1. The undesired effects of four configurations (Far-field and narrow-band/Far-field and wide-band/near-field and narrow-band/near-field and wide-band) on a particular point-like target located at a squint angle from broadside are demonstrated, by simulation, in Section 2.2. The authors exhibit the undesired effects before and after applying the near-field, wide-band or both compensation methods from the literature on the four configurations. It is revealed that in the near-field wide-band configuration, working directly on the raw data is not suitable to give full capability to spectral estimation methods. Then in Section 2.3, a SAR focusing technique (the back-projection algorithm), to reconstruct a focusing SAR image and to be able to use spectral estimation methods even for squint angle configuration, is applied. The projection over a Cartesian and a Polar grid is shown, and spectral estimation methods are applied and compared for the two projections. The remainder of this paper is organized as follows.

Section 3 presents the data model, some spectral estimation methods and a flowchart of the proposed processing algorithm. Finally, these spectral estimation methods are applied on measured SAR data at millimeter wave with a wide frequency bandwidth and a large aperture in Section 4.

2. NEAR-FIELD/WIDE-BAND COMPENSATION

2.1. Geometry Configuration

Consider the case where a transmitting antenna is located at a fixed position in a 3-dimensional spatial space in a Cartesian coordinate system (see Fig. 1). The location of its phase center is (x_t, y_t, z_t) . A receiving synthetic array is located over an aperture of length L and aligned with the x -axis. The location of each phase center is (x_a, y_a, z_a) with $x_a \in [-L/2, L/2]$. The radiation pattern of each element is considered as isotropic. N_s point scatterers are considered and identified by their Cartesian coordinates (x_i, y_i, z_i) . The index i is for the i th point scatterer P_i . The equivalent distance from the radar to the i th point scatterer is given by:

$$\begin{aligned} d_i(x_a) &= (dT x_i + dR x_i)/2 \\ &= \left(\sqrt{(x_t - x_i)^2 + (y_t - y_i)^2 + (z_t - z_i)^2} + \sqrt{(x_a - x_i)^2 + (y_a - y_i)^2 + (z_a - z_i)^2} \right) / 2 \end{aligned} \quad (1)$$

A transmitted baseband signal $u(\tau)$ in the time domain with its counterpart in the frequency domain $U(f)$ is used, with a frequency diversity over a frequency bandwidth B_f . The signal has been transposed around the carrier frequency f_c before being sent through the medium. The received signals along the aperture $S_r(x_a, f)$ in Eq. (2) are modeled as a sum of transmitted signals, which are weighted and delayed. The complex weighting s_i represents all the attenuations that occur during the round-trip propagation and the reflectivity of the i th point-like target. The delays arise from the round-trip distance determined in Eq. (1).

$$S_r(x_a, f) = \sum_{i=1}^{N_s} s_i U(f) e^{-j4\pi(f+f_c)d_i(x_a)/c} + n(x_a, f) \quad (2)$$

with c being the speed of light and $n(x_a, f) \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2)$ being a Gaussian white noise with zero mean and variance σ^2 . The focused received signals $(S(x_a, f))$ by adapted filtering [1] along the aperture is expressed as:

$$S(x_a, f) = S_r(x_a, f) U^*(f) = \sum_{i=1}^{N_s} s_i H(f) e^{-j4\pi(f+f_c)d_i(x_a)/c} + n_f(x_a, f) \quad (3)$$

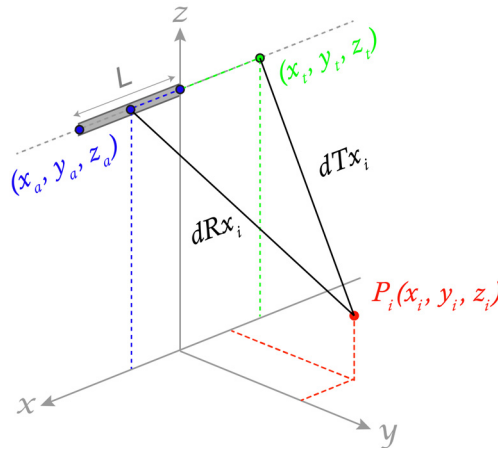


Figure 1. Geometry of the imaging RADAR configuration.

where $H(f)$ is the resulting transfer function which defines the properties of the focused signal and $n_f(x_a, f)$ the filtered white noise by adapted filtering. $(\cdot)^*$, $(\cdot)^t$ and $(\cdot)^H$ are the conjugate, transpose and conjugate transpose operator, respectively.

Its counterpart in the wavenumber domain is expressed as:

$$S(x_a, k) = \sum_{i=1}^{N_s} s_i H(k) e^{-j(k+k_c)d_i(x_a)} + n_f(x_a, k) \quad (4)$$

The round-trip carrier wavenumber is described as $k_c = 4\pi f_c/c$, whereas the baseband wavenumber domain is covered by the signal spectrum such as $k = 4\pi f/c$.

The range focused received signals in the spatial domain $s(x_a, d)$ after an inverse Fourier transform along the wavenumber domain, is expressed as:

$$s(x_a, d) = \sum_{i=1}^{N_s} s_i h(d - d_i(x_a)) e^{-j\varphi_x(x_a)} + n_f(x_a, d) \quad (5)$$

$h(d)$ is the range ambiguity function in the spatial domain. Where the transmitted signal has a flat spectrum over the frequency band B_f and zero elsewhere, the range ambiguity function corresponds to a sinc function characterized by its range resolution (δ_r) that is inversely proportional to the frequency band used such as $\delta_r = \frac{c}{2B_f}$.

$\varphi_x(x_a) = k_c d_i(x_a)$ corresponds to the phase variation along the aperture.

2.2. Near-Field and Wide-Band Configurations

The range focused received signals $s(x_a, d)$ have exact phase history information about the area of interest. Nonetheless, approximations may be used in various cases.

A particular target is considered to be in a far-field region if it is located far enough from the radar so that the backscattered spherical wave arriving upon the aperture can be considered as a plane wave. It is highly dependent on the range between the target and the aperture length. A simple rule of thumbs is that a backscattered spherical wave from a broadside target is considered as a plane wave if $d_i(x_a = 0) > (2L^2)/(\lambda_c)$ with $x_a = 0$ the center of the receiving array. It corresponds to a phase variation along the aperture lower than $\pi/8$ with λ_c the wavelength at the carrier frequency.

Four different configurations are studied and simulated with the corresponding simulated parameters of each configuration detailed in Table 1. In the case of a far-field and narrow-band configuration (see Fig. 2(a)), the range and cross-range focusing are linked to a Fourier transform thanks to the linear phase variations that occur.

Table 1. Simulation parameters for the different configurations (FF: Far-field, NF: Near-field, NB: Narrow-band and WB: Wide-band).

Parameter [unit]	FF/NB	NF/NB	FF/WB	NF/WB
f_c/B_f [GHz]	50/0.5	50/0.5	50/20	50/20
L [m]	0.2	0.2	0.2	0.6
$d(x_a = 0)$ [m]	50	1.5	50	1.5
Farfield criterion: $2l^2/\lambda_c$ [m]	13.3	13.3	13.3	120
Narrowband criterion: $c/(2L \sin \theta_M)$ [GHz]	2.2	2.2	2.2	0.73
Nonlinear range migration condition: $(L - 4\delta_r^2)/(8\delta_r)$ [m] if $\delta_r < L/2$	$\delta_r > L/2$	$\delta_r > L/2$	0.66	6

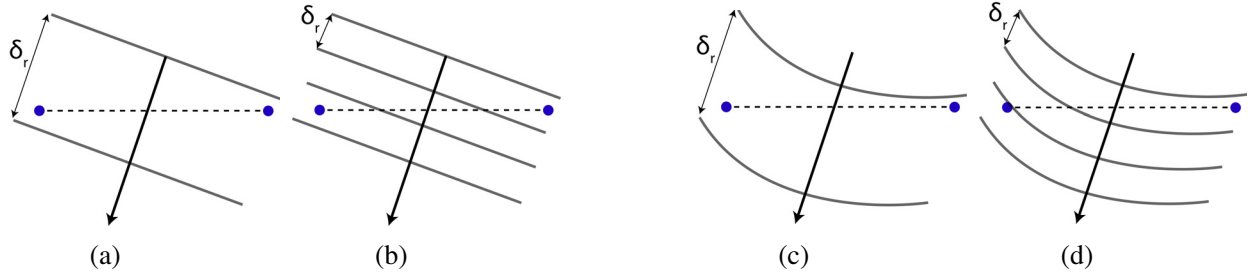


Figure 2. Sketch of the propagation of the range resolution front for a point-like target in (a) far-field narrow-band configuration, (b) far-field wide-band configuration, (c) near-field narrow-band configuration, (d) near-field wide-band configuration. The blue dots represent the extreme sides of the aperture represented by the dashed line. The arrow gives the angle of incidence of the wave.

