The Research of W-Band High Order Frequency Multiplier Based on Avalanche Diode

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Abstract—A research of millimeter wave high order frequency multiplier based on the fierce inductive nonlinearity of avalanche diode is presented. The operation of high order frequency multiplication is introduced, and the high order harmonics generation character under external RF field modulation is analyzed. The characteristic of multiplier circuit is also discussed. Maximum output power of 6 mW and minimum conversion loss of 17 dB are obtained at output frequencies of 94 GHz and 96 GHz with 15th multiplication order. The phase noise of output 94 GHz signal is about $-90\,\mathrm{dBc/Hz}$ and $-94.33\,\mathrm{dBc/Hz}$ at $10\,\mathrm{kHz}$ and $100\,\mathrm{kHz}$ offset.

1. INTRODUCTION

High quality millimeter wave frequency source is an important component in many millimeter and submillimeter wave systems. Traditionally, millimeter wave signals are obtained by oscillators, such as avalanche oscillator and Gunn oscillator [1–10]. However, they have some shortages such as relatively poor phase noise and frequency stability, which need to be improved by subsidiary method limitedly. Frequency multiplication technology is an effective way to generate highly stable and low phase noise millimeter wave signals [11–15]. However, general frequency multiplier could not realize a direct frequency multiplication at a harmonic order greater than four due to the limitation of nonlinearity of capacitance or resistance. So millimeter wave signals are usually obtained by multistage frequency multiplication and amplification chains, which have to solve these problems of interstage match and filter.

High order frequency multiplication in a single stage is a simple and high efficiency method to generate microwave and millimeter wave signals, which have demonstrated its advantages by Step Recover Diode at microwave band. Taking advantage of the fierce inductive nonlinearity in avalanche effects, avalanche diode could generate high order harmonics up to short millimeter wave band with effective harmonic power in a single stage [16], which implies that avalanche diode could translate high stable microwave reference source into millimeter wave band without additional amplification [17, 18]. Good performance of the avalanche diode high order multiplier have been achieved in millimeter wave band [16–24].

Initial research has demonstrated that the high nonlinearity of avalanche effects plays an important role in high order frequency multiplication [16, 22–24]. Later the modulation effect of external injected RF field to avalanche current nonlinearity is analyzed [20]. However, the generation characteristic of avalanche current under external RF field modulation has not been discussed. This paper first introduces the operation of high order frequency multiplication and gives a detailed analysis of high order harmonic generation under external injected RF field modulation. Then the characteristic of the multiplier circuit is discussed. Satisfied results have been obtained in experiment research.

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2. THE OPERATION OF HIGH ORDER FREQUENCY MULTIPLICATION

The dynamic operating characteristic of avalanche diode can be best understood by dividing the active zone of the device into two parts: (1) the avalanche area, where the electric field is strong enough to cause carriers avalanche multiplication, and it behaves as a nonlinear inductor from the analysis of Read's equation [16, 25, 26]; (2) the drift area, where no carriers are generated, and carriers travel at their saturated velocities. In the avalanche breakdown process of avalanche area, carrier ionization rate is the sensitive nonlinear function of electric field. When external injected RF field reacts on avalanche diode, avalanche breakdown will take place once the total field intensity is larger than the breakdown intensity. During the periodic variation of external RF field, the carrier ionization rate is modulated by the variation of RF field. As a result, the waveform of avalanche current, which is generated from the avalanche multiplication of ionized carriers, is also modulated by external RF field for high order harmonic generation. The generation of avalanche current can be described by carrier continuity equations and current density equation as follows [29]:

$$\frac{\partial n}{\partial t} = \frac{1}{q} \frac{\partial J_n}{\partial x} + \alpha_p p v_s + \alpha_n n v_s \tag{1}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} + \alpha_p p v_s + \alpha_n n v_s \tag{2}$$

$$J = J_n + J_p = qnv_s + qpv_s \tag{3}$$

where J_n and J_p are the electron and hole current densities; n and p are the electron and hole densities; α_n and α_p are the electron and hole ionization rates; v_s is the saturation velocity; q is the electronic charge. The second and third terms on the right-hand side of Eqs. (1) and (2) correspond to the generation rate of the electron-hole pairs by avalanche multiplication. Since the electron-hole pairs are just generated in the very narrow avalanche area, these parameters, which are the function of space position x and time t, are considered as having no space variation in the avalanche area and just changing with the time t. Supposing that the electron and hole have equal ionization rates, adding Equations (1) and (2) to obtain a new equation and integrating the new equation over the extent of avalanche area, the relationship between avalanche current and ionization rate is obtained by considering the boundary condition of avalanche area [26]:

