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Spatial Power Combining of VLF Umbrella Antenna Arrays with Multi-Delay Lines

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Abstract—The output power of antennas is an important factor affecting the radiation performance of umbrella antenna arrays. Considering the power limit of very-low-frequency (VLF) umbrella arrays and the uncontrollable directivity, we propose a novel method for the spatial power-combining (SPC) of VLF umbrella arrays. Using multiple groups of feeders, the problem of phase shifting of the signals can be solved for VLF arrays. In the high frequency portion of the VLF range (25–30 kHz), this novel method can improve the efficiency of VLF arrays by 26% in the special directivity. A model of a trideco-tower umbrella antenna array is established in the FEKO simulation software. The simulation results show that compared with the in-phase feeding, the VLF transmitting antenna array forms the main beam in all directions. The array gain of the umbrella phased array in the 0° (180°) beam position is larger than 1.1 dB. The front-to-back ratio of the arrays is 3.7 dB. Compared with the in-phase feed mode, the directivity of the phased array enhances and the efficiency increases markedly. The simulation results demonstrate the effectiveness of the proposed method.

1. INTRODUCTION

Umbrella transmitting antennas are highly efficient and have been widely applied to very-low-frequency (VLF) (3–30 kHz) communication [1]. The umbrella array types include two-element arrays, triangle arrays and ring arrays. Cutler antenna is the most common umbrella antenna among the arrays. The efficiency of the array can reach more than 70% when the two identical and symmetrical umbrella antenna structures are supported by 13 towers. Generally, a practical VLF transmitting antenna usually operates at high-voltage and high output power [2]. However, the radiation performance of the VLF antenna is limited by the solid state power-combining efficiency and the propagation characteristics of the ionosphere [3–5]. Overcoming the power limitations and compensating for the communication limits in high noise areas are the challenges in VLF communication [6]. Power-combining technology represents an important electromagnetic synthesis method for traditional solid-state single station transmitters and the method is widely used in telemetry, communications, and radar fields [7–10]. Staiman firstly proposed the idea of free spatial combining [11]. Currently, spatial power-combining (SPC) is widely applied in phased arrays and multi-beam systems and is mainly applied to microwave equipment [12–17]. The efficiency of power-combining is in the range of 70–90%. The advantage of SPC is that there is less power loss in the free space compared to the traditional power synthesis circuit. In [18], the input impedance of a two element trideco-tower umbrella array was changed with the frequency and the inter-element spacing. However, the influence of the element feeding phase on the antenna performance was not considered. In [19], the author provided a single frequency signal synthesis model of multiple sparse arrays based on the time reversed electromagnetic wave and focused on the effect of the phase error on the synthesis efficiency. However, factors such as signal frequency and the array formation

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have not received sufficient attention. In order to consider the areal environment, the analysis has to be performed under real conditions. To date, the application of SPC technology to VLF transmitting antenna arrays has not been investigated.

This study focuses on limiting the power restrictions and enhancing the targeted regional radiation ability of VLF transmitting arrays. We propose a novel method for achieving SPC with multi-delay feeders in the VLF band. Using two element trideco-tower umbrella arrays as an example, a multi-delay feeder is set up on the high/low-voltage feeders through the umbrella arrays and is applied to the phase shift of the VLF signals. In a comparison with the traditional in-phase feeding method, the array mode and beam control method of VLF transmitting antenna are discussed and the feasibility of the new method is determined. Using the FEKO simulation software, a model of a trideco-tower umbrella antenna array is established and the influence of the SPC technology on the gain and pattern of the VLF transmitting arrays is calculated. We demonstrate via simulations that the proposed method improves the radiation performance of the umbrella VLF antenna array as a result of SPC.

2. NUMERICAL CALCULATIONS FOR SPATIAL POWER-COMBINING

In a VLF transmitting antenna, the transmitter needs to provide higher power for the front end of the transmitting antenna within the acceptable power range (usually less than 1000 kW). By optimizing the matching network, the reflection coefficient of the transmission network can be reduced, preventing the back-flow of the energy. In a three-dimensional space, the radiation power of the antenna is superimposed and synthesized in space AVLF umbrella antenna can be regarded as a whip antenna with top wires. The down-lead wires are the main radiators. The top wires increase the capacitance of the antenna and reduce the antenna voltage [20], thereby increasing the power capacity of the VLF antenna. The symmetrical structure of the trideco-tower umbrella array determines that the arrays are isotropic. The power density is given as [21]:

$$W_0 = e_t \frac{P_t}{4\pi R^2} \tag{1}$$

where R is the distance between the center point of the array and the receiving point, e_t the radiation efficiency of the umbrella antenna, and P_t the input power of the umbrella antenna. The received power is related to the efficiency of the receiving antenna e_r and the direction coefficient D_r. When the transmitting antenna and the receiving antenna are matched, the polarization between the incident wave and the receiving antenna is respectively matched. The ratio of the input power to the output power can be expressed as follows:

$$\frac{P_r}{P_t} = e_r e_r \frac{\lambda^2 D_r(\theta_r, \phi_r) D_t(\theta_t, \phi_t)}{16\pi^2 R^2} \tag{2}$$

where λ denotes the wavelength, and D_r denotes the directivity coefficient of the transmitting antenna. For a drag-whip antenna, the receiving antenna cannot be matched with the umbrella antenna. Therefore, the reflection coefficient and standing wave coefficient caused by the mismatch of the antennas should be taken into account. Then, the ratio in the Equation (2) can be obtained as follows:

$$\frac{P_r}{P_t} = e_r e_r \left(1 - |G_r|^2\right)\left(1 - |G_t|^2\right) \times \frac{\lambda^2 D_r(\theta_r, \phi_r) D_t(\theta_t, \phi_t)}{16\pi^2 R^2} \tag{3}$$

where G_t denotes the reflection coefficient of the transmitting antenna, and G_r denotes the reflection coefficient of the receiving antenna.

Compared with traditional power combining circuits, the free SPC has a lower-power loss [22]. In a modern phased array method, the spatial variation of the radiation pattern is achieved by using several array matrices with certain amplitudes and phases that correspond to a particular relationship. In the far field, due to the long wavelength of the VLF, the inter-element spacing is relatively close. Therefore, the propagation paths of the radiation waves of the two elements are approximately parallel. The superposed electromagnetic wave can be regarded as a parallel beam. Moreover, the receiving electric strength is an induced electric field in the near field. The horizontal plane pattern of an individual trideco-tower umbrella antenna is circular and the structure of the two element trideco-tower umbrella array is symmetrical. When the array elements are organized linearly with equal spacing, the horizontal pattern is no longer circular. In a Cartesian coordinate system, the directivity function of a two-element array is expressed as [23]:

$$f(\theta, \varphi) = \cos\left(\frac{\xi + kd\cos\delta}{2}\right) \tag{4}$$

where ξ denotes the initial phase difference of the feed current of the two elements, $k = 2\pi/\lambda$ the wave number, d the inter-element spacing, and δ the angle between the radial direction and the positive direction of the x axis of the antenna array. In the horizontal plane, the array factor of the two-element array can be simplified as:

$$f(\delta) = \cos\left(\frac{\xi + kd\cos\delta}{2}\right) \tag{5}$$

The initial phase difference between the beam positions can be determined by Equation (5) and can be controlled by the length of the phase shift feeder.

3. BEAM CONTROL FOR THE UMBRELLA ARRAY ANTENNA

Generally, VLF transmitting antenna arrays are bulky and have a complex structure. In the example of the two element trideco-tower umbrella array, the array elements are arranged linearly. The inter-element spacing is about 2000 m, which approximates a wavelength of $\lambda/4$. Therefore, a different feed phase corresponds to a different array pattern. The key to achieving the beam control is the approach for feeding the phase shift. The phase of the current can be delayed by the delay line which produces a wave path-difference so that there is a certain phase difference between the two elements. There are two types of phase shift feeders: a low-voltage phase shift feeder and a high-voltage phase shift feeder. The switching of the phase shift feeder is achieved by the beam controller in a central console and different phase shift feeders can be selected to control the pointing of the antenna array. Using the delay line in the phased array antenna the delay time of the phase of the electric signal can be achieved. The relative delay of the delay line is given as:

$$\Delta\tau = \frac{\Delta L \cdot \tilde{n}}{c} \tag{6}$$

where $\Delta\tau$ denotes the relative delay, ΔL the relative length of the delay line, $\bar{n}$ the refractive index of the delay line, and $c = 3 \times 10^8$ m/s the speed of light in a vacuum. The relative length of the delay line can be obtained by the relationship between the phase shift and the relative delay. The expression of the relative length can be simplified as:

$$\Delta L = \frac{2\pi d}{360^\circ} \cdot \frac{\cos\varphi}{\bar{n}} \tag{7}$$

It is known from Equation (7) that the relative length ΔL is independent of the frequency, which is related to the inter-element spacing d, the beam pointing angle φ and the refractive index of the delay line $\bar{n}$. When the spacing of the elements and the form of the feeder are fixed, the relative length ΔL is a function of the beam pointing angle φ.

