Progress In Electromagnetics Research M, Vol. 63, 195–206, 2018

Random Radiation Source Optimization Method for Microwave Staring Correlated Imaging Based on Temporal-Spatial Relative Distribution Entropy

Qingquan Meng*, Tingting Qian, Bo Yuan, and Dongjin Wang

Abstract—Microwave Staring Correlated Imaging (MSCI) is a high-resolution radar imaging modality, whose resolution is mainly determined by the randomness of radiation source. To optimize the design of random radiation source, a novel concept of temporal-spatial relative distribution entropy (TSRDE) is proposed to describe the temporal-spatial stochastic characteristics of radiation source. The TSRDE can be utilized as the optimization criterion to design the array configuration and signal parameters by means of optimization algorithms. In this paper the genetic algorithm is applied to search for the best design. Numerical simulations are performed and the results show that the TSRDE is an effective method to characterize the randomness of radiation source, and the source parameters optimized by this method can dramatically improve the imaging resolution.

1. INTRODUCTION

Radar imaging technology utilizes the electromagnetic wave to get object information and obtain the images of the target [1, 2]. Compared with optical imaging, microwave radar imaging has many advantages, such as large coverage, strong permeability, and the capability of working at all time and under all-weather conditions. In recent years, kinds of radar imaging methods [3–6] are investigated. In [3], a microwave imaging method based on particle swarm algorithm for reconstructing two-dimensional dielectric scatterers is presented. Donelli et al. [4] proposed a novel stochastic microwave method based on evolutionary algorithm for the detection, location and reconstruction of electric properties of breast cancer. An iterative multi-resolution method for the reconstruction of the unmeasured components of the equivalent current density is investigated in [5]. An innovative two-step methodology for the microwave imaging is mentioned in [6].

Different from these methods above, Microwave Staring Correlated Imaging (MSCI) is a newly proposed high resolution radar imaging modality [7–10]. By transmitting temporal-spatial stochastic radiation field (TSSRF) and correlating the received signals and TSSRF, MSCI could achieve high-resolution imaging results while its platform remains stationary. The MSCI system can be carried on the floating balloon and other air stationary platforms for continuous high-resolution staring observation, and it has good potential applications in disaster monitoring, remote sensing, resource exploration and other fields.

The resolution of MSCI depends on the orthogonality of TSSRF [9], thus to obtain high resolution imaging results we should optimize the design of microwave random source. In recent years, several efficient methods to construct the random microwave source are proposed. The pseudo-random gold sequence is used as the emitting signals in [7–9]. The zero-mean Gaussian noise is utilized to modulate

Received 29 September 2017, Accepted 2 December 2017, Scheduled 13 January 2018
* Corresponding author: Qingquan Meng (mengqq@mail.ustc.edu.cn).
The authors are with the Key Laboratory of Electromagnetic Space Information, Chinese Academy of Sciences, University of Science and Technology of China, Hefei 230027, China.

the frequency, phase and amplitude in [10–12]. The random frequency hopping signals are applied to construct microwave source in [13]. But the methods to design the array geometry and signal parameters are not involved. The researches in [14] investigate design method of array configuration based on the effective rank theory. But this method is based on the randomness evaluation of radiation field, and the computational complexity dramatically increases when we calculate the effective rank.

This paper focuses on the effective optimization method to design the array geometry and signal parameters in order to construct high-performance random source. Different from traditional radar system, in MSCI the array configuration and the signal parameters must be stochastic to radiate the TSSRF. The methods to describe the randomness of stochastic radiation field have been deeply studied in [9, 15]. However, to the best of our knowledge, the feasible method that can quantitatively characterize the random performance of the radiation source have not been investigated. So we have to define an effective quantity to evaluate the randomness of radiation source.

Entropy is an effective method to characterize the chaotic degree of a system. In information theory, information entropy or Shannon entropy is used to characterize the uncertainty degree of the random variables, and is widely used in radar system designing. In [16, 17], mutual information is utilized to design radar waveform. The researches in [18, 19] apply relative entropy to select the waveform in the point target model.

In this paper, we propose a new quantity named temporal-spatial relative distribution entropy (TSRDE) to describe the randomness of stochastic radiation source. The TSRDE applies information theory to calculate the probabilistic distribution of relative parameter vector, and utilizes the entropy method to characterize the chaotic degree of relative parameter vector distribution in multi-dimensions including the position of transmitters, emission frequency, amplitude, etc.. Thus the TSRDE can quantitatively describe the random performance of the radiation source. Then utilizing TSRDE as the optimization criterion, we can optimize the design of the array configuration and signal parameters by means of optimization algorithms. As for such complex optimization problem, the traditional optimization algorithms easily fall into the local optimum. Compared with the traditional optimization algorithms, evolutionary algorithms [20] inspired by the biological evolution are global optimization methods with high robustness and wide applicability. Genetic Algorithm (GA) is a kind of evolutionary algorithm which has the property of fast global searching [21]. In [22, 23], GA is used to design the waveform and antenna of MIMO radar. In this paper we utilize the GA as the optimization algorithm to search for the best design.

The biggest advantage of this design method is that it can make full use of the limited array aperture and limited frequency bandwidth resources to construct the TSSRF as random as possible. Another merit of this method is that the spatial arrangement and signal parameters of radiation sources can be integratedly designed to guarantee the optimization results are global optimal. In the past, these parameters are designed independently so the overall effect can not be ensured as the best. The simulations verify the superiority of this method.

The rest of this paper is organized as follows. In Section 2, the mathematical model of MSCI is established. The novel design method based on temporal-spatial relative distribution entropy (TSRDE) is proposed in Section 3. In Section 4, the extensive numerical simulations are presented to evaluate the performance of this method. In Section 5 we conclude this paper.

2. MATHEMATICAL MODEL OF MSCI

Consider the Microwave Staring Correlated Imaging system which contains L transmitters and a receiver, as illuminated in Fig. 1.