$$\frac{\tau_a}{2} \frac{dI_a}{dt} = I_a \left(\int_0^{l_a} \alpha(E) \, dx - 1 \right) + I_s \tag{4}$$

where I_a is the avalanche current, I_s the thermally generated reverse saturation current, $\alpha(E)$ the carrier ionization rate, and τ_a and l_a are the transit time and length of avalanche area. In avalanche multiplication I_s is so small compared to I_a that it could be neglected [26–28]. Solving the differential equation of (4), the expression of I_a is derived:

$$I_a = I_0 \exp\left\{ \int \frac{2}{\tau_a} \left[\int_0^{l_a} \alpha(E) dx - 1 \right] dt \right\}$$
 (5)

Equation (5) indicates that avalanche current I_a is the sensitive nonlinear function of ionization rate $\alpha(E)$, which is also the nonlinear function of the sum of static electric field and external injected RF field. The modulation effects of external RF field to avalanche current for high order harmonic generation can be analyzed by expanding $\alpha(E)$ by Taylor series:

$$\alpha(E) = \alpha(E_c) + \alpha'(E_c) E_t + \frac{\alpha''(E_c)}{2!} E_t^2 + \frac{\alpha'''(E_c)}{3!} E_t^3$$
(6)

where

$$\alpha (E_c) = A \cdot \exp(-b/E_c)$$

$$E_t = E - E_c = E_{RF} \sin \omega t$$

Both A and b are the ionization rate constants, which are decided by semiconductor material and structure, E_c is the static electric field and E_t the external injected RF field. The third order term in Taylor series is retained for analyzing the modulation effects of external RF field to high order harmonic generation accurately. Substituting Eq. (6) into Eq. (5), the expression of I_a under RF field modulation is obtained:

$$I_{a} = I_{0} \exp \left\{ \frac{2}{\tau_{a}} \left[\alpha \left(E_{c} \right) \times l_{a} \times m \left(t \right) \right] - t \right\}$$
 (7)

where m(t) is defined as the RF modulation function, which is determined by waveform of external RF field and the expanded terms of Taylor series. The form is given as follows:

$$m(t) = t - \frac{b \cdot E_{RF}}{E_c^2} \left[1 + \frac{E_{RF}^2}{6E_c^2} \left(\frac{b^2}{E_c^2} - \frac{6b}{E_c} + b \right) \right] \frac{\cos \omega t}{\omega}$$

$$+ \frac{b \cdot E_{RF}^2}{4E_c^3} \left(\frac{b}{E_c} - 2 \right) \left(t - \frac{\sin 2\omega t}{2\omega} \right)$$

$$+ \frac{b \cdot E_{RF}^3}{6E_c^4} \left(\frac{b^2}{E_c^2} - \frac{6b}{E_c} + 6 \right) \left(\frac{(\cos \omega t)^3}{3} \right)$$
(8)

Thus the generation and waveform of avalanche current under the modulation of external RF field can be calculated according to Eqs. (7) and (8), depicted in Fig. 1.

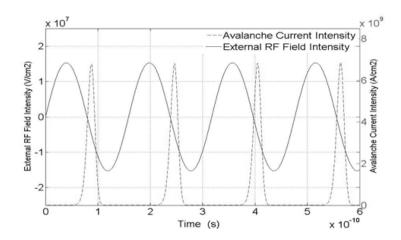


Figure 1. Avalanche current generation under RF field modulation.

In Fig. 1, the quick rise of current peak and down to a few picoseconds indicates that avalanche current contains rich high order high frequency harmonics up to short millimeter wave band. The harmonics characteristic can be analyzed by a Fourier series expansion of avalanche current as:

$$I_a = I_0 + \sum_n I_{an} \sin(n\omega t + \varphi_n) \tag{9}$$

where I_0 is the constant current, I_{an} the nth harmonic current amplitude, and φ_n the phase delay between harmonic frequency and input RF frequency. According to the analysis of [28, 30], the harmonic impedance Z_n of avalanche diode at frequency $n\omega$ can be expressed:

$$Z_{n} = R_{n} + jX_{n}$$

$$= \frac{l_{d}^{2}}{v_{d}\varepsilon A} \left[\frac{1}{1 - \frac{\omega_{n}^{2}}{\omega_{a}^{2}}} \right] \frac{1 - \cos(\omega_{n} \times \tau_{d})}{\frac{(\omega_{n} \times \tau_{d})^{2}}{2}} + j \frac{l_{d}}{\omega_{n}\varepsilon A} \left[\left(\frac{\sin(\omega_{n} \times \tau_{d})}{\omega_{n} \times \tau_{d}} - 1 \right) - \frac{\frac{\sin(\omega_{n} \times \tau_{d})}{\omega_{n} \times \tau_{d}} + \frac{l_{a}}{l_{d}}}{1 - \frac{\omega_{a}^{2}}{\omega_{n}^{2}}} \right]$$
(10)

where R_n and X_n are the resistance and reactance at the *n*th harmonic frequency, and l_a and l_d are the length of avalanche area and drift area. v_d is the saturation drift velocity of carriers, A the diode

cross-sectional area, ε the dielectric permittivity of semiconductor, τ_d the transit time of drift area, ω_a avalanche frequency, and ω_n the harmonic frequency. So the generated high order harmonic power can be calculated according to Eqs. (9) and (10):

$$P_n = I_{an}^2 \times \operatorname{Re}\left(Z_n\right) \tag{11}$$

3. THE ANALYSIS OF MULTIPLIER CIRCUIT

Considering the difference of operation frequency range between input and output signals, the avalanche diode high order frequency multiplier is designed and built with microstrip-waveguide circuit structure. The input matching, filtering and bias circuit are built on microstrip substrate with thickness of 0.508 mm, relative dielectric constant of 3.2 and connected to avalanche diode by gold tape. Avalanche diode is mounted in the center of multiplier cavity with a T-type tuning circuit of three stubs around it as shown in Fig. 2. The input RF signal is injected into avalanche diode mounted in the T-type tuning multiplier cavity through input microstrip circuit and excites the avalanche breakdown of the avalanche diode to generate the avalanche current, which contains rich high order harmonic frequency up to millimeter wave band as discussed in Section 2.

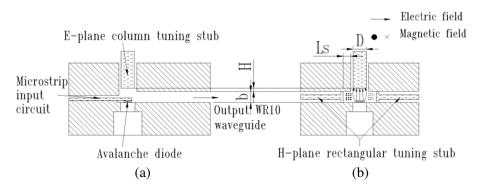


Figure 2. The circuit structure of avalanche diode high order frequency multiplier, (a) longitudinal section view, (b) cross section view.

In frequency multiplier cavity, the E-plane column tuning stub is assembled above avalanche diode exactly, and the inner structure is close to the cylinder resonant cavity since the inner cavity height is much smaller than the transverse dimension of cavity. In the operation of high order frequency multiplication mode, avalanche diode excited by external RF signal will generate electromagnetic field radiation at different harmonic frequencies in the multiplier cavity due to the generated avalanche current in the avalanche diode as discussed in Section 2, which can be expressed by the sum of the field of existed electromagnetic modes. Once one of the generated harmonic frequencies is close to the intrinsic resonant frequency of particular mode in multiplier cavity, the field intensity of this resonant mode at the harmonic frequency will increase largely. Thus the field distribution in multiplier cavity can be approximately expressed by the field character of this resonant mode at single mode operation [31].

In frequency multiplier cavity, the particular resonant mode, which can excite TE_{10} mode in output waveguide direction maximally, will be built by changing the structure dimension of multiplier cavity through tuning the position of three side stubs. Because the inner structure of frequency multiplier cavity approaches cylinder resonant cavity, the most effective mode to excite TE_{10} mode in output waveguide direction is the circular symmetry mode of TM_{010} according to the analysis of [31]. Once the output frequency range is determined, a full-wave simulation tool (such as HFSS or CST) can be used to optimize the dimensions of multiplier cavity and tune the stubs to realize the desired TM_{010} resonant mode at output frequency range in multiplier cavity for realizing the field transformation between multiplier cavity and output waveguide. The equivalent circuit model of the multiplier is shown in Fig. 3.

In frequency multiplier cavity, the diameter of the E-plane column tuning stub can be changed and tuned by replacing different column tuning stubs, and the two H-plane rectangular tuning stubs have

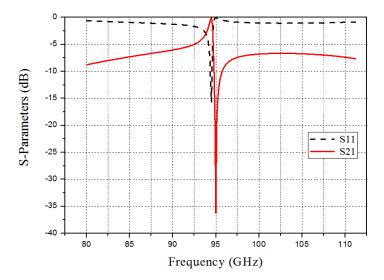


Figure 3. The simulation result of T-type tuning multiplier cavity.