3.1. The Phase Shift with a Low-Voltage Feeder

The array patterns, formed by the trideco-tower umbrella arrays, consist of broadside array, end-fire array and other pattern modes. The phase shift of the high-power feed signal is controlled by the low-voltage feed cage. Thus a feed phase difference between the two element arrays is formed, which controls the steering deflection of the SPC. As shown in Figure 1(a), the phase difference between the elements occurs at the low-voltage side. The low-voltage phase shift system (LPSS) uses a low-voltage feed control mode, which is connected to the helix houses by setting the switch statuses in the LPSS mode (shown in Table 1). The signals are tuned and matched at the helix houses to ensure that the electromagnetic waves create the wave path difference in space to achieve the SPC.

As shown in Figure 2, the phase shift switch is set on the low-voltage feed cage based on the length of the delay line. Under the control of the beam controller at the central console, we can choose the phase shift feed cage with different lengths. The arrays can generate a special phase difference in the

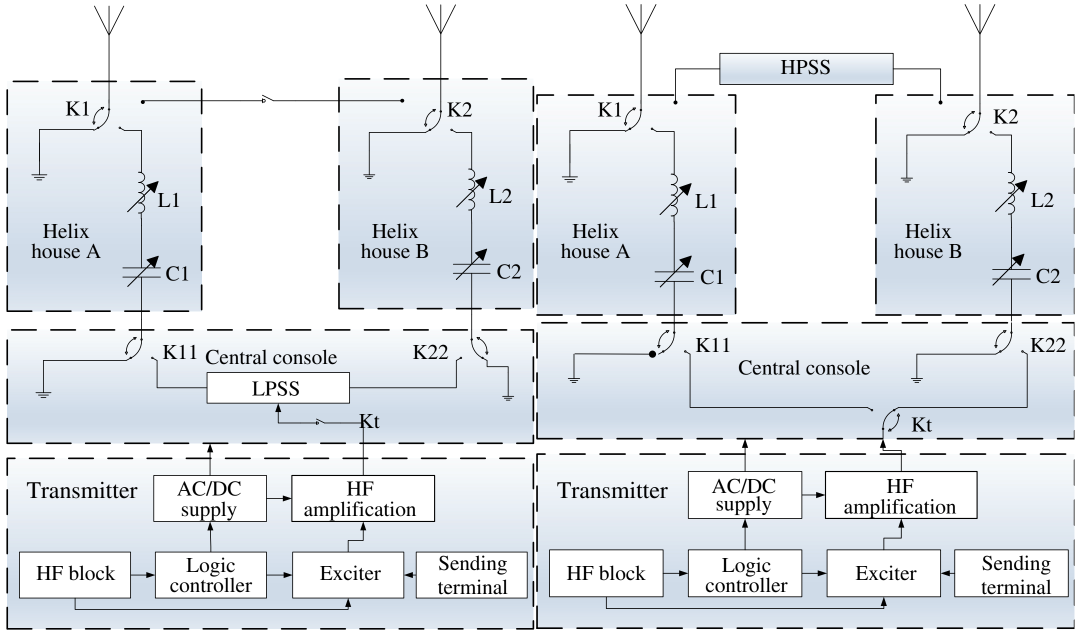


Figure 1. Schematic diagrams of the phase shift for a multi-delay feeder of the VLF umbrella antenna arrays. (a) The phase shift with a low-voltage feeder. (b) The phase shift with a high-voltage feeder.