Let (x, y, z) be the Cartesian coordinate with the origin O. The MSCI radar system is placed on a stationary platform. The aperture of radar array is denoted as D, the imaging region is labeled as S. $\vec{r}_i'$ is the position coordinate vector of the i-th transmitter. D_i represents the antenna aperture of the i-th transmitter. $f_i(t)$ is the emission signal of the i-th transmitter. The receiver is located at $\vec{r}_0'$. The coordinate vector of the targets on imaging region is $\vec{r}$.

In MSCI model mentioned above, using the time-domain Green function, the stochastic radiation

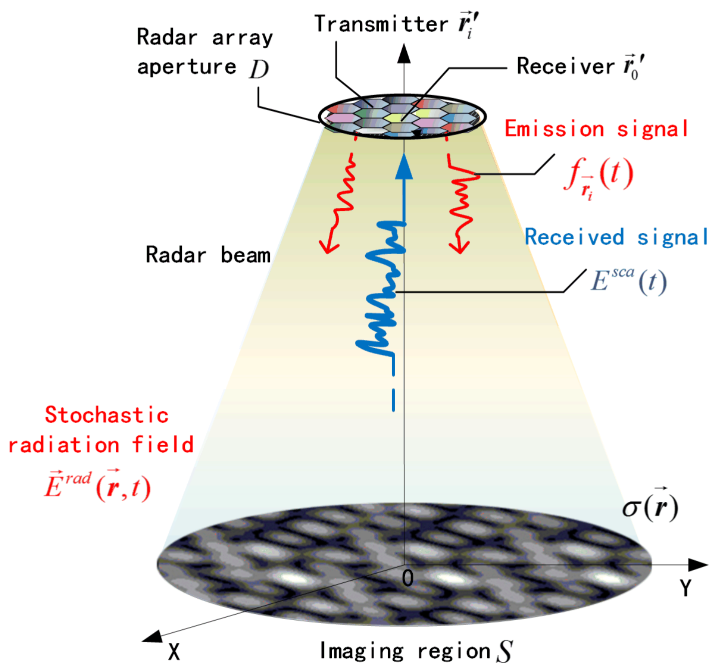

Figure 1. The imaging scenario of microwave staring correlated imaging (MSCI) system.

field $\vec{E}^{rad}(t,\vec{r})$ generated by random radiation source is expressed as follows [8, 24]:

$$\vec{E}^{rad}(t,\vec{r}) = \sum_{i=1}^{L} \int_{D_i} \frac{1}{4\pi|\vec{r}-\vec{r}'|} \vec{E}^{ape}\left(t - \frac{|\vec{r}-\vec{r}'|}{c}, \vec{r}'\right) d\vec{r}' \tag{1}$$

where c is the speed of light, and $\vec{E}^{ape}(t,\vec{r}')$ is the aperture field of transmitter antenna.

Using the first-order Born approximation, the received echo signal $E^{sca}(t)$ can be expressed as follows [8]:

$$E^{sca}(t) = \int_{S} \frac{\vec{E}^{rad}(t - |\vec{r}-\vec{r}_0'|/c, \vec{r})}{4\pi|\vec{r}-\vec{r}_0'|} \sigma(\vec{r}) d\vec{r} \tag{2}$$

where $\sigma(\vec{r})$ denotes the backscattering coefficient distribution on the imaging region S. We define the modified radiation field $\vec{E}_c^{rad}(t,\vec{r})$ as:

$$\begin{aligned} \vec{E}_c^{rad}(t,\vec{r}) &= \frac{\vec{E}^{rad}(t - |\vec{r}-\vec{r}_0'|/c, \vec{r})}{4\pi|\vec{r}-\vec{r}_0'|} \\ &= \frac{1}{(4\pi)^2|\vec{r}-\vec{r}_0'|} \sum_{i=1}^{L} \int_{D_i} \frac{1}{|\vec{r}-\vec{r}'|} \vec{E}^{ape}\left(t - \frac{|\vec{r}-\vec{r}'| + |\vec{r}-\vec{r}_0'|}{c}, \vec{r}'\right) d\vec{r}' \end{aligned} \tag{3}$$

In this paper, we assume that the transmitter antennas are point sources, thus the aperture field of the i-th transmitter antenna can be expressed as:

$$\vec{E}^{ape}(t,\vec{r}') = \delta(\vec{r}' - \vec{r}_i') \cdot f_i(t) \tag{4}$$

Further assumption can be made that the emission signal $s_i(t)$ is random frequency hopping pulse signal [13] described as follows:

$$s_i(t) = \sum_{m=1}^{M} \text{rect}\left(\frac{t-mT}{T}\right) \exp\{j2\pi f_{mi} t\} \tag{5}$$

where T is the period of pulse, and f_{mi} is the emission frequency of the i-th transmitter in the m-th pulse. Substituting Eq. (4) and Eq. (5) into Eq. (3), we can get the specific expression of radiation field:

$$\vec{E}_c^{rad}(t,\vec{r}) = \frac{1}{(4\pi)^2|\vec{r}-\vec{r}_0'|}\sum_{i=1}^{L}\sum_{m=1}^{M}\frac{1}{|\vec{r}-\vec{r}'|}\mathrm{rect}\left(\frac{t-mT}{T}\right)\exp\left\{j2\pi f_{mi}\left(t-\frac{|\vec{r}-\vec{r}'|+|\vec{r}-\vec{r}_0'|}{c}\right)\right\} \quad (6)$$

Therefore, the received echo signal can be rewritten as:

$$E^{sca}(t) = \int_S \vec{E}_c^{rad}(t,\vec{r})\sigma(\vec{r})d\vec{r} \quad (7)$$