Table 1. The optimized dimension parameters of multiplier cavity.

ĺ	Broadside	Narrow side	Diameter of	The position	The position
	of WR10	of WR10	column tuning	of column	of rectangular
	Waveguide (a)	waveguide (b)	stub (D)	tuning stub (H)	tuning stub (Ls)
	$2.54\mathrm{mm}$	$1.27\mathrm{mm}$	$2.44\mathrm{mm}$	$1.9\mathrm{mm}$	$0.05\mathrm{mm}$

the fixed size of $2.54\,\mathrm{mm} \times 1.27\,\mathrm{mm}$. After the optimization of the dimensions of multiplier cavity by the full-wave simulation tool of HFSS, the optimized dimension parameters of multiplier cavity can be obtained and are shown in Table 1.

According to the optimized dimension parameters of multiplier cavity, the transmission characteristic of the T-type tuning multiplier cavity at output frequency range can be simulated by calculating the S-parameters of multiplier cavity, which is shown in Fig. 3.

In Fig. 3, it is shown that at the output frequency around 94.5 GHz, S_{21} is about -0.3 dB, and S_{11} is about -15 dB, which illustrate that the generated high order harmonic frequency near 94.5 GHz of avalanche diode can have a good transmission characteristic based on the optimized frequency multiplier cavity. In addition, by tuning the position of column tuning stub (H) and the position of rectangular tuning stub (Ls), the optimum output frequency of frequency multiplier cavity can be tuned and changed, which is very beneficial for avalanche diode high order frequency multiplier to operate at a broad output frequency range.

According to above analysis and simulation of the frequency multiplier circuit structure, the qualitative circuit model of avalanche diode high order frequency multiplier can be analyzed and is shown in Fig. 4.

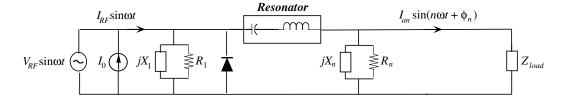


Figure 4. The circuit model of avalanche diode high order multiplier.

As shown in the circuit model, the frequency multiplier cavity with a T-type resonant tuning circuit can be equal to a series resonator. In the resonator, the output nth harmonic frequency will build stable electromagnetic field distribution by resonating in the T-type tuning multiplier cavity and radiate to output waveguide direction effectively.

In the circuit model, the input and output signals are operated at different frequency bands. Because the input RF signal is operated at microwave band, the input microstrip circuit is matched to the avalanche diode with resistance of R_1 and reactance of X_1 at input frequency range. In addition, the output high order harmonic signal of avalanche diode is operated at millimeter wave band, and the output waveguide circuit is matched to the avalanche diode by the T-type tuning multiplier cavity with resistance of R_n and reactance of X_n at output frequency range. The impedance match between avalanche diode and output waveguide has been achieved by the transformation of electromagnetic field between the T-type tuning multiplier cavity and output waveguide.

An actual circuit photograph of the avalanche diode high order multiplier is shown in Fig. 5.



Figure 5. A photograph of avalanche diode high order frequency multiplier.

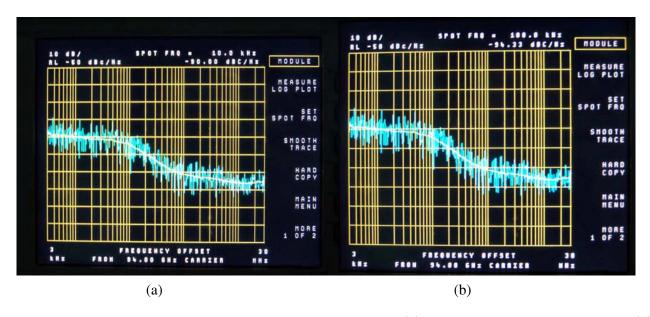


Figure 6. The measured phase noise of output 94 GHz signal, (a) the phase noise at 10 kHz offset, (b) the phase noise at 100 kHz offset.

