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Design of Miniaturized Rat-Race Couplers with Arbitrary Power Division Ratios

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Abstract—A miniaturized rat-race coupler with arbitrary power division ratio is proposed in this paper. The design formulas of the rat-race coupler with arbitrary power division ratio are derived using even-odd decomposition analysis. The proposed structure demonstrates miniaturized size and perfect isolation due to adding phase inverter to the branch. For demonstration, a 20 dB rat-race coupler operating at 1 GHz with 81.34% size reduction is designed and fabricated. There is good agreement between measured and simulated results.

1. INTRODUCTION

In modern communication systems, rat-race coupler is one of the most basic devices used for microwave and millimeter-wave applications. Different structures of arbitrary power division rat-race coupler have been reported widely. Rat-race couplers for arbitrary power division using 50 Ω lines only regardless of the power division ratio [1], using composite right/left-handed transmission lines [2–4] and using π-type stepped-impedance section [5] have been presented. The key of the large power division ratio is the realization of high-impedance line in the traditional structure [6].

A phase inverter (PI) is a device that changes the phase of a signal by 180° and is a crucial component in microwave circuits. It has been applied in many microwave components such as power divider [7], rat-race hybrids and feeding network of antenna arrays. In recent years, efforts have been directed to the development of novel phase inverters such as broad-band uniplanar phase inverter [8], dual-band phase inverter [9] and reconfigurable phase inverter [10]. The phase inverter is crucial components in microwave circuits for size reduction and perfect isolation.

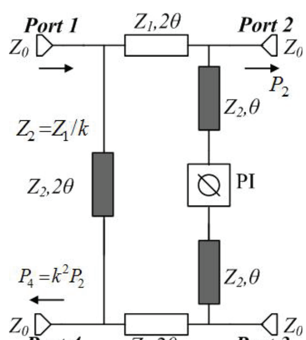

Figure 1. The proposed circuit model of the miniaturized rat-race coupler.

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In this paper, a novel design of miniaturized rat-race couplers with arbitrary power division ratios is proposed, and the closed-form design formulas are presented. The arbitrary power division ratios are attained by controlling the characteristic impedance of the branch lines. Figure 1 shows the circuit model of the miniaturized rat-race coupler. The proposed structure has features of miniaturized layout and perfect isolation because of using a phase inverter. The rat-race coupler with transmission lines (Z_1, Z_2) and a phase inverter is designed. For verification, a 20 dB rat-race coupler operating at 1 GHz with 81.34% size reduction is designed and fabricated.

2. DESIGN OF ARBITRARY POWER DIVISION RATIO RAT-RACE COUPLER

The closed-form design formulas of the rat-race coupler with the ideal phase inverter in Figure 1 can be derived using the even-odd-mode analysis. (Z_0 is the impedance of port, and θ is the electrical length of corresponding line at the central frequency f_a). Figure 2 shows the even-odd-mode circuits (the PI in this structure cause the difference of line connected to the ground). The ABCD matrices of even- and odd-mode are Eqs. (1) and (2), respectively. $Y_1 = 1/Z_1$ and $Y_2 = 1/Z_2$ are the characteristic admittances of the branches in the rat-race coupler.

$$\begin{bmatrix} A_e & B_e \\ C_e & D_e \end{bmatrix} = \begin{bmatrix} \cos 2\theta + 2Y_2/Y_1 \cos^2\theta & j\sin 2\theta/Y_1 \\ -j2Y_2\cos 2\theta \cot 2\theta + j(Y_1^2+Y_2^2)/Y_1 \sin 2\theta & \cos 2\theta - 2Y_2/Y_1 \sin^2\theta \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} A_o & B_o \\ C_o & D_o \end{bmatrix} = \begin{bmatrix} D_e & B_e \\ C_e & A_e \end{bmatrix} \tag{2}$$

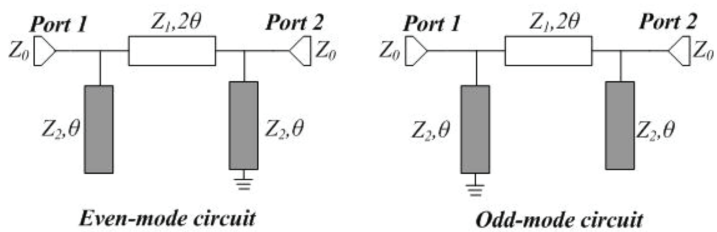


Figure 2. The even-mode and odd-mode circuit of the coupler.