Equation (7) is the imaging equation of integral type. To solve it by numerical computation, Eq. (7) should be discretized as a matrix equation [7]. The time domain is discretized as $\{t_1, t_2, \ldots, t_m, \ldots, t_M\}$ and the imaging region is discretized as $\{\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_n, \ldots, \vec{r}_N\}$, $\vec{r}_n$ is the position vector of the center of the n-th imaging grid cell. The imaging region is divided into N imaging grid cells according to the minimum unit to be resolved. Let $\sigma_n = \sigma(\vec{r}_n)$ which stands for the backscattering coefficient of the n-th imaging grid cell. The scattering coefficient vector is expressed as:

$$\sigma = [\sigma_1, \sigma_2, \ldots, \sigma_n, \ldots, \sigma_N]^T \quad (8)$$

Then Eq. (7) can be discretized as:

$$\begin{bmatrix} E^{sca}(t_1) \\ E^{sca}(t_2) \\ \vdots \\ E^{sca}(t_M) \end{bmatrix} = \begin{bmatrix} E_c^{rad}(t_1,\vec{r}_1) & E_c^{rad}(t_1,\vec{r}_2) & \cdots & E_c^{rad}(t_1,\vec{r}_N) \\ E_c^{rad}(t_2,\vec{r}_1) & E_c^{rad}(t_2,\vec{r}_2) & \cdots & E_c^{rad}(t_2,\vec{r}_N) \\ \cdots & \cdots & \cdots & \cdots \\ E_c^{rad}(t_M,\vec{r}_1) & E_c^{rad}(t_M,\vec{r}_2) & \cdots & E_c^{rad}(t_M,\vec{r}_N) \end{bmatrix} \cdot \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_N \end{bmatrix} \quad (9)$$

Here $E_c^{rad}(t_m, \vec{r}_n)$ is the radiation field located at $\vec{r}_n$ and at the time t_m, corresponding to the matrix element at the m-th row and n-th column. Eq. (9) can be written as matrix form:

$$E^{sca} = E_c^{rad} \cdot \sigma \quad (10)$$

Equation (10) is called the imaging equation of MSCI. It is the mathematical model that describes the relationship among received echo signal, backscattering coefficient and modified radiation field. Target reconstruction can be achieved by solving Eq. (10) using various correlation algorithms.

3. TEMPORAL-SPATIAL RELATIVE DISTRIBUTION ENTROPY

The concept of temporal-spatial relative distribution entropy (TSRDE) is inspired by the entropy in thermodynamics. The entropy in thermodynamics is utilized to describe the disorder degree of the molecules system. In MSCI, we can find that a given stochastic radiation source can be thought as a set of discrete points of parameter vectors in a multidimensional space which includes space, time and other parameters. The more chaotic the distribution of the point set of parameter vectors, the better the performance of stochastic radiation sources will be. So it is feasible to utilize the concept of entropy to describe the chaotic degree of the radiation source.

The main idea of TSRDE is to regard the relative parameter vectors as a random vector and then compute the probabilistic distribution of relative parameter vectors. Thus we can obtain the entropy of relative parameter vectors, we define it as temporal-spatial relative distribution entropy because the relative parameter vector includes both the array geometry in spatial domain and the signal parameters in temporal domain.

The calculation flow of the TSRDE is described as follows:

Step 1: Calculate the relative parameter vectors of any different time slices and antennas according to the position coordinates, frequency, phase and amplitude.

Step 2: Make equispaced division to the range of each dimension of the relative parameter vectors.

Step 3: Work out the normalized probabilistic distribution of each dimension of the relative parameter vector.

Step 4: Calculate the distribution entropy of each dimension according to the normalized probabilities.

Step 5: Make weighted summation of each dimensional distribution entropy to get the TSRDE which quantitatively describes the randomness of the radiation sources.

In practice, the transmitters, whose positions are not changed with time, are usually distributed in a plane. What's more, the polarization and phase modulation are generally not considered, and the amplitude modulation lacks effectiveness in the process of designing the stochastic radiation sources. Therefore, in this paper, we only consider the optimization of transmitter position and emission frequency of the random frequency hopping signal.

The parameter vector of random radiation source is defined as:

$$\vec{v}_{ij} = (x_i, y_i; f_{ij}) \quad i = 1, \ldots, N; j = 1, \ldots, M \tag{11}$$

N is the number of transmitters, M the number of time slices, and x_i and y_i are x coordinate and y coordinate of the i-th transmitter. f_{ij} is the emitting frequency of the i-th transmitter and j-th time slice. All of the parameter vectors make up a set labeled as V.

$$V = \{\vec{v}_{ij} = (x_i, y_i; f_{ij}) |\ i = 1, \ldots, N; j = 1, \ldots, M\} \tag{12}$$

Calculate the relative parameter vector set U based on the parameter vector set.

$$\begin{aligned} U = \{&\vec{u}_{ijkl} = (x_i - x_k, y_i - y_k; f_{ij} - f_{kl}) | \\ &i = 1, \ldots, N; j = 1, \ldots, M; k = 1, \ldots, N; l = 1, \ldots, M; i \neq k \,\text{or}\, j \neq l\} \end{aligned} \tag{13}$$

Decompose each dimension of the set U into sub-set.

$$\begin{cases} U_x = \{\Delta x_{ik} | i = 1, \ldots, N; k = 1, \ldots, N; i \neq k\} \\ U_y = \{\Delta y_{ik} | i = 1, \ldots, N; k = 1, \ldots, N; i \neq k\} \\ U_f = \{\Delta f_{ijkl} | i = 1, \ldots, N; j = 1, \ldots, M; k = 1, \ldots, N; l = 1, \ldots, M; i \neq k \,\text{or}\, j \neq l\} \end{cases} \tag{14}$$

Make equispaced division to the range of U_x, U_y, U_f. A_q is the q-th sub-interval of the range of U_x, U_y. B_q is the q-th sub-interval of U_f.