4. EXPERIMENT RESEARCH

The silicon avalanche diode used in high order multiplier has $p^+ - n - n^+$ single drift diode structure with high punch through factor operating at 7–8 V and 50–100 mA. In experiment research, it is found that the optimum operating point with maximum output power is about at input frequency of 6.27 GHz with 90 mA bias current, and the optimum operating point with minimum conversion loss is about at input frequency of 6.4 GHz with 60 mA bias current. The maximum output power is about 6 mW with 21.8 dB conversion loss at output frequency of 94 GHz with 15th multiplication order operating at 7.8 V and 90 mA. And output power of about 2.5 mW with minimum 17 dB conversion loss is obtained at output frequency of 96 GHz with 15th multiplication order operating at 7.1 V and 60 mA. Changing the input frequency from 6.25 GHz to 6.42 GHz, effective output power above 1 mW from 93.75 GHz to 96.3 GHz is obtained with 15th multiplication order by careful circuit tuning and tuning the bias current from 60 mA to 90 mA.

The phase noise of output 94 GHz signal with 15th multiplication order is about $-90\,\mathrm{dBc/Hz}$ and $-94.33\,\mathrm{dBc/Hz}$ at $10\,\mathrm{kHz}$ and $100\,\mathrm{kHz}$ offset while the phase noise of input $6.27\,\mathrm{GHz}$ reference signal is about $-113.33\,\mathrm{dBc/Hz}$ and $-118.5\,\mathrm{dBc/Hz}$ at $10\,\mathrm{kHz}$ and $100\,\mathrm{kHz}$ offset, which are similar to the research of [32]. The number of phase deteriorations is very close to the theoretically predicted value, $20\,\mathrm{log}\,15 = 23.52$. The measured phase noise is shown in Fig. 6.

5. CONCLUSION

A W-band avalanche diode high order frequency multiplier based on the fierce inductive nonlinearity of avalanche effect is presented. Theory analysis of the operation of high order multiplication and the harmonic generation characteristic under external RF field modulation according to the physical property of avalanche diode indicate the capability of generating high order high frequency harmonics up to short millimeter wave band. Good experiment results have demonstrated it and indicated that the avalanche diode high order multiplier is very suitable for high quality millimeter wave frequency synthesizer, radar and communication systems.

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REFERENCES

- 1. Vahdati, H. and A. Abdipour, "Nonlinear stability analysis of an oscillator with distributed element resonator," *Progress In Electromagnetics Research*, Vol. 80, 241–252, 2008.
- 2. Vahdati, H. and A. Abdipour, "Nonlinear stability analysis of microwave oscillators using the periodic averaging method," *Progress In Electromagnetics Research*, Vol. 79, 179–193, 2008.
- 3. Shi, Z.-G., S. Qiao, and K. S. Chen, "Ambiguity functions of direct chaotic radar employing microwave chaotic colpitts oscillator," *Progress In Electromagnetics Research*, Vol. 77, 1–14, 2007.
- 4. Mokari, H. and P. Derakhshan-Barjoei, "Numerical analysis of homojunction gallium arsenide avalanche," *Progress In Electromagnetics Research B*, Vol. 7, 159–172, 2008.
- 5. Seyedi, M. H., "Numerical analysis of homojunction avalanche photodiodes (APDs)," *Progress In Electromagnetics Research C*, Vol. 3, 45–56, 2008.
- 6. Akbarzade, M., D. D. Ganji, and M. H. Pashaei, "Analysis of nonlinear oscillators with U force by He's energy balance method," *Progress In Electromagnetics Research C*, Vol. 3, 57–66, 2008.
- 7. Zhang, H., J. Wang, and C. Tong, "Progress in theoretical design and numerical simulation of high power terahertz backward wave oscillator," *PIERS Online*, Vol. 4, No. 3, 311–315, 2008.
- 8. Lin, M.-C. and P.-S. Lu, "An injection-locked millimeter wave oscillator based on field-emission cathodes," *PIERS Online*, Vol. 4, No. 3, 371–375, 2008.

9. Peidaee, P. and A. Baghai-Wadji, "On the calculation of polynomially perturbed harmonic oscillators," *PIERS Online*, Vol. 3, No. 4, 485–489, 2007.