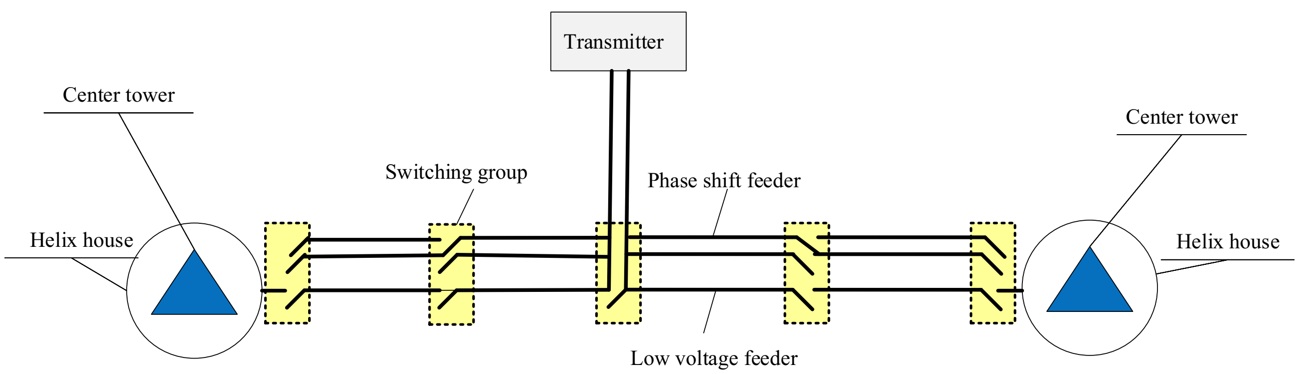


Figure 2. The LPSS for the VLF umbrella arrays.

broadside array, the end-fire array and the other modes. Tuned at the helix houses, the signals are fed to the down-lead wires to form the beam pointing. The mutual coupling of the arrays and the reliable operation of the transmitter are inevitable. The mutual coupling of the VLF umbrella antenna arrays can be improved by optimizing the inter-element spacing and the working modes [24]. When the low-voltage feeder is applied to the phase shift, the output power of the transmitter, which is divided by the power divider, is independently fed into the antennas. The influence of the circuit coupling on the phase can be effectively avoided by the power divider.

3.2. The Phase Shift with a High-Voltage Feeder

The function of the high-voltage feeder of the VLF transmitting antenna is to send the high-frequency signal tuned by the tuning coils to each antenna element. In the high-voltage phase shift system (HPSS), the current-carrying capacity of the tuning coil in the helix house falls to half of the carrying capacity of the dual umbrella arrays in the single tuning mode. Therefore, it is essential for the HPSS that the output power is two times the output power of the dual tuning mode so that the dual antenna elements can operate at the rated power.

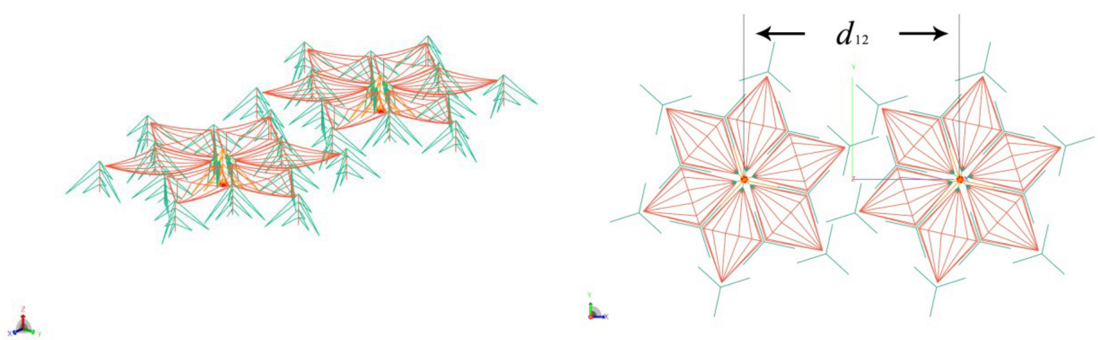
As shown in Figure 1(b), the high-voltage feeder phase shifting occurs at the high-voltage side after the antennas are tuned by setting the switch statuses in the HPSS mode (Table 1). The feed signals are transmitted and phased by the dual high-voltage feeders with different lengths. The difficulty of this method is that the strong coupling of the inter-elements at the high-voltage side is uncontrollable, which causes the phase deviation. Therefore, when the HPSS is implemented, the coupling between the antennas and cables should be taken into account.

Table 1. List of the switch statuses in different operating modes.