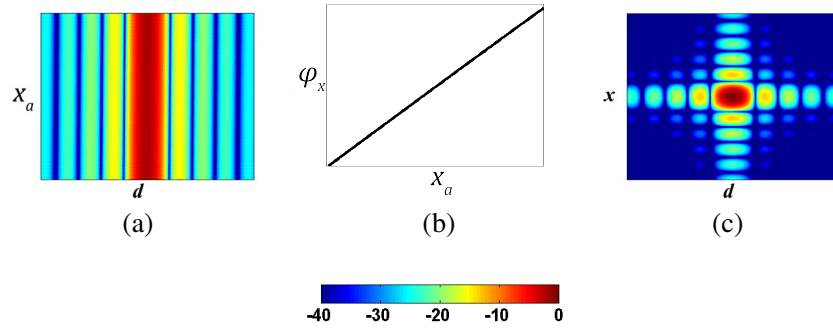


Figure 3. Simulation results of the matrix $\mathbf{S}$ for a point-like target in a far-field narrow-band configuration after (a) the range focusing and (c) the range and the cross-range focusing (Fourier SAR image without compensations). (b) shows the phase variation along the aperture.

Figure 3 shows a simulated raw data matrix for a far-field narrow-band configuration. It has been simulated using only one point-like target located at an angle $\theta_i = 20^\circ$ from broadside with θ defining the cross-range look-angle. The intensity of each image is normalized, expressed in decibels, to peak at 0 dB and clipped at -40 dB. It has to be noticed that the final images are centered on the 2D point-spread function of the point-like target.

Figure 3(a) shows the raw data after range focusing in Eq. (5). It permits to show that no range migration occurs.

The spatial frequency domain $k_x = [(k_c + k) \sin \theta]/2$ corresponds to the projection of the wavenumber $(k_c + k)$ on the aperture plane. It is also referred as the slow-time frequency. Because the point-like target is located relatively far from the aperture ($d_i(x_a) \gg L$), the phase variation along the aperture is considered linear (see Fig. 3(b)). By taking the center of the synthetic array ($x_a = 0$) as a reference and by means of the first order Taylor expansion, the phase variation can be approximated by

$$\varphi_x(x_a) = k_c d_i(x_a) \approx k_c (d_i(0) - x_a \sin \theta_i/2) \quad (6)$$

Figure 3(c) shows the raw data matrix after range and cross-range focusing using a 2-D Discrete Fourier Transform (DFT). The point-like target response is then properly focused. Using Eq. (6) in Eq. (5) and omitting the noise yields:

$$s(x_a, d) = \sum_{i=1}^{N_s} \alpha_{c_i} h(d - d_i(x_a)) e^{-jk_c x_a \sin \theta_i/2} \quad (7)$$

with $\alpha_{c_i} = s_i e^{-jk_c d_i(0)}$.

For a given angular sector that is illuminated (i.e., $|\theta_i| \leq \theta_M$), a Radar is considered in a narrow-band condition if the variation of the range Radar ambiguity function ($h(d - d_i(x_a))$) along the receiving

aperture is lower than the range resolution (δ_r), avoiding any range cell migration. In other words, the phase shift generated by the back-scattered plane wave from a point-like target located at an angle θ_M from broadside must be lower than δ_r . It yields the following criterion: $B_f < c/(2L \sin \theta_M)$.

In the narrow-band case, $k + k_c \approx k_c$, that allows for removing the frequency dependency in Eq. (4).

Contrarily, in a wide-band configuration (see Fig. 2(b)), the exponential term depends on both the wavenumber frequency $k + k_c$ and the receiving antenna location x_a . Using the wide-band compensation algorithm in [16, 17], the wavenumber frequency dependency can be eliminated using spatial re-sampling, such as:

$$x_a = \frac{k_c - 2\pi B_f/c}{k + k_c} \tilde{x}_a \quad (8)$$

Substituting the expression of $\tilde{x}_a$ into Eq. (4) and according to the approximation in Eq. (6) gives:

$$\tilde{S}(\tilde{x}_a, k) = \sum_{i=1}^{N_s} \alpha_{c_i} H(k) e^{-jkd_i(0)} e^{j(k_c - 2\pi B_f/c) \tilde{x}_a \sin \theta_i/2} \quad (9)$$

This removes the frequency dependency of the signal, and by means of interpolation, it permits the rearrangement of the raw data in such a way that one particular target response is seen in the same range cell along the aperture. Fig. 4 corresponds to the far-field wide-band configuration. In this particular configuration, range migrations occur (see Fig. 4(a)). Hence, the point-like target response is shared among multiple range cells. The selection of one particular range row, where the target is present, gives partial information about the target response. The target response is not properly focused as shown in Fig. 4(c). Red dashed lines are added in Fig. 4(b) to show the location where a range migration occurs. It can also be used for the others configurations.

After applying the wide-band compensation using Eq. (9), the range migration is removed (see Fig. 4(d)) that allows for properly focusing the point-like target response (see Fig. 4(f)).

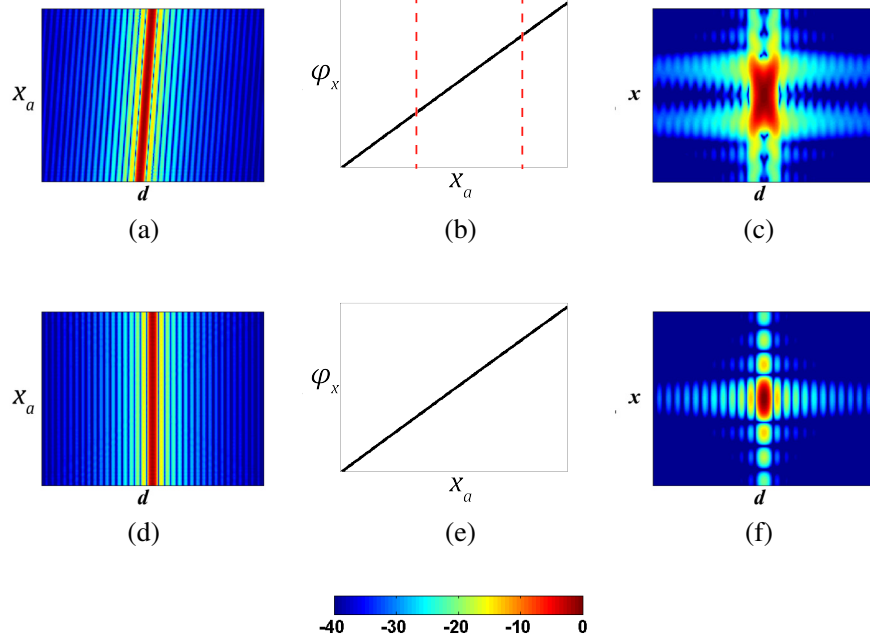


Figure 4. Simulation results of the raw data matrix $\mathbf{S}$ in the far-field wide-band configuration (a), (b), (c) without the wide-band compensation and (d), (e), (f) with the wide-band compensation. The results in (a) and (d) are shown after the range focusing and (c), (f) after the range and the cross-range focusing (Fourier SAR image). (b) and (e) show the phase variation along the aperture after the range focusing.

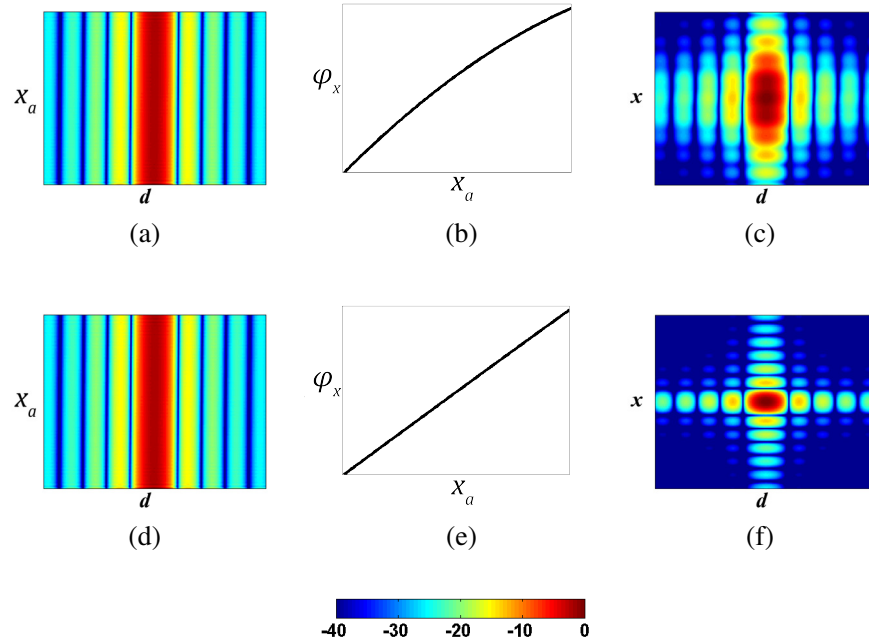


Figure 5. Simulation results of the raw data matrix $\mathbf{S}$ in the near-field narrow-band configuration (a), (b), (c) without the near-field compensation and (d), (e), (f) with the near-field compensation. The results in (a) and (d) are shown after the range focusing and (c), (f) after the range and the cross-range focusing (Fourier SAR image). (b) and (d) show the phase variation along the aperture after the range focusing.