- 10. Lin, M.-C. and P. S. Lu, "Interaction mechanism of a field emission based THz oscillator," *PIERS Online*, Vol. 3, No. 7, 1011–1015, 2007.
- 11. Chen, Z. and J. Xu, "Design and characterization of a W-band power-combined frequency tripler for high-power and broadband operation," *Progress In Electromagnetics Research*, Vol. 134, 133–150, 2013.
- 12. Siles, J. V., C. Lee, R. Lin, et al., "A high-power 105–120 GHz broadband on-chip power-combined frequency tripler," *Microwave and Wireless Components Letters*, Vol. 25, No. 3, 157–159, IEEE, 2015.
- 13. Bao, M., R. Kozhuharov, and H. Zirath, "A high power-efficiency D-band frequency tripler MMIC with gain up to 7 dB," *Microwave and Wireless Components Letters*, Vol. 24, No. 2, 123–125, IEEE, 2014.
- 14. Kim, S. K., C. Choi, C. Cui, et al., "A W-band signal generation using N-push frequency multipliers for low phase noise," *Microwave and Wireless Components Letters*, Vol. 24, No. 10, 710–712, IEEE, 2014.
- 15. Siles, J. V., C. Lee, R. Lin, et al., "A high-power 105–120 GHz broadband on-chip power-combined frequency tripler," *IEEE Microwave and Wireless Components Letters*, Vol. 25, No. 3, 157–159, 2015.
- 16. Rolland, P. A., J. L. Vaterkowski, E. Constant, and G. Salmer, "New modes of operation for avalanche diodes: Frequency multiplication and upconversion," *IEEE Trans. Microwave Theory Tech.*, Vol. 24, 768–775, 1976.
- 17. Ermak, G. P. and A. V. Varavin, "2-mm wave vector network analyzer upon high-order IMPATT multipliers," *International Journal of Infrared and Millimeter Waves*, Vol. 27, 681–686, 2006.
- 18. Ermak, G. P., A. V. Varavin, and E. A. Alekseev, "Phase locking of 2-mm wave sources upon high-order IMPATT multipliers," *International Journal of Infrared and Millimeter Waves*, Vol. 24, 1609–1615, 2003.
- 19. Huang, J., T. Gan, and Y. Zou, "A novel W-band fully coherent solid-state radar transceiver,", 2001 CIE International Conference on Proceedings Radar, 907–911, 2001.
- 20. Zhao, M., Y. Fan, and Y. Zhang, "The W-band high order avalanche diode frequency multipliers," *International Journal of Infrared and Millimeter Waves*, Vol. 28, 663–669, 2007.
- 21. Rolland, P. A., G. Salmer, A. Derycke, and J. Michel, "Very-high-rank avalanche diode frequency multiplier," *Proceedings of the IEEE*, Vol. 61, 1757–1758, 1973.
- 22. Rolland, P. A., E. Constant, A. Derycke, and J. Michel, "Multiplication de frequence par diode a avalanche en ondes millimetriques," *Acts Electronics*, Vol. 17, 213–228, 1974.
- 23. Kramer, B. M., A. C. Derycke, A. Farrayre, and C. F. Masse, "High-efficiency frequency multiplication with GaAs avalanche diodes," *IEEE Trans. Microwave Theory Tech.*, Vol. 24, 861–863, 1976.
- 24. Venger, A. Z., A. N. Ermak, and A. M. Yakimenko, "Frequency multiplier based on an avalanche-and-transit diode," *Instruments and Experimental Techniques*, Vol. 23, 691–692, 1980.
- 25. Haddad, G. I., P. T. Greiling, and W. E. Schroeder, "Basic principles and properties of avalanche transit-time devices," *IEEE Trans. Microwave Theory Tech.*, Vol. 18, 752–772, 1970.
- 26. Read, W. T., "A proposed high frequency negative resistance diode," *Bell System Tech. Journal*, Vol. 37, 400–446, 1958.
- 27. Constant, E., E. Allamando, and A. Semichon, "Transit-time operation of an avalanche diode driven by a subharmonic signal and its application to frequency multiplication," *Proceeding of the IEEE*, Vol. 58, 483–484, 1970.
- 28. Gilden, M. and M. E. Hines, "Electronic tuning effects in the read microwave avalanche diode," *IEEE Transactions on Electron Devices*, Vol. 13, 169–175, 1966.
- 29. Sze, S. M., Physics of Semiconductor Devices, 3rd edition, Wiley, New York, 2006.

- 30. Zhao, M., J. Zhan, Y. Fan, Z. He, and Y. Zhang, "A novel W-band microstrip integrated avalanche diode high order frequency multiplier," *International Journal of Infrared and Millimeter Waves*, Vol. 29, 741–747, 2008.
- 31. Wu, Z., "Electromagnetic analysis of the oscillator with a cap structure," *Journal of Chengdu Institution of Radio Engineer*, Vol. 4, 68–77, 1981.
- 32. Zhao, M., Y. Fan, D. Wu, and Z. He, "The investigation of W band microstrip integrated high order frequency multiplier based on the nonlinear model of avalanche diode," *Progress In Electromagnetics Research*, Vol. 85, 439–453, 2008.