$$\begin{cases} A_q = ((q-1)a, qa], \quad q = 1, \ldots, N \\ B_q = ((q-1)b, qb], \quad q = 1, \ldots, C_{NM}^2 \end{cases} \tag{15}$$

where a is interval of A_q, and b is the interval of B_q. Denote that the number of elements of U_x, U_y, U_f that fall in the q-th sub interval is g_q:

$$\begin{cases} g_q^{(x)} = \wp\{\Delta x_{ik} \in A_q | i = 1, \ldots, N; k = 1, \ldots, N; i \neq k\} \\ g_q^{(y)} = \wp\{\Delta y_{ik} \in A_q | i = 1, \ldots, N; k = 1, \ldots, N; i \neq k\} \\ g_q^{(f)} = \wp\{\Delta f_{ijkl} \in B_q | i = 1, \ldots, N; j = 1, \ldots, M; k = 1, \ldots, N; l = 1, \ldots, M; i \neq k \,\text{or}\, j \neq l\} \end{cases} \tag{16}$$

where $\wp\{\bullet\}$ is the operator that numerates the number of elements in a set. The normalized probabilities of each dimension can be calculated as:

$$\begin{cases} p_q^{(x)} = g_q^{(x)} \Big/ \sum\limits_{m=1}^{(N-1)N/2} g_m^{(x)}, q = 1, \ldots, (N-1)N/2 \\ p_q^{(y)} = g_q^{(y)} \Big/ \sum\limits_{m=1}^{(N-1)N/2} g_m^{(y)}, q = 1, \ldots, (N-1)N/2 \\ p_q^{(f)} = g_q^{(f)} \Big/ \sum\limits_{m=1}^{(NM-1)NM/2} g_m^{(f)}, q = 1, \ldots, (NM-1)NM/2 \end{cases} \tag{17}$$

The distribution entropy of each dimension can be obtained from the respective normalized probabilities:

$$\begin{cases} H_x = -\sum\limits_{q=1}^{(N-1)N/2} p_q^{(x)} \log p_q^{(x)} \\ H_y = -\sum\limits_{q=1}^{(N-1)N/2} p_q^{(y)} \log p_q^{(y)} \\ H_f = -\sum\limits_{q=1}^{(NM-1)NM/2} p_q^{(f)} \log p_q^{(f)} \end{cases} \tag{18}$$

where H_x, H_y, H_f are the distribution entropies of the dimensions. Assume that the weights of the dimensional distribution entropies are w_x, w_y, w_f. The TSRDE is defined as:

$$H = w_x H_x + w_y H_y + w_f H_f \tag{19}$$

The larger value the TSRDE has, the more random the distribution of source parameters is. Thus the TSRDE quantitatively describes the stochastic performance of radiation sources.

Utilizing the TSRDE as the optimization criterion, we can design the array geometry and signal parameters by means of optimization algorithm. The optimization problem can be described as:

$$\begin{aligned} &\vec{v}_{ij} = \arg \max_{\vec{v}} H(\vec{v}) \quad (i = 1, \ldots, N \; j = 1, \ldots, M) \\ &\text{st. } \vec{v}_{ij} \in [x_{\min}, x_{\max}] \cup [y_{\min}, \mathrm{y}_{\max}] \cup [f_{\min}, f_{\max}] \end{aligned} \tag{20}$$

where the $[x_{\min}, x_{\max}]$ $[y_{\min}, \mathrm{y}_{\max}]$ $[f_{\min}, f_{\max}]$ are the ranges of x_i, y_i, f_{ij}. The genetic algorithm (GA), which has the property of fast global searching, will be utilized under the principle of maximizing the TSRDE.

The algorithm flow of optimization is described in detail as follows:

Step 1: Parameter initialization. Initial the aperture area of random radiation source s, the diameter of array element d, the number of array elements N_a, the frequency range, the number of individuals in a population N_g, the number of generations N_p, the probability of crossover p_c, and the probability of mutation p_m.

Step 2: Individual coding. Denote $I_k = v_{11} v_{12} \ldots v_{1M} \ldots v_{N1} v_{N2} \ldots v_{NM}$ as a individual. One individual represents a certain parameter setting of the radiation source, then generate N_g individuals randomly to constitute the initial population $T_0 = \{I_k^{(0)} | k = 1, \ldots, N_g\}$.

Step 3: The TSRDE of individual $H(I_k)$ is denoted as its fitness, and calculate the fitness of initial population according to the definition of TSRDE.

Step 4: Crossover operation. Generate a random number δ between [0,1]. If $\delta < p_c$, the crossover point $d_1 d_2$ is confirmed by the roulette wheel selection method [25]. Assume that the father individual is $I^{(father)} = v_{11}^{(f)} v_{12}^{(f)} \ldots v_{1M}^{(f)} \ldots v_{N1}^{(f)} v_{N2}^{(f)} \ldots v_{NM}^{(f)}$, the mother individual is $I^{(mother)} = v_{11}^{(m)} v_{12}^{(m)} \ldots v_{1M}^{(m)} \ldots v_{N1}^{(m)} v_{N2}^{(m)} \ldots v_{NM}^{(m)}$, the new individual after crossover operation is $I^{(son)} = v_{11}^{(f)} \ldots v_{d_1 d_2}^{(f)} v_{d_1 (d_2+1)}^{(m)} \ldots v_{NM}^{(m)}$. The other individuals are generated in the same way, finally a new population $T_q = \{I_k^{(q)} | k = 1, \ldots, N_g\}$ is constructed.

Step 5: Mutation operation. Generate a random number η between [0, 1]. If $\eta < p_m$, the mutation point w is randomly selected, and the value of the $w - th$ element of the individual I_k is assigned a new random number.

Step 6: Calculate the fitness of each individual in the new population T_q.