As regards the near-field narrow-band configuration (see Fig. 2(c)), the linear phase variation approximation in Eq. (6) is not valid anymore. In Figs. 5(a) & 5(d), no range migration occurs, but the nonlinear phase shift generated by the spherical wave-front shown in Fig. 5(b) does not allow to properly focus the point-like target response (see Fig. 5(c)). Removing the spherical component, by only keeping the linear phase shift variation from the plane wave assumption, is desired. However, the near-field compensation has to be determined for each SAR measurement cell. In fact, each target location has its own wave-front that is dependent on the pair of a range and a cross-range location. At one particular location $\hat{P}_i(\hat{x}_i, \hat{y}_i, \hat{z}_i)$, the distance variation along the aperture $\hat{d}_i(x_a)$ is determined using Eq. (1). The phase shift generated by the distance variation is then removed from the received raw data in Eq. (5) and replaced by a linear phase variation from the plane-wave assumption determined by the angular location of $\hat{P}_i$ from broadside (i.e., $\hat{\theta}_i$).

$$\hat{s}(x_a, d) = s(x_a, d) e^{jk_c \hat{d}_i(x_a)} e^{-jk_c x_a \sin \hat{\theta}_i / 2} \quad (10)$$

Figure 5(e) shows the linear phase variation after near-field compensation. After being focused in range and cross-range, the point-like target is properly focused (see Fig. 5(f)).

Finally, Figs. 2(d) & 6 correspond to the near-field wide-band case. The range migration in Fig. 6(a) is a combination of the linear range migration due to the wide-band signal used and a nonlinear range migration due to the near-field location of the target. According to Fig. 6(c), it is apparent that the target response is not properly focused. Figs. 6(c) & 6(f) & 6(i) show the point-like target responses without compensations, with the wide-band compensation and with wide-band and near-field compensations, respectively. Exploiting the two previous algorithms together does not properly focus the point-like target due to the nonlinear range migration generated by targets in a near-field environment. The nonlinear range migration from a broadside target in near-field occurs if $d_i(x_a = 0) < (L^2 - 4\delta_r^2)/(8\delta_r)$ and $\delta_r < L/2$.

In the following section, focusing SAR techniques, such as the back-projection algorithm, is used to consider the exact geometry of the problem and to combine coherently the received signals to properly focus the point-like targets.

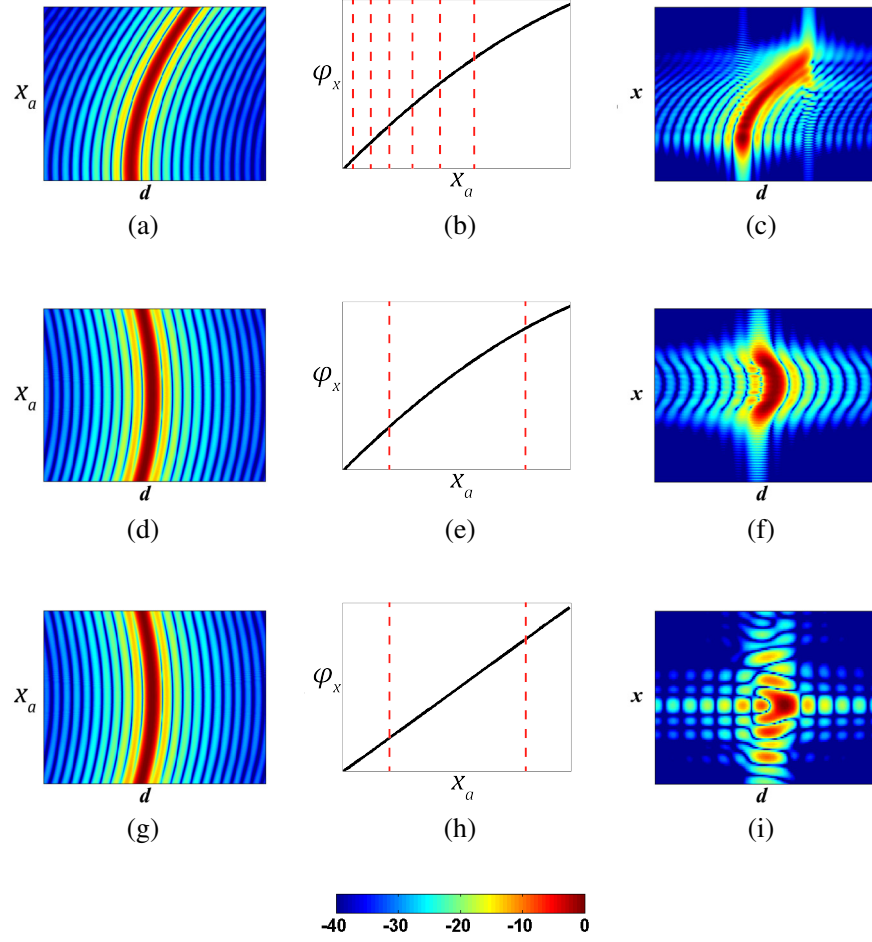


Figure 6. Simulation results of the raw data matrix $\mathbf{S}$ in the near-field wide-band configuration after (a), (d), (g) the range focusing and after (c), (f), (i) the range and the cross-range focusing (Fourier SAR image). (b), (e) and (h) show the phase variation along the aperture after the range focusing. In (a), (b), (c), no compensation occurs. In (d), (e), (f), only the wide-band compensation is applied, and finally in (g), (h), (i), the near-field and wide-band compensations are applied.

2.3. Compensation Using Focusing Techniques

To compensate for the near-field and wide-band behaviors on the raw-data, a focusing technique is employed to project the received signals over a 2D Cartesian grid to have an estimate of the complex 2D reflectivity field. The 2D plane, which is regularly sampled, usually follows a Cartesian grid having the origin located at the center of the synthetic receiving array. Each pixel corresponds to a 2D spatial area of size δ_y and δ_x with δ_y and δ_x perpendicular and parallel to the array aperture respectively. The i th pixel p_i has a spatial coordinate $(x_i = \alpha\delta_x, y_i = \beta\delta_y)$ with α and β real numbers. The regular grid is shown in Fig. 7(a).

The back-projection algorithm is used for generation of the focused SAR image $f(x, y; z = z_0)$ with z_0 a constant. In what follows, the projection is performed over a 2D-plane at one particular height from the synthetic aperture. For one particular range cell, it takes the received signal from a given position along the aperture x_a in Eq. (5) and back-projects it over a spherical arc corresponding to all the possible contributing image pixels. Once the back-projection is performed on the remainder received signals from the other ranges and the others positions along the aperture, then accumulated, the focused SAR image is obtained. One particular pixel p_i of the focused SAR image spanning the

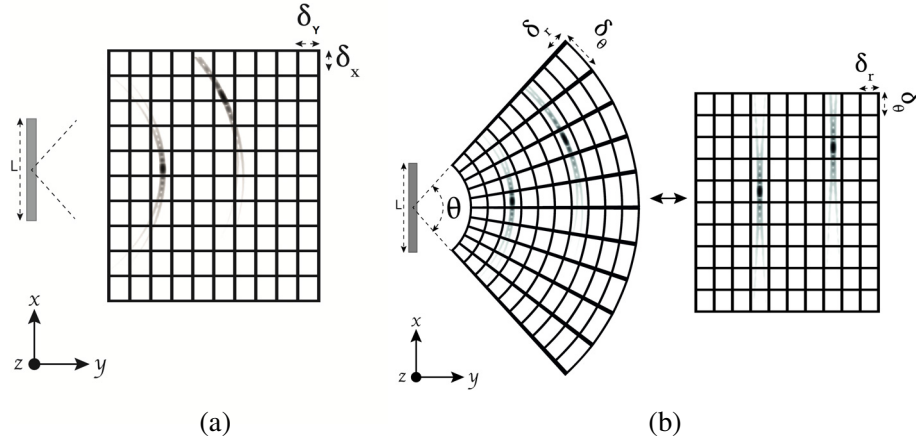


Figure 7. Imaging geometry of the synthetic aperture RADAR with the received raw data focused over (a) a 2D Cartesian grid, (b) a 2D polar grid.