System mode	Connection status	Kt	K11	K22	K1	K2
LPSS	Connected	√	√	√	√	√
	Grounded					
HPSS	Connected	Helix house A	√		√	√
	Grounded			√		

4. NUMERICAL EXAMPLES

According to the structural parameters of the umbrella arrays described in [24], a trideco-tower umbrella array model with a perfect electrical conductor (PEC) is established in the simulation software FEKO 7.0. As shown in Figure 3, the center tower is 298 m high and is insulated from the top wires by high-voltage insulators. The excitation point at the bottom is connected to the down-lead wires. The heights of the middle towers and the outer towers are respectively 262.2 m and 243.5 m. The two elements are identical in form and symmetrical in structure. The main beam of the directional pattern is wider because of the relatively close distance. As shown in Figure 3, d_{12} denotes the inter-element spacing, and the initial inter-element spacing is 1623 m. When axis of the elements is regarded as the positive x-axis in a Cartesian coordinate system, the beam position can be divided into three groups: the 0° (180°) beam position, the 45° (135°) beam position and the 90° (270°) beam position.



(a) (b)

Figure 3. The geometry of the VLF umbrella antenna array. (a) 3D view. (b) Plane view.

4.1. Example One: Gains for Different Beam Positions

In order to verify the usefulness of the proposed SPC method, simulations are performed for each beam position. To obtain the gains for different frequencies, we maintain the initial inter-element spacing of the umbrella arrays and set the operation frequencies from 10 kHz to 30 kHz. Similarly, the gains of the phased arrays for the different inter-element spacings can be obtained by increasing the spacing of the center towers Figure 4(a), Figure 4(b), and Figure 4(c) represent the maximum gains at the 0° (180°) beam position, the 45° (135°) beam position and the 90° (270°) beam position respectively for different

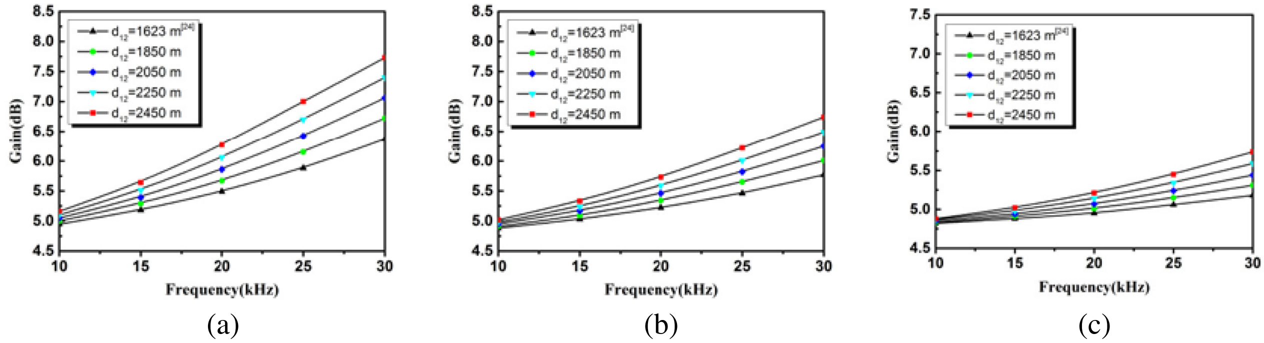


Figure 4. Maximum gains of the VLF umbrella antenna array for different beam positions. (a) 0° (180°) beam position. (b) 45° (135°) beam position. (c) 90° (270°) beam position.

frequencies. Thus, it is note worthy that a 1 V voltage source is implemented in the simulations of the VLF model.

As shown in Figure 4, the gains of the umbrella antenna array are increasing with increasing frequencies at the 0° (180°) and 45° (135°) beam positions. When the main beam points to the 0° (180°) beam position, the gain reaches the maximum value. At 25 kHz, the antenna gains of the main beam in [24] are 5.8 dB, 5.4 dB, and 5 dB at the 0° (180°), 45° (135°) and 90° (270°) beam positions respectively and increase by 0.7 dB, 0.4 dB and 0.2 dB at 15 kHz. Compared with the in-phase feeding model described in [24] the gains of the antenna array at the 0° (180°) beam position are 1.3 dB, 0.8 dB and 0.4 dB respectively larger when the inter-element spacing is increased to 2450 m. The gains of the in-phase feeding do not increase considerably with increasing frequencies and distances (Figure 4(c)). Apart from the nondirectional gains, the gains of the umbrella antenna array for the initial inter-element spacing are shown in Table 2.