Step 7: Determine whether the maximum generation number is reached. If the termination criterion is satisfied, the algorithm is stopped, the individual whose fitness is maximal is the final result. Else return to Step 4.

4. NUMERICAL SIMULATIONS

In this section, we present several numerical simulation results to evaluate the performance of the proposed method. Firstly, the verification simulations that apply the proposed method to optimize the array configuration are done. Then we deeply investigate the relationship between the TSRDE and the number of effective singular value of TSSRF [14] which characterizes the randomness of radiation field. Finally, we simulate the imaging results of different radiation source design methods to test the effectiveness of the proposed radiation source optimization method.

4.1. Simulation 1

In simulation 1, we apply the proposed method to optimize the array configuration. The aperture area is 1.6 m × 1.6 m, the number of transmitters is 25. Firstly we work out the TSRDE of uniform array configuration, that is 3.1368, shown in Fig. 2(a), then we utilize the genetic algorithm to calculate the

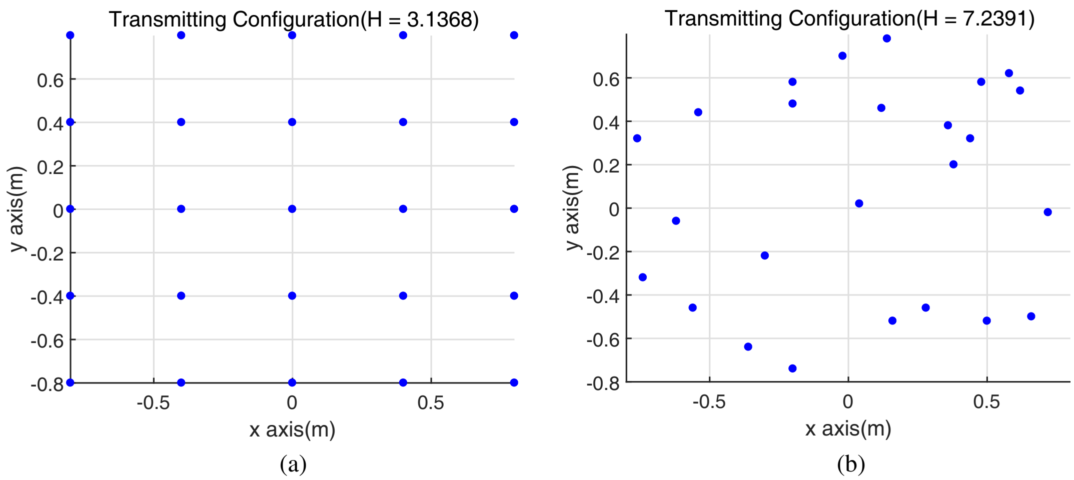

Figure 2. The optimization of position configuration based on TSRDE. (a) The uniform array configuration, whose TSRDE is 3.1368. (b) The optimal array configuration, whose TSRDE is 7.2391.

optimal transmitter position configuration, the optimal results are shown in Fig. 2(b). The TSRDE of optimal result is 7.2391. After optimization the TSRDE is greatly promoted.

4.2. Simulation 2

In MSCI radar system, the stochastic characteristics of radiation sources determine the orthogonality of random radiation field. In this subsection, we aim to verify the effectiveness of TSRDE by investigating the relationship between TSRDE and the orthogonality of radiation field. Here the number of effective singular value of TSSRF [14] is utilized to quantitively characterize the orthogonality of radiation field.

In simulation 2, firstly we apply the proposed method to optimize the design of radiation source in three different source configuration strategies under various time slices number, aperture length and bandwidth. Then we calculate the number of effective singular value of the radiation field constructed by the optimal source configuration. Finally by comparing the relationship between the TSRDE and number of effective singular value, we can evaluate the effectiveness of TSRDE.

The three strategies to configure the source parameters are described as follows:

Strategy A: All of the transmitters emit the same frequency in the same time slice, while the emission frequencies are stochastic in different time slices. The array geometry is uniformly arranged.

Strategy B: Generate M different subintervals in given bandwidth as the frequency interval of each time slice. The length of subintervals are determined by Fibonacci sequence. And then select N different frequencies randomly in each subinterval as the emission frequency of the transmitters. The array geometry is randomly distributed.

Strategy C: In each time slice, select N random frequencies in total bandwidth as the emission frequency of the transmitters. That is to say the frequency intervals of each time slice are the total bandwidth. The array configuration is randomly arranged.

The aperture length and total bandwidth of the three strategies are all the same.

Figure 3 shows the TSRDE and the effective singular value number under various time slices number, aperture length and bandwidth. The number of transmitters remains 25. The number of time slices changes from 200 to 2000. The aperture length ranges from 1 m to 4 m. The bandwidth ranges from 100 MHz to 1 GHz. Fig. 3(a), Fig. 3(c) and Fig. 3(e) show the TSRDE curve of the three strategies under various time slices number, aperture length and bandwidth. Fig. 3(b), Fig. 3(d) and Fig. 3(f) show the curve of effective singular value number of the three strategies under various time slices number, aperture length and bandwidth.

It is obvious that the TSRDE rises with the increase of time slices number, aperture length and bandwidth. This result is coincident with the theory. And the TSRDE of Strategy C is the highest among all of the strategies, which means the best stochastic performance, because the frequency intervals of each time slices are the biggest in the three strategies. The number of effective singular value also rises with the increasing of time slices number, aperture length and bandwidth, the variation trends are almost the same as that of the TSRDE under different conditions, that is to say TSRDE can effectively reveal the orthogonality of random radiation field. These simulation results prove that the TSRDE is an effective quantity to describe the randomness of radiation source.