Cartesian grid is constructed by:

$$\hat{f}(x_i, y_i; z_i = z_0) = \int_{x_a = -L/2}^{L/2} s(x_a, d_i(x_a)) e^{jk_c d_i(x_a)} dx_a \quad (11)$$

Spectral Estimation Methods may be applied to the focused SAR image to improve the spatial resolution. However, because the two point-like scatterers are in a near-field wide-band configuration, the 2D ambiguity function of each point scatterer spreads on multiple rows and multiple columns (see Fig. 7(a)). Extracting the phase history of each point scatterer by the selection of one row or one column is not valid anymore as only partial information is selected.

To overcome this issue, a better solution, in the authors' opinion, is to project the received signals over a polar grid that follows the 2D ambiguity function of point-like scatterers anywhere in the considered area from the radar system perspective. The polar coordinates are considered with nonuniform sampling of the range projected over a 2D plane and a nonuniform sampling of the angle taking into account the decrease of cross-range resolution at squint angles. Such sampling can be transformed to Cartesian coordinates that are dense near the array and sparse away from the array. Hence, the i th pixel p_i covers a 2D spatial area of size δ_r the range resolution and δ_θ the angular resolution with spatial coordinates over the Cartesian coordinates ($x_i = r_i \cos \theta_i$, $y_i = r_i \sin \theta_i$), as shown in Fig. 7(b). By doing so, the selection of one particular row or column gives full information about the target behavior. Because the 2D SAR image resolution is inherently limited by the frequency band used and the synthetic aperture dimensions, the polar grid is then critically sampled to match the SAR system resolution, reducing the size of the 2D SAR image to avoid long time calculation during the spectral estimation method process.

Figure 8(a) shows the projection of the range resolution δ_r over the 2D-plane. As it is the projection of the range resolution over the ground in SAR configuration, it is named as ground-range resolution. As regards a synthetic aperture located at a height $z_0 = H$ from the 2D plane, the ground range resolution (δ_{rg}) at the i th pixel location is defined as:

$$\delta_{rg}(\phi_i) = \frac{\delta_r}{\cos \phi_i} \quad (12)$$

with ϕ_i being the elevation angle. In the same manner, Fig. 8(b) shows the azimuth angular resolution. When the beam of the synthetic aperture is digitally steered at a broadside angle ($\theta_i = 0$), the angular resolution is defined as $\delta_\theta = \lambda_c/L$ with λ_c the carrier wavelength. When the beam of the synthetic

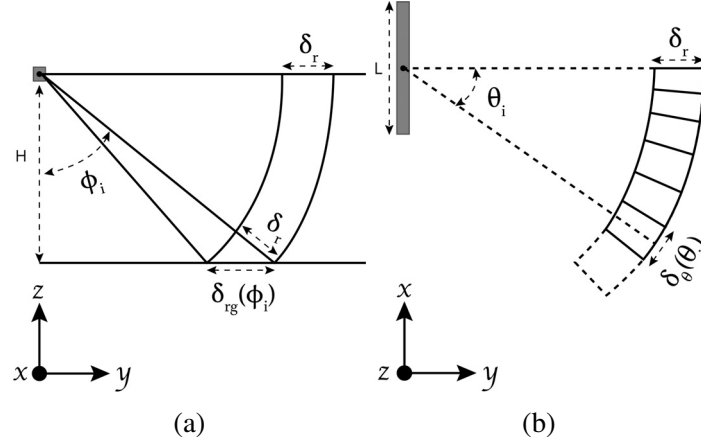


Figure 8. Cutting views of the imaging geometry showing the variation of (a) the range resolution along the plane considered (b) the azimuth angular resolution along the range of observation angles.

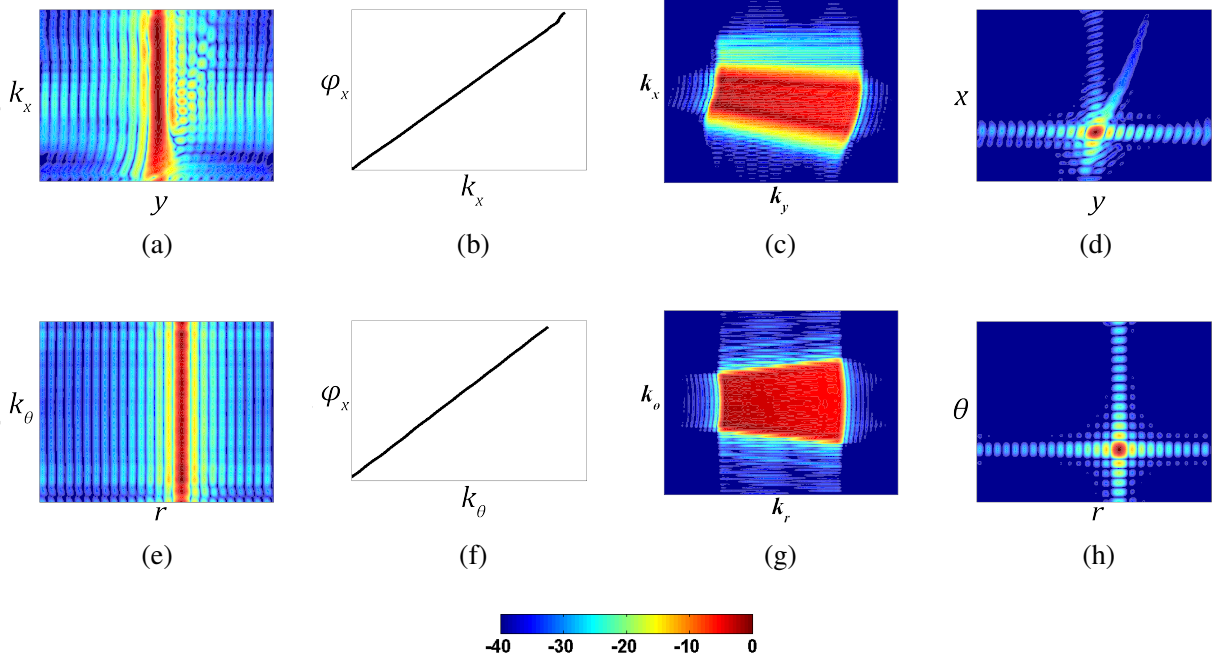


Figure 9. Simulation results of the raw data matrix $\mathbf{S}$ after applying the focusing technique in the near-field wide-band configuration. The raw data are projected over (a), (b), (c), (d), a Cartesian grid (see Fig. 7(a)), and over (e), (f), (g), (h), a polar grid (see Fig. 7(b)), after (a), (e) the range focusing, after (d), (h) the range and the cross-range focusing (Fourier SAR image). (b) and (f) show the phase variation after the range focusing, and (c), (g) show the 2D spectrum of the reconstructed image.

aperture is digitally scanned, the length of the synthetic aperture seen from a θ_i angular point of view is reduced, given an aperture length of $L \cos \theta_i$. The azimuth angular resolution is then:

$$\delta_\theta(\theta_i) = \frac{\lambda_c}{L \cos \theta_i} \quad (13)$$

Considering the variation of the spatial resolution over the 2D-plane, the polar grid is critically sampled at the system resolution. Hence, each pixel has its own set of range and cross-range angular resolution. The i th pixel covers a spatial area of size $(\delta_{rg}(\phi_i), \delta_\theta(\theta_i)d_i(0))$ that corresponds to the area

covered by the synthetic aperture beam at this particular location. As for the regular grid, the received raw data are then projected using focusing algorithms over the irregular sampled polar grid as shown in Eq. (11), given the critically sampled 2D complex SAR matrix $\mathbf{Y} = f(x, y; z = z_0)$. Hence, the data are refocused so that the phase of the Radar ambiguity function is linear. This allows image formation via DFTs. The near-field and wide-band behaviors are compensated over the entire area of interest. Fig. 9 shows the simulated results in the same near-field wide-band configuration used in Fig. 6 where the raw data are projected over a Cartesian and a polar grid. Thanks to the focusing technique, the projection over both the Cartesian and polar coordinates permits to have a linear phase variation; however, the 2D spectrum of reconstructed images after using a focusing technique show that the projection over a polar coordinate permits to have a convenient spectrum to apply 2-D spectral estimation methods unlike the projection over a Cartesian grid. If large amount of data is to be processed, it may be of interest to apply spectrum analysis algorithms only on overlapping sub-images and then reconstruct the final results [21].