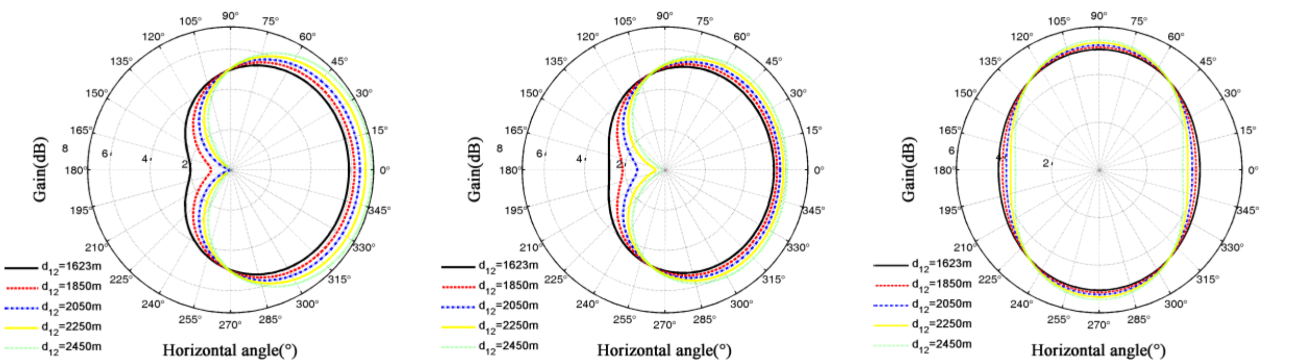
Table 2. List of the array gains of the VLF umbrella antenna array.

Frequency	Array gain		
	0°/180° beam position	45°/135° beam position	90°/270° beam position
10 kHz	0.2 dB	0.1 dB	0.1 dB
15 kHz	0.4 dB	0.3 dB	0.1 dB
20 kHz	0.7 dB	0.5 dB	0.2 dB
25 kHz	1.1 dB	0.7 dB	0.3 dB
30 kHz	1.6 dB	1.0 dB	0.4 dB

Table 2 shows relatively large differences in the array gain for the different frequencies under the PEC condition. At 25 kHz, the array gain of the antenna at the 0° (180°) beam position is larger than 1 dB, which improves the radiation efficiency of the antenna array by about 26% in the end-fire direction. Below 20 kHz, the array gain is small and increases slightly as the angle of the beam increases. At the same operating frequency, the array gain in the in-phase feed mode at a minimum and the directivity of the antenna array is not obvious. When the beam of the antenna arrays cans to the 0° (180°) wave position, the array gain attains the maximum value. These results indicate that the SPC technique significantly improves the directional gain of the umbrella phased arrays in the high-frequency portion of the VLF range (25–30 kHz), especially in the end fire direction.

4.2. Example Two: Directivity Patterns for Different Inter-element Spacings

The second example is aimed at providing an indication of the directivity patterns for different inter-element spacings of the VLF umbrella antenna array. To control the study variables (e.g., distance), we



(a) (b) (c)

Figure 5. Horizontal patterns of the VLF umbrella antenna array for different inter-element spacings at 25 kHz. (a) 0° (180°) beam position. (b) 45° (135°) beam position. (c) 90° (270°) beam position.

retain the other model parameters. The array spacing increases to 1850 m, 2050 m, 2250 m and 2450 m. The horizontal patterns of the umbrella arrays are shown in Figure 5.

In the figures, the two-element arrays form a precise pointing of the main beam in each beam position. The main lobes are wide and obvious. Figure 5(a) and Figure 5(b) clearly show that the main beam of the dual-element umbrella arrays is wide at the 0° (180°) and 45° (135°) beam positions. Note that the front-to-back ratio of the pattern approaches 3.7 dB at the 0° (180°) beam position. Compared with the gain of the in-phase feeding model in [24], the maximum gain of the phased arrays is about 1.2 dB higher than the gain of the in-phased feeding model at $d_{12} = 2450$ m. Figure 5(c) shows that the horizontal pattern of the antenna array changes little when the arrays are fed by the in-phase current. The horizontal side direction gain increases slightly with the increase in the inter-element spacing. However, it is noteworthy that the inter-element spacing should not be too wide, which could result in increased feeder losses. These results indicate that the novel SPC method compensates the weak radiation in the end-fire direction when the in-phase feeding mode is used.