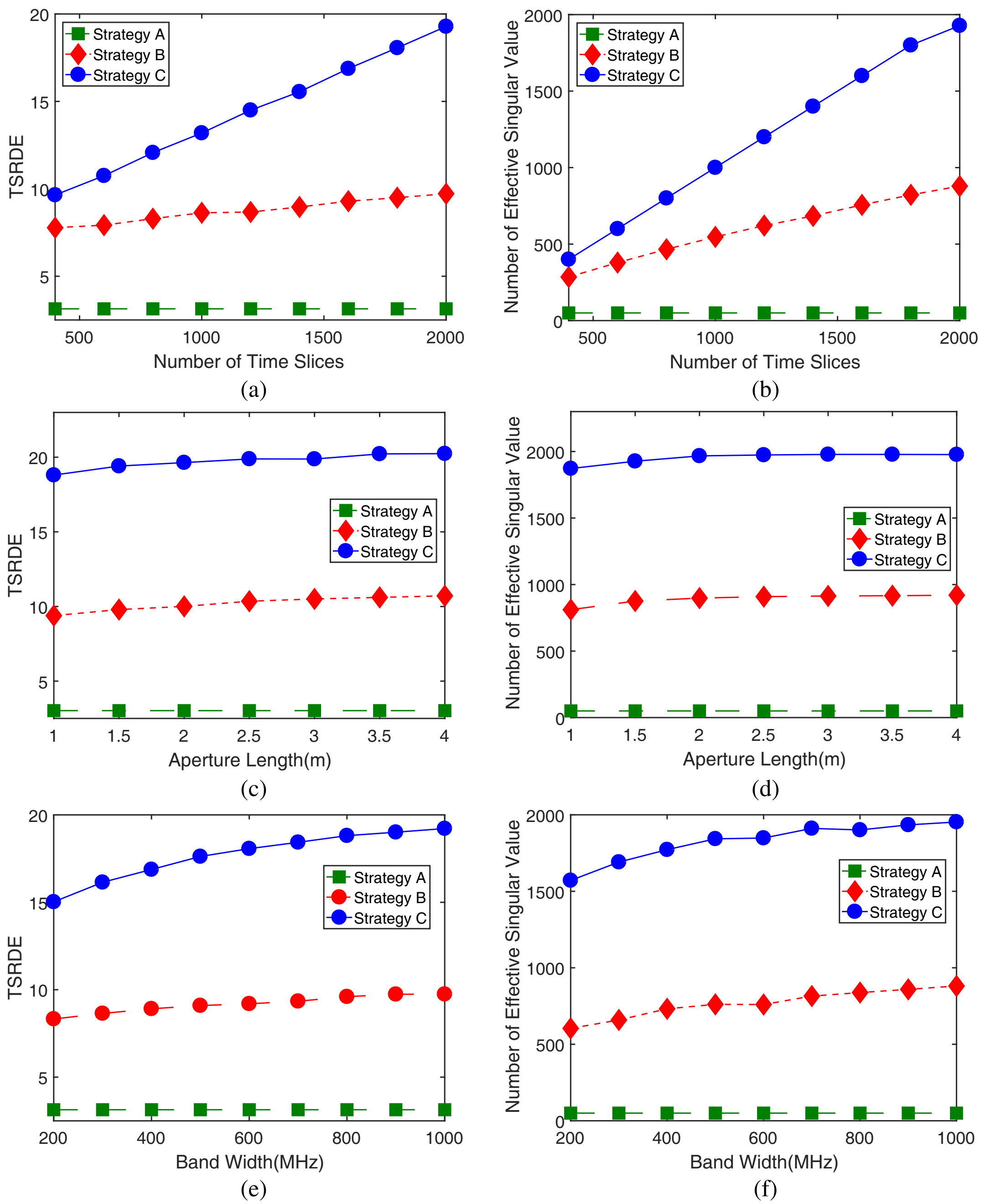

Figure 3. The TSRDE and the number of effective singular value under different conditions. (a) The TSRDE under different time slices number. (b) The number of effective singular value under different time slices number. (c) The TSRDE under different aperture length. (d) The number of effective singular value under different aperture length. (e) The TSRDE under different bandwidth. (f) The number of effective singular value under different bandwidth.

4.3. Simulation 3

In simulation 3, we investigate the effectiveness of the optimization method based on TSRDE by comparing the imaging performance of different radiation source design methods. Theoretically speaking, the radiation source with better randomness can achieve higher imaging resolution. In this