To be digitally processed, the continuous received signals $s(x_a, d)$ are sampled to give the 2-D data sequence $Y \in \mathbb{C}^{N_a \times N_f}$. N_f corresponds to the number of frequency components taken after the sampling process in the frequency (wavenumber) domain to be adapted to the analysis of the observed scene (avoiding any range ambiguity). The continuous aperture x_a is sampled in a group of N_a receiving elements with element spacing Δ_x . It gives $x_a(m) = (m - \frac{N_a+1}{2})\Delta_x$ with $\{x_a(m)\}_{m=0}^{N_a-1}$. The range vector $\{d(n)\}_{n=0}^{N_f-1}$ is sampled at the range resolution with $d_{amb} = N_f \delta_r$, the ambiguous distance. Since the range vector d is a discrete vector, obtaining $d_i(x_a)$ from d for applying the back-projection algorithm in Eq. (11) requires an interpolation.

3. SPECTRAL ANALYSIS ALGORITHMS

Once the near-field and wide-band signal features have been accounted for, spectral estimation methods can be used to improve resolution and contrast with reduced speckle effects if the appropriate data model is used. In practice, the construction of an adequate data model based on a finite number of observations cannot be perfectly achieved. The data model remains an estimated data model that attempts to match as far as possible the reality.

3.1. Signal Model

Spectral estimation methods may be used along both the range and the cross-range direction. Along the range direction, the spectral diversity is achieved from multiple frequency components in a finite frequency band. Once spectral estimation methods are applied, this gives improved range location estimates. Along the cross-range direction, the spectral diversity is achieved from a sampled aperture in the spatial domain. Applying spectral estimations methods on the spatial frequency domain gives improved Direction of Arrival (DoA) estimates. An established signal model of SAR data [21, 22] along the cross-range direction consists of N_s waves arriving from distinct directions θ_i , each corresponding to a specific scatterer. The complex amplitude of the i^{th} signal is s_i , and an array of N_a elements is considered. The additive white Gaussian noise vector $\mathbf{n} \sim \mathcal{N}_{\mathbb{C}}(0, \sigma_n^2)$ has zero mean and variance σ^2 . The array output vector $\mathbf{y} \in \mathbb{C}^{N_a}$ is given by:

$$\mathbf{y} = \sum_{i=1}^{N_s} \mathbf{a}(\theta_i) s_i + \mathbf{n} = \mathbf{A} \mathbf{s} + \mathbf{n} \quad (14)$$

where

$$\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_{N_s})] \in \mathbb{C}^{N_a \times N_s} \quad (15)$$

is the received signal steering matrix, which contains the steering vectors. It corresponds to the compensations applied to the synthetic array for a range of observation angles (DoAs). The signal vector, which represents the complex reflectivities of the scatterers, is given by:

$$\mathbf{s} = [s_1, s_2, \dots, s_{N_s}]^T \in \mathbb{C}^{N_s} \quad (16)$$

In the narrow-band plane-wave case (far-field zone), the steering vector of the i th signal using Eq. (6) is:

$$\mathbf{a}(\theta_i) = e^{-j\xi} [1, e^{-jk_c \Delta'_x \sin(\theta_i)}, \dots, e^{-j(N_a-1)k_c \Delta'_x \sin(\theta_i)}]^T \quad (17)$$

where ξ is an arbitrary phase, and $\Delta'_x = \Delta_x/2$ represents the half of the array inter-element spacing and accounts for the radar definition of the round-trip wavenumber given earlier.

The data covariance matrix $\mathbf{R} \in \mathbb{C}^{N_a \times N_a}$ is expressed as:

$$\mathbf{R} = \mathbf{E} \{ \mathbf{y} \mathbf{y}^H \} \in \mathbb{C}^{N_a \times N_a} = \mathbf{A} \mathbf{R}_{ss} \mathbf{A}^H + \sigma^2 \mathbf{I} \quad (18)$$

with $\mathbf{E}\{\cdot\}$ being the expectation operator, $\mathbf{R}_{ss}$ the source covariance matrix, and $\sigma^2 \mathbf{I}$ the noise covariance matrix.

The covariance matrix may be decomposed onto its signal and noise subspaces such as [27]:

$$\mathbf{R} = \mathbf{U} \mathbf{\Lambda} \mathbf{U} = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \mathbf{U}_n \mathbf{\Lambda}_n \mathbf{U}_n^H \quad (19)$$

with $\mathbf{\Lambda} = \text{diag}(\lambda_1, \dots, \lambda_{N_a})$ being the eigen-value matrix with $\lambda_i \geq \lambda_{i+1}$. Considering uncorrelated source and signal terms, $\mathbf{\Lambda}_s = \text{diag}(\lambda_1, \dots, \lambda_{N_s})$ is the source eigen-value matrix, and $\mathbf{\Lambda}_n = \text{diag}(\lambda_{N_s+1}, \dots, \lambda_{N_a})$ is the noise eigen-value matrix. $\mathbf{U}$ is the eigen-vector matrix with $\mathbf{U}_s$ and $\mathbf{U}_n$, the corresponding signal-space and noise-space eigen-vector matrices.

The non-singularity of the covariance matrix estimate requires a pre-processing scheme, named spatial smoothing, which is, in practice, widely employed [28] to guarantee this property. It is based on a diversity of the ξ phase in Eq. (17) and consists in extracting the array covariance matrix as the average of a group of smaller overlapping subarray covariance matrix. The number of subarrays is $K = N_a - N_u$ of length N_u . It corresponds to a spatial smoothing ratio $\eta = N_u/N_a$. Using $\mathbf{y} \in \mathbb{C}^{N_a}$, the estimated covariance matrix after spatial smoothing $\hat{\mathbf{R}} \in \mathbb{C}^{N_u \times N_u}$ is then determined as:

$$\hat{\mathbf{R}} = \frac{1}{K} \sum_{k=1}^K \mathbf{y}_k \mathbf{y}_k^H \quad (20)$$

with $\mathbf{y}_k = [y_k, \dots, y_{k+N_u-1}]^T$. y_k being the received signal from the k th element of $\mathbf{y}$. The objective is to estimate the DoAs (θ_i) and the reflectivities (s_i).

Various spectral analysis algorithms exist in the literature [7, 8, 10, 14]. The capability of separation of two or more closely spaced scatterers is, in practice, mainly affected by the Signal-To-Noise ratio (SNR), the number of receiving elements (N_a), and the kind of observed target responses (stochastic or deterministic/correlated or uncorrelated...) [29, 30]. In this section, two non-parametric and one parametric spectral analysis algorithms are summarized that are the conventional beamforming, the CAPON and the MUSIC methods. The authors do not wish to compare the performance of these algorithms, but rather to show that the proposed method may be applied to any existing spectral analysis approach. The results are then demonstrated using the three algorithms mentioned above.

3.2. Spectral Estimation Methods

All the methods considered belong to the filter bank category [8]. Given, $\mathbf{h}(\omega)$ a spatial filter and $\mathbf{Q}$ a square matrix, the methods follow the generic equation:

$$\hat{P}(\omega) = (\mathbf{h}^H(\omega) \mathbf{Q} \mathbf{h}(\omega))^\alpha \quad (21)$$

The DoA estimation, on a finite range of observation angles, is applied with $\theta \in [\theta_{\min}, \theta_{\max}]$ with N_θ scanning angles. In the plane wave assumption, the phase shift generated by a backscattered plane-wave arriving at an angle θ from broadside between two receiving antennas is $\omega = k_c \Delta'_x \sin \theta$. $\mathbf{h}(\omega)$ is built from the steering matrix defined in Eq. (15). The methods are summarized in Table 2.

The coordinates of the local maxima of $\hat{P}(\omega)$ are linked to the angular positions of the observed scatterers. As regards the Beamforming and CAPON algorithms, $\hat{P}(\omega_i)$ provides an estimate of $\mathbf{E}\{|s_i|^2\}$ whereas MUSIC does not provide amplitude information. $\mathbf{E}\{|s_i|^2\}$ may be estimated using a least-square approach [8]. The maximum outputs of the beamforming algorithm provide an estimate of the signal power s_i , and the signal parameter estimate is given by the value of ω that achieves this maximum

Table 2. Spectral estimation methods.