4.3. Example Three: Input Impedances for Different Feeding Phase Differences

There are many factors affecting the performances of the SPC arrays, such as element number, radiation power and feeding phase. Among these factors, the feeding phase of the array elements is the critical factor. There is no point in controlling the phase differences as the performance of the antenna array degrades. The performance of the VLF antenna array is mainly reflected by the input impedance. We primarily focus on the input impedance values of the VLF phased arrays at 25 kHz. The input impedance values of the VLF phased arrays are shown in Figure 6.

Figure 6(a) depicts the input resistance of the VLF umbrella antenna with respect to the initial phase differences at 25 kHz. When the in-phase feed mode is applied to the arrays, the input resistance of the two-element arrays is 0.4 Ω. With increasing differences in the feeding phase, the input resistance values of the element one increase gradually to a peak value, while the input resistance values of the other element gradually decrease. When the phase difference between the two elements is 30°, the input resistance of the array element is double that of the arrays in the in-phase feeding mode and the input resistance of the other element tends to be 0 Ω. At this phase difference, the mutual resistance produced by the element one is equal to the negative self-resistance of the other element. Figure 6(b) shows that the phase differences have little effect on the input reactance of the VLF phased arrays. Compared with the wavelength of the arrays the inter-element spacing is relatively short. The range of the phase differences of the SPC arrays in the space is not large. Also, the beam width of the radiation pattern is wide and the directivity is not strong. Hence, the proposed SPC method improves the input resistance of the single array element and satisfies the tuning requirement of the VLF arrays.

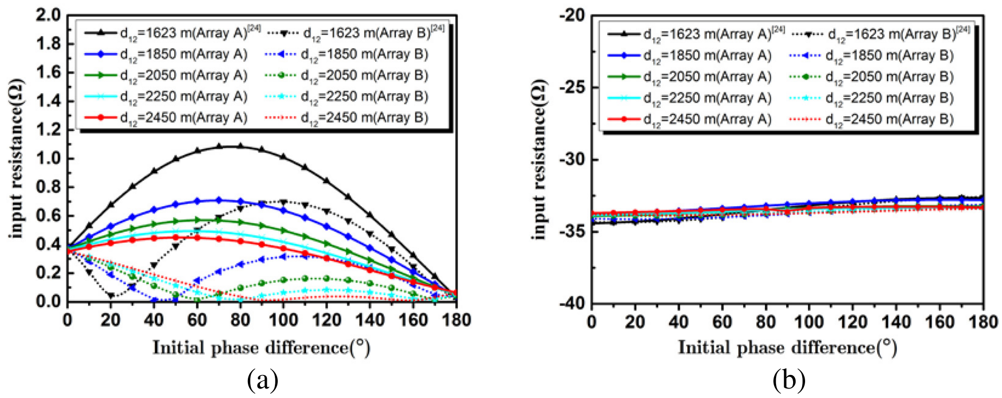


Figure 6. Input impedance values of the VLF umbrella antenna array versus the initial phase differences. (a) Input resistance. (b) Input reactance.

5. CONCLUSIONS

In this study, a novel SPC method for VLF transmitting antenna arrays is proposed. The proposed method can be divided into two approaches: the first approach is to separate two signals from a transmitter to the two element antenna; the signals are fed by the tuned arrays via a low-voltage feed cage and a phase-shifted feed cage. In the second approach, the two elements are connected in parallel with the high-voltage feeder. The element directly connected to the transmitter is tuned directly. The other element, shifted by the phase shift of the high-voltage feeder, is tuned to the ground. This study firstly applies the SPC technique in the VLF communication field. Considering the characteristics of the main beam of the VLF arrays, we divide the beam positions and arrange the multi-delay lines for the low-voltage or high-voltage feeder. A simulation of the proposed method shows outstanding radiation performance due to the phase shift of the VLF arrays. The array gain, directivity and impedance characteristics of the VLF umbrella array are obtained. Compared with the in-phase feeding mode described in [24], the proposed method improves the radiation efficiency by 26% at the end-fire direction, which enhances the efficiency of the VLF arrays and solves the problem of the beam control. It is verified by a performance analysis and simulations that the proposed SPC method is more robust than the conventional feeding method when applied to the VLF umbrella antenna arrays. Future work in this area will include an evaluation of the coupling between the antenna and the feeder and its influence on the phase of the current and application of the SPC method to other VLF antenna arrays.

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