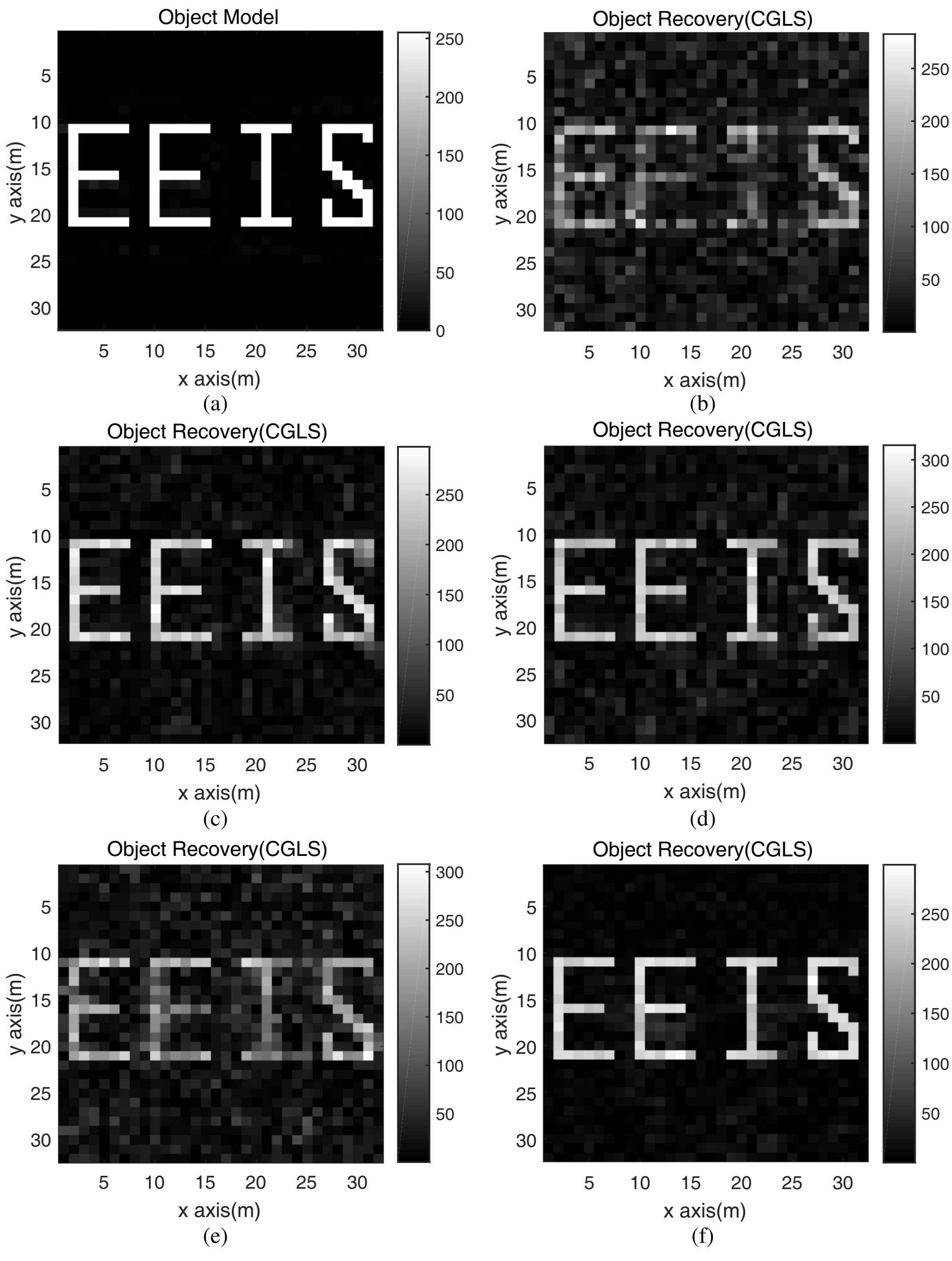

Figure 4. The imaging results of different design methods. (a) The imaging object. (b) The imaging result of design method 1. (c) The imaging result of design method 2. (d) The imaging result of design method 3. (e) The imaging result of design method 4. (f) The imaging result of design method 5.

subsection we utilize the normalized mean square error (NMSE) to evaluate the imaging resolution. NMSE denotes the differences between the recovery result and the true object. Thus the lower NMSE represents the better imaging performance. The random source design methods include:

Method 1 [13]: The transmitters emit random hopping frequency signal, the signal frequency obeys uniform distribution. The array geometry is uniformly arranged.

Method 2 [7]: The transmitters emit pseudo-random gold sequence signal. The array geometry is randomly arranged.

Method 3 [10]: The frequency of emitting signal is modulated by zero-mean Gaussian noise. The array geometry is randomly arranged.

Method 4 [14]: The transmitters emit random hopping frequency signal, the signal frequency obeys uniform distribution. The array configuration is optimized based on the effective rank theory, the optimization algorithm is GA.

Method 5: The transmitters emit random hopping frequency signal. The emitting frequency and array geometry are integratedly designed by optimization method based on TSRDE.

Figure 4 shows the recovery images of the five design methods using CGLS algorithm [26]. The radiation source parameters of these design methods are all the same. The signal bandwidth is 1 GHz and carrier frequency is 10 GHz. The SNR is 20 dB. The number of time slices is 2000 and the number of transmitters is 25. The aperture area is $1.6\,\mathrm{m} \times 1.6\,\mathrm{m}$. The imaging object is given in Fig. 4(a), the recovery results of design method 1, design method 2, design method 3, design method 4 and design method 5 are respectively shown in Fig. 4(b), Fig. 4(c), Fig. 4(d), Fig. 4(e) and Fig. 4(f). The NMSE of recovery results obtained from 20 Monte-Carlo trials are listed in Table 1.

Table 1. The NMSE of different design methods obtained from 20 monte-carlo trials.

Design Method	NMSE
Method 1	1.2135
Method 2	0.7256
Method 3	0.6993
Method 4	1.1976
Method 5	0.5188

It can be seen that the imaging result of design method 5 achieves the best resolution performance, whose NMSE is the lowest of all. The reason is that the randomness of radiation source is effectively characterized by TSRDE and the frequency and array geometry are integratedly optimized. The recovery result of design method 4 is slightly better than that of design method 1, because the array configuration is designed by the method proposed in [14]. The results of design method 2 and 3 are not as good as that of design method 5, because the frequency and array geometry are randomly generated without optimization process. These simulation results verify the effectiveness of the radiation source optimization method based on TSRDE.

5. CONCLUSION

In this paper, a novel concept of temporal-spatial relative distribution entropy (TSRDE) is proposed to describe the temporal-spatial stochastic characteristics of radiation source in the microwave staring correlated imaging system. We can utilize it as the optimization criterion to design the array configuration and signal parameters by means of optimization algorithms. Numerical simulations are performed and the results show that the TSRDE is an effective method to characterize the randomness of radiation source, and the source parameters optimized by the proposed algorithm dramatically improve the imaging resolution. In this paper we only consider the optimization of the array geometry and the frequency modulation. In the future we will take more parameters into consideration to realize the joint optimization of frequency, phase, amplitude and polarization, thus the spatial resolution can be further increased.

ACKNOWLEDGMENT

This work is supported by National Natural Science Foundation of China under Grant No. 61431016 and the Hi-Tech Research and Development Program of China under Grant Project No. 2013AA122903.