Methods	Spectrum estimation
Beamforming [7]	$\mathbf{Q} = \hat{\mathbf{R}}, \alpha = 1, \mathbf{h}(\omega) = \mathbf{a}(\omega) / \hat{P}_{bf}(\omega) = \mathbf{a}^H(\omega) \hat{\mathbf{R}} \mathbf{a}(\omega)$
CAPON [31]	$\mathbf{Q} = \hat{\mathbf{R}}, \alpha = 1, \mathbf{h}(\omega) = \frac{\mathbf{a}(\omega) \hat{\mathbf{R}}^{-1}}{\mathbf{a}(\omega)^H \hat{\mathbf{R}}^{-1} \mathbf{a}(\omega) / \hat{P}_{\text{CAPON}}(\omega) = \frac{1}{\mathbf{a}^H(\omega) \hat{\mathbf{R}}^{-1} \mathbf{a}(\omega)}}$
MUSIC [27]	$\mathbf{Q} = \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H, \alpha = -1, \mathbf{h}(\omega) = \mathbf{a}(\omega) / \hat{P}_{\text{MUSIC}}(\omega) = \frac{1}{\mathbf{a}^H(\omega) \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}(\omega)}$

($\omega_i = k_c \Delta'_x \sin(\theta_i)$). It produces coherent complex image that represents the output of banks of narrow-band filters where each filter output is tuned to a given DoA. The beamforming does not require any spatial smoothing pre-processing. Nevertheless, smoothing is useful for reducing speckle effects. CAPON uses selective adaptive filters around the current frequency (ω) and minimizes the total power subject to the constraint that the filter passes the frequency ω undistorted. It permits to improve the spatial resolution while maintaining high image contrast.

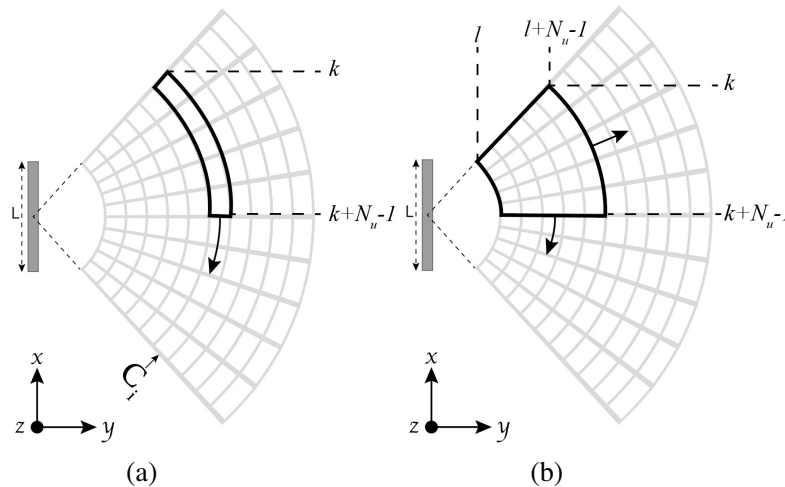
MUSIC uses the sub-space decomposition in Eq. (19) to indicate the presence of sinusoidal components in the studied signal. The main sinusoidal components are selected (model selection) by estimating the size of the noise sub-space. The Akaike Information Criterion [32] provides a means for model selection (to estimate N_s). This implies that $N_s < N_a$. Further, MUSIC only extracts the main components with the strongest responses. An extension of Table 2 for the 2-Dimensional case is shown in [33, 34].

3.3. Near-Field Wide-Band Configuration

In a near-field configuration with wide-band signals, it has been explained in Section 2.2 that the phase variation depends on the range and the DoA. This implies that the steering vectors have to be modified for each pair of locations θ and r . To overcome this important limitation, we propose here to apply the spectral analysis algorithms on the complex 2D focused SAR image where the near-field and wide-band effects have been compensated. A discrete 2-D SAR image is built from Eq. (11) as $[\mathbf{Y}]_{i,j} = \hat{f}(x_i, y_j; z = z_0)$ with $\mathbf{Y} \in \mathbb{C}^{N_n \times N_m}$. Then 1-D or 2-D spectral estimation techniques may be applied to improve the focusing over the ground-range (y), azimuth (x) or both domains.

The 1-D spatial smoothing in Eq. (20) is then applied to the DFT of one particular column $y = C_i$ as shown in Fig. 10(a).

In the same manner as the 1-D spectral analysis case, 2-D spectral estimation methods can be

**Figure 10.** (a) 1D, (b) 2D spatial smoothing applied on the irregular grid.

used on the focused SAR image as in [21–24]. Unlike the approach proposed in [21], which consists in applying spectral estimation methods from a SAR image sampled in Cartesian coordinates, we propose to deal with SAR images projected in a polar format (r, θ) as depicted in Fig. 7. This permits to avoid limitations encountered by Cartesian sampling for the particular fan-beam configuration.

From the $N_n \times N_m$ matrix $\mathbf{Y}$, 2-D spectral estimation methods are applied. Given $\mathbf{Z}$ the 2D Fourier transform of the matrix $\mathbf{Y}$ and given a $(N_n N_m) \times 1$ vector as

$$\mathbf{z} \triangleq \text{vec}(\mathbf{Z}) \quad (22)$$

with $\text{vec}(\cdot)$ being the operation of stacking the columns of a matrix on top of each other, the covariance matrix is:

$$\mathbf{R} = \mathbf{E} \{ \mathbf{z} \mathbf{z}^H \} \quad (23)$$

The spatial smoothing stated in Eq. (20) is also used for the 2D case (see Fig. 10(b)). The estimated covariance matrix is:

$$\hat{\mathbf{R}} = \frac{1}{KL} \sum_{k=1}^K \sum_{l=1}^L \mathbf{z}_{k,l} \mathbf{z}_{k,l}^H \quad (24)$$

with

$$\mathbf{z}_{k,l} = \text{vec} \left(\begin{bmatrix} [\mathbf{Z}]_{k,l} & \cdots & [\mathbf{Z}]_{k,l+N_u-1} \\ \vdots & \ddots & \vdots \\ [\mathbf{Z}]_{k+N_u-1,l} & \cdots & [\mathbf{Z}]_{k+N_u-1,l+N_u-1} \end{bmatrix} \right). \quad (25)$$

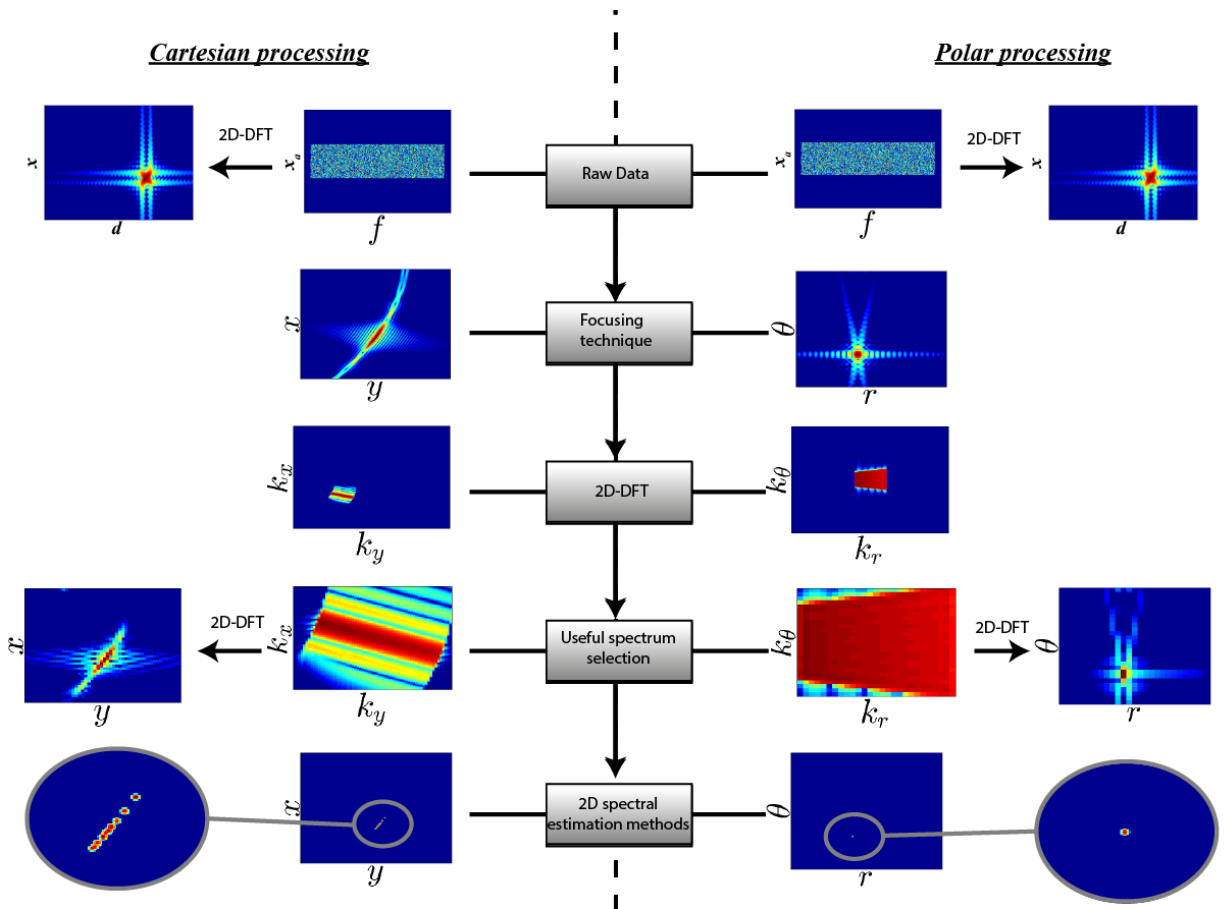


Figure 11. Flow chart of applying the spectral analysis algorithms in a near-field environment with wide-band signals using a Cartesian grid ([21] → left part) and a polar grid (proposed method → right part).

Hence, $\mathbf{z}_{k,l}$ is a $(N_u \cdot N_u) \times 1$ vector.

The elements of the 2-D steering vector $\mathbf{a}_{\omega_1, \omega_2} = \mathbf{a}_{\omega_1} \otimes \mathbf{a}_{\omega_2}$ constitute the Kronecker product of the two 1-D steering vectors associated with the exponential coefficients of the discrete Fourier transform applied on $\mathbf{Y}$ to obtain $\mathbf{Z}$ with $\omega_1, \omega_2 \in [0, 2\pi[$. The filter $\mathbf{h}(\omega)$ in the Table 2 is replaced by $\mathbf{a}_{\omega_1, \omega_2}$, and the estimated covariance matrix in Eq. (24) is used to apply 2D spectral estimation methods. The corresponding spectral estimation results may be represented using a re-sampling of the reflectivity maps $\hat{P}(\omega_1, \omega_2) \rightarrow \hat{P}(x, y)$ with $x = r \cos \theta$ and $y = r \sin \theta$.

Considering an aperture length $L = 0.1$ m and one point-like target located at an angle $\theta_i = 20^\circ$ from broadside at a range $d_i(x_a = 0) = 1.5$ m from the radar, the steps for applying spectral analysis algorithms in a near-field wide-band configuration are outlined in Fig. 11 in the general 2D case and listed as follows:

- From simulation or measurement, the received signals are used to build the complex raw data matrix $(S(x_a, f))$.
- The complex raw data matrix is projected onto an over-sampled Cartesian or polar grid (complex 2D focused SAR image) by applying focusing techniques such as the back-projection algorithm $(\hat{f}(x, y; z = z_0))$.
- the complex 2D focused SAR image is transformed to the spectral domain using a 2D DFT.
- The useful spectrum is selected ($\mathbf{Z}$). Its limits are defined as the spectral region with a sufficiently high signal to noise ratio.
- 2D spatial smoothing is applied on the useful spectrum.
- 2D spectral estimation methods are applied on the spatially smoothed spectrum to give the final result $(\hat{P}(\omega_1, \omega_2))$.

The projection over a polar grid permits to have a symmetric and centred 2D spectrum unlike the projection over a Cartesian grid. Then it provides full capability to spectral estimation methods as shown in Fig. 12 in which the spectral estimation methods are applied on both projection methods to validate the principle. The final complex image results correspond to projections over a polar grid ($x = r \cos \theta$, $y = r \sin \theta$). To have a physical interpretation of the final images, interpolations and denormalizations have to be applied to project the final results over a regular grid, which matches the Cartesian coordinates $(\hat{P}(x, y))$.

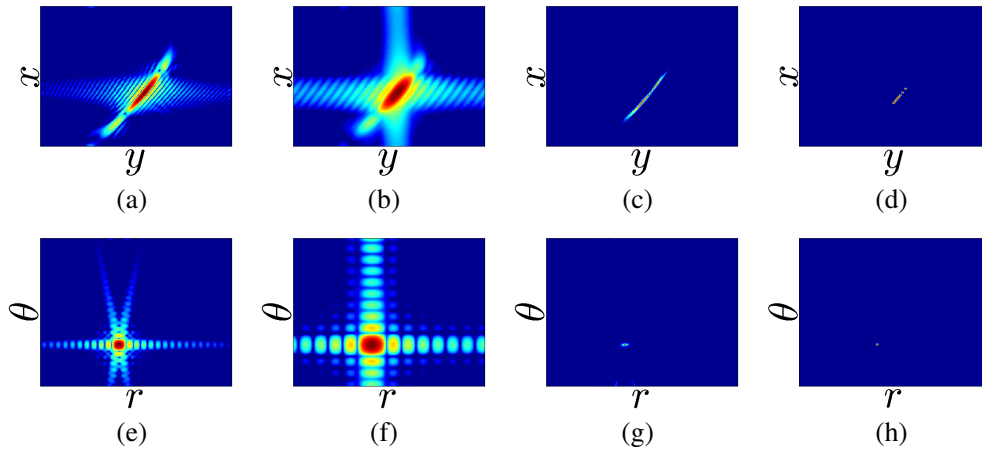


Figure 12. Simulated results of 2D spectral estimation methods using (a), (b), (c), (d) a Cartesian grid and (e), (f), (g), (h) a polar grid with (a), (e) the beamforming method, (b), (f) spatially smoothed beamforming, (c), (g) the CAPON method and (d), (h) the MUSIC method.

4. MEASUREMENTS

The measurements were performed in the IETR (Institute of Electronics and Telecommunication of Rennes) facility DIADEM (DIagnostic, Analysis and Dosimetry of EM fields). This facility dedicated to Electro-magnetic imaging is based on a $600 \times 600 \times 600 \text{ mm}^3$ xyz scanner located in an anechoic chamber. The RF measurement system uses a classical architecture with a VNA (Vector Network Analyzer) and external VDI (Virginia Diodes, Inc) transmitter and receiver frequency extenders. In this configuration, the emission part is fixed to illuminate the scene to be imaged with an incident elevation angle of 30° . The 300 elements reception array is synthesized moving the RF reception module to 2 mm spaced discrete positions thanks to the scanner (see Fig. 13). The objects, to be imaged, are settled at 1 m distance on a 1.5 m height foam support. Thanks to this support whose relative electrical permittivity is close to one, the sources of diffraction are minimized and mainly limited to the backscattering of the anechoic chamber. A 20 GHz bandwidth signal with a 50 GHz central frequency has been used for all the tests. To speed up the acquisition of the 1001 frequency points obtained for each reception position,

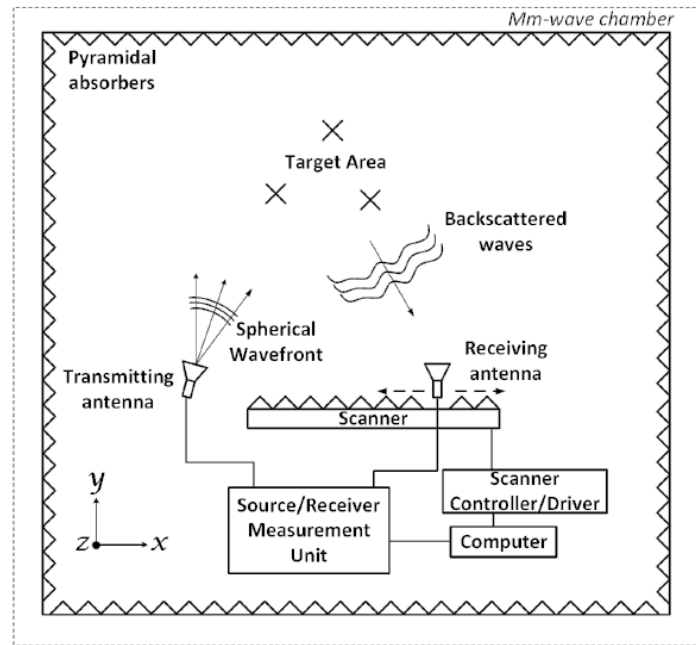


Figure 13. Measurement setup.