REFERENCES

1. Ausherman, D. A., A. Kozma, J. L. Walker, H. M. Jones, and E. C. Poggio, "Development in radar imaging," *IEEE Trans. Aerospace and Electronic Systems*, Vol. 20, 363–397, 1984.
2. David, S. S., "Remote sensing with imaging radar: A review," *Geoforum*, Vol. 1, 61–74, 1970.
3. Caorsi, S., M. Donelli, A. Lommi, and A. Massa, "Location and imaging of two-dimensional scatterers by using a particle swarm algorithm," *Journal of Electromagnetic Waves and Applications*, Vol. 18, 481–494, 2004.
4. Donelli, M., I. Craddock, D. Gibbins, and M. Sarafianou, "A three-dimensional time domain microwave imaging method for breast cancer detection based on an evolutionary algorithm," *Progress In Electromagnetics Research M*, Vol. 18, 179–195, 2012.
5. Rocca, P., M. Donelli, G. L. Gragnani, and A. Massa, "Iterative multi-resolution retrieval of non-measurable equivalent currents for the imaging of dielectric objects," *Inverse Problems*, Vol. 25, 1–15, 2009.
6. Franceschini, G., M. Donelli, R. Azaro, and A. Massa, "Inversion of phaseless total field data using a two-step strategy based on the iterative multiscaling approach," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 44, 3527–3539, 2006.
7. Guo, Y., X. He, and D. Wang, "A novel super-resolution imaging method based on stochastic radiation radar array," *Measurement Science and Technology*, Vol. 24, No. 7, 31–36, 2013.
8. He, X., B. Liu, and D. Wang, "A novel approach of high spatial-resolution microwave staring correlated imaging," *Proceedings of 2013 Asia-Pacific Conference on Synthetic Aperture Radar*, 75–78, Tsukuba, Japan, September 2013.
9. Ma, Y., X. He, and D. Wang, "Microwave staring correlated imaging and resolution analysis," *Proceedings of 2013 Geo-Informatics in Resource Management and Sustainable Ecosystem International Symposium*, 75–78, Wuhan, China, November 2013.
10. Li, D., X. Li, Y. Cheng, Y. Qin, and H. Wang, "Radar coincidence imaging: An instantaneous imaging technique with stochastic signals," *IEEE Transactions on Geoscience Remote Sensing*, Vol. 52, No. 4, 2261–2271, 2014.
11. Zhu, S., A. Zhang, Z. Xu, and X. Dong, "Radar coincidence imaging with random microwave source," *IEEE Antennas and Wireless Propagation Letters*, Vol. 14, 1239–1242, 2015.
12. Li, D., X. Li, and Y. Cheng, "Three dimensional radar coincidence imaging," *Progress In Electromagnetics Research M*, Vol. 33, 223–238, 2013.
13. Zhou, X., H. Wang, Y. Cheng, Y. Qin, and H. Chen, "Radar coincidence imaging for off-grid target using frequency hopping waveforms," *International Journal of Antennas and Propagation*, Vol. 2016, 1–16, 2016.
14. Zha, G., H. Wang, and Z. Yang, "Effect analysis and design on array geometry for coincidence imaging radar based on effective rank theory," *Proceedings of 2015 ISPRS International Conference on Computer Vision in Remote Sensing*, 1–8, Xiamen, China, April 2015.
15. Guo, Y., D. Wang, and C. Tian, "Research on sensing matrix characteristics in microwave staring correlated imaging based on compressed sensing," *Proceedings of 2014 IEEE International Conference on Imaging Systems and Techniques*, 1–6, Island of Santorini, Greece, October 2014.
16. Bell, M. R., "Information theory and radar waveform design," *IEEE Trans. on Information Theory*, Vol. 9, 1578–1597, 1993.
17. Luo, Y., Z. Zhao, and C. Luo, "MIMO-OTHR waveform optimization based on the mutual information theory," *Progress In Electromagnetics Research M*, Vol. 46, 69–80, 2016.
18. Tang, B., J. Tang, and Y. Peng, "MIMO radar waveform design in colored noise based on information theory," *IEEE Transactions on Signal Processing*, Vol. 58, No. 9, 4684–4697, 2010.

19. Maherin, I. and Q. Liang, "Radar sensor network for target detection using Chernoff information and relative entropy," *Physical Communication*, Vol. 13, 244–252, 2014.
20. Liu, W., Y. Lu, and M. Lesturgie, "Evolutionary algorithms based sparse spectrum waveform optimization," *Principles of Waveform Diversity and Design*, Vol. 2011, 152–162, 2011.
21. Mishra, A. and A. Shukla, "Mathematical analysis of schema survival for genetic algorithms having dual mutation," *Soft Computing*, Vol. 1, 1–9, 2011.
22. Boudamouz, B., P. Millot, and C. Pichot, "MIMO antenna design with genetic algorithm for TTW radar imaging," *Proceedings of 2012 EuRAD 9th European Radar Conference*, 150–153, Amsterdam, the Netherlands, October 2012.
23. Lellouch, G. and A. Mishra, "Multi-carrier based radar signal optimization using genetic algorithm," *Advances in Intelligent Systems and Computing*, Vol. 258, 525–534, 2014.
24. Liu, B. and D. Wang, "Orthogonal radiation field construction for microwave staring correlated imaging," *Progress In Electromagnetics Research M*, Vol. 57, 139–149, 2017.
25. Cerf, R., "The quasispecies regime for the simple genetic algorithm with roulette wheel selection," *Advances in Applied Probability*, Vol. 49, No. 3, 903–926, 2017.
26. Rubae, T., P. M. Meaney, P. Meincke, and K. D. Paulsen, "Nonlinear microwave imaging for breast-cancer screening using Gauss-Newton's method and the CGLS inversion algorithm," *IEEE Transactions on Antennas and Propagation*, Vol. 55, 2320–2331, 2007.