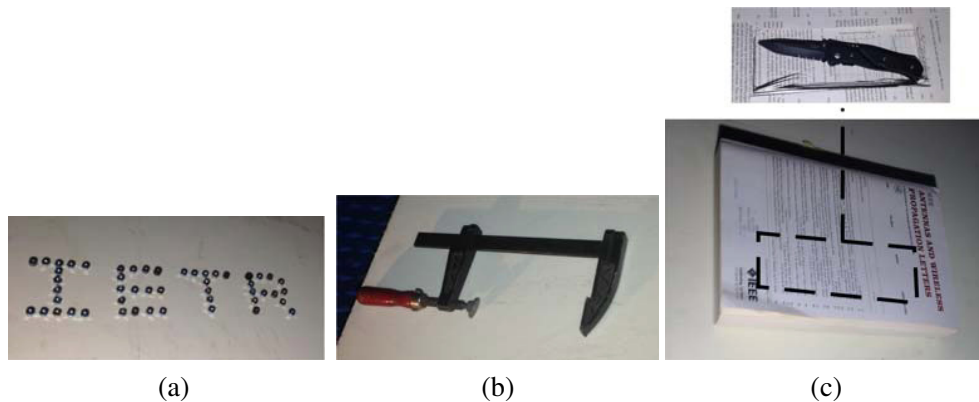


Figure 14. Picture of the three scene configurations with (a) 50 bolts of 5 mm diameter configured to spell IETR, (b) a screw clamp, and (c) a knife hidden inside a thick book.

an IF (intermediate frequency) Filter bandwidth of 100 kHz was applied. It has to be noticed that even if the IF filter is broadband, the dynamic range provided by the VDI modules is high enough for the purposes of these tests. In such a configuration, a maximal range resolution of 7.5 mm can be obtained, with an azimuth angular resolution of 0.57° corresponding to a 1 cm cross-range resolution at 1 m range distance. Moreover, the 7.5 m ambiguous distance, longer than the anechoic chamber length, enables the compensation for the chamber backscattering in an efficient way. Considering the 30° elevation orientation of the emission part, the ground range resolution is limited to 8.7 mm.

Three complex examples of targets that have been imaged are reported in Fig. 14: a canonical point

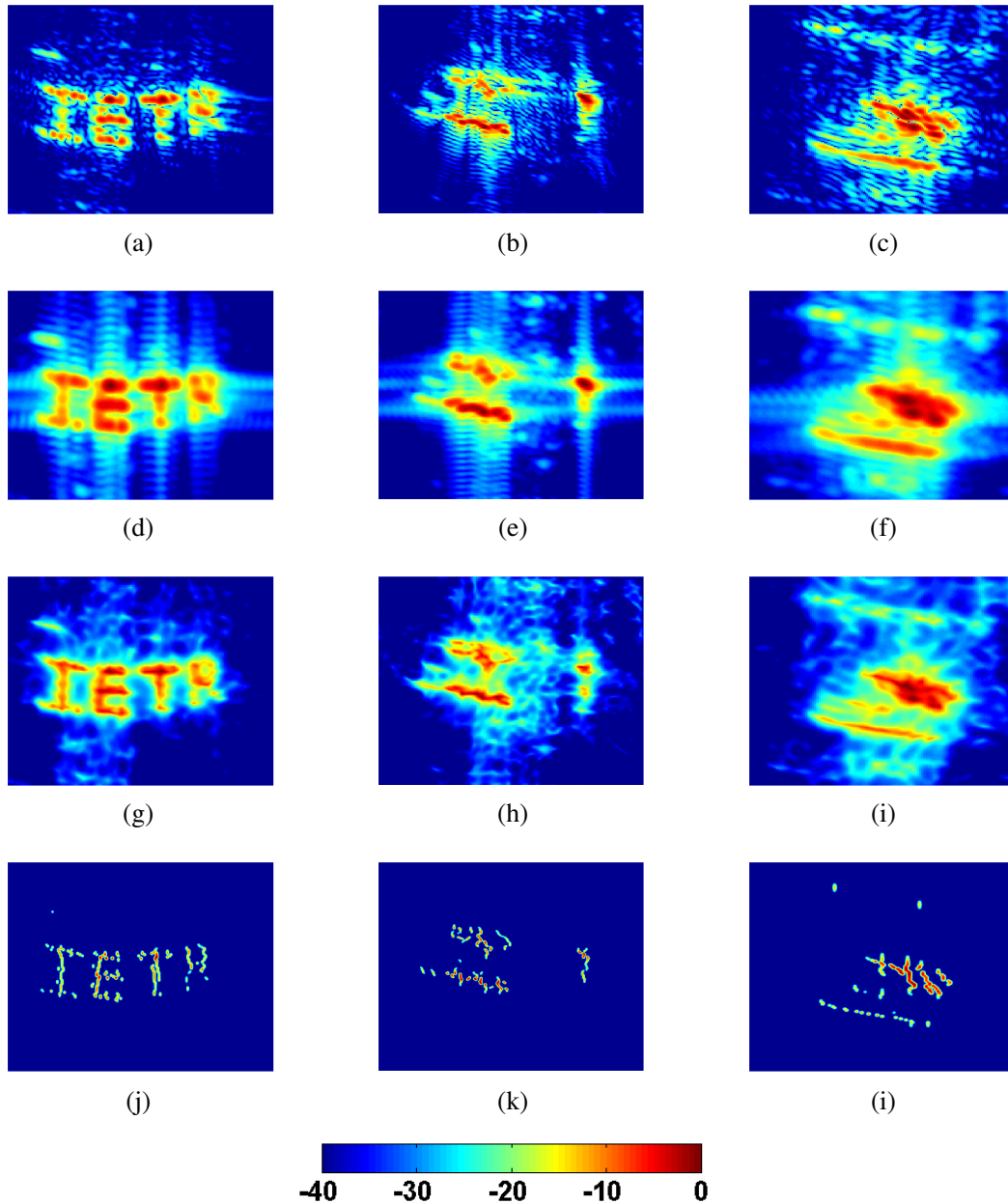


Figure 15. 2D spectral estimation methods applied on the (a), (d), (g), (j) bolts configured to spell IETR; (b), (e), (h), (k) the screw clamp and (c), (f), (i), (l) the knife hidden in a book. (a), (b), (c) show the beamforming method and (d), (e), (f) show the beamforming method after spatial smoothing, while (g), (h), (i) show the CAPON method and (j), (k), (l) show the MUSIC methods.

scatterers scene made of dozen of 5 mm diameter bolts positioned with a 10 mm spacing to write the word IETR; a realistic scene with a screw clamp; a realistic scene with a 1 mm blade stainless steel knife hidden inside a book. The estimation results of these three scenes applying the imaging techniques are reported in Fig. 15. The twelve images correspond to the results of four spectral estimation methods that are applied on the three complex targets. From top to bottom, the spectral estimation methods are: the beamforming, the spatially smoothed beamforming, CAPON and MUSIC. A spatial smoothing ratio $\eta = 50\%$ is used. Since the spatial smoothing pre-processing reduces the size of the covariance matrix, the spatial resolution of the spatially smoothed beamforming method is reduced by twice as compared to the beamforming method. Nonetheless, the spatial smoothing pre-processing becomes useful for CAPON and MUSIC algorithms. Further, since MUSIC does not provide an amplitude estimate, the amplitude is estimated using a least-square approach [8], and the Akaike Information Criterion [32] is used to provide a means for model selection. The bolt scene images show that the methods give good results as the word is readable, and the bolts can be discriminated from each other. The other test cases are more challenging as the point scatterers hypothesis is not fulfilled in such a straight way as the bolts scene, and considering the complexity of the surrounding of the target of interest. The results obtained with the beamforming method show that the screw clamp and the knife can be detected, but the image contrast, including ripples effects, is not sufficient to apply an easy recognition of the target. The CAPON method gives a better result, even if the contrast remains poor. The best result is obtained by applying the MUSIC algorithm: the knife is well localized, and the contrast is very high. However, it only extracts the main components with the strongest responses.

5. CONCLUSION

This study discusses the use of spectral estimation methods rather than the Fourier transforms to form SAR image with an improved spatial resolution in a near-field region with wide-band signals. Because the spectral estimation methods have been developed considering plane-wave narrow-band assumptions, the complex data need to be compensated from near-field region with wide-band signals in order not to change the core of the algorithms. The authors have proposed a solution to perfectly compensate these effects with the aim of giving full capability to the spectral estimation methods. It has been used in measurements to image three scenes at millimeter-wave and at a range of 1 m. After the validation exploiting 2-D spectral estimation methods using a linear array for 2-D imaging, the authors work on a 2-D array to exploit 3-D spectral estimation methods for 3-D imaging.

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