Then

$$S_{11} = \frac{1}{2}\left(\frac{A_e+B_e-C_e-D_e}{A_e+B_e+C_e+D_e} + \frac{A_o+B_o-C_o-D_o}{A_o+B_o+C_o+D_o}\right) \tag{3}$$

$$S_{21} = \frac{1}{A_e+B_e+C_e+D_e} + \frac{1}{A_o+B_o+C_o+D_o} \tag{4}$$

$$S_{41} = \frac{1}{2}\left(\frac{A_e+B_e-C_e-D_e}{A_e+B_e+C_e+D_e} - \frac{A_o+B_o-C_o-D_o}{A_o+B_o+C_o+D_o}\right) \tag{5}$$

With power input from port 1, the power dividing ratio can be expressed as ($k \geq 1$):

$$P_2 : P_4 = |S_{21}|^2 : |S_{41}|^2 = 1 : k^2 \tag{6}$$

The characteristic impedances' value can be expressed from Eqs. (4)–(6) as follows:

$$Z_2 = \frac{\sqrt{1+k^2}}{\sqrt{\rho_M}} Z_0 \tag{7}$$

$$Z_1 = \frac{Z_2}{k} = \frac{\sqrt{1+k^2}}{k\sqrt{\rho_M}} Z_0 \tag{8}$$

where $\rho_M = (1+S_{11})/(1-S_{11})$ ($\rho_M \geq 1$) is the voltage standing wave ratio at f_a. The power division ratio is independent of the value of ρ_M and influenced by the values of Z_1 and Z_2.

When $\rho_M = 1$, the rat-race coupler is perfectly matched at f_a, and this is the case of the traditional design method. Equations (7) and (8) are the same as the traditional one [5].

When $\rho_M \neq 1$, there are two different operation frequencies (f_{01} and f_{02}, and f_{01} can be the working frequency of the structure proposed by this paper). The lower and upper frequencies can be expressed as follows:

$$f_{01} = \frac{2}{\pi} \cdot \cot^{-1} \sqrt{\frac{1+k^2}{2k}\left(1-\frac{1}{\rho_M}\right)} \cdot f_a \tag{9}$$

$$f_{02} = 2f_a - f_{01} \tag{10}$$

f_a can be deduced from Equation (9) as follows:

$$f_a = \frac{\pi}{2} f_{01} / \cot^{-1} \sqrt{\frac{1+k^2}{2k}\left(1-\frac{1}{\rho_M}\right)} \tag{11}$$

When $k = 1$, an equal division rat-race coupler can also be designed by Eqs. (7)–(10). The equal division design method [10] is the special case of the proposed one, and the proposed method is much simpler. Moreover, the proposed design method can be used in arbitrary division ratio design. The influences of Z_1 and Z_2 on the division ratio k are shown in Figure 3. The maximum power ratio is 20.5 dB ($k = 10.6$) when $Z_1 = 160\,\Omega$ and $Z_2 = 15\,\Omega$, considering that $15\,\Omega$–$160\,\Omega$ is the range impedance of microstrip line in reality.

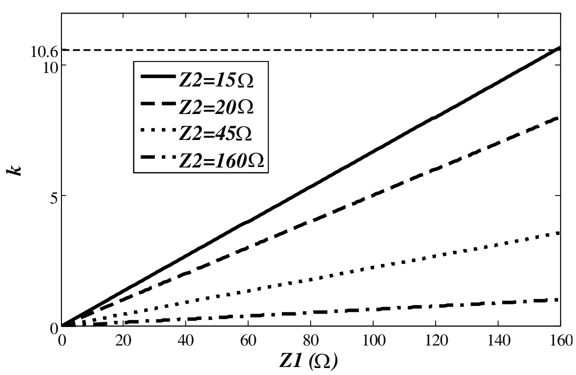


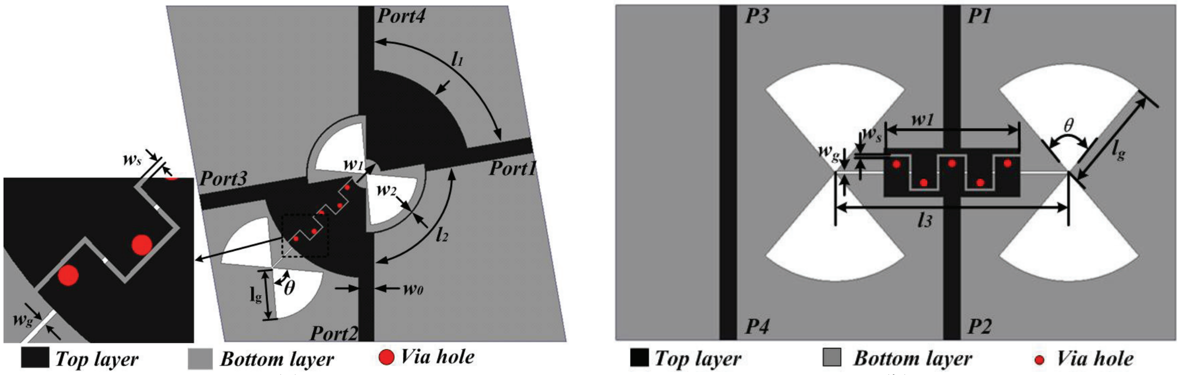
Figure 3. The influences of Z_1 and Z_2 to the division ratio k.

For verification, using the analytical design method, when $\rho_M = 10$, $Z_0 = 50\,\Omega$, for designing a 20 dB ($k = 10$) rat-race coupler operating at $f_{01} = 1$ GHz. From Equations (7)–(10), the design parameters can be calculated, and the values are $Z_1 = 15.89\,\Omega$, $Z_2 = 158.9\,\Omega$, $f_a = 3.5814$ GHz and $f_{02} = 6.16$ GHz. The structure of the miniaturized rat-race coupler is shown in Figure 4.

For further demonstration of the proposed analytical design method, the design parameters for different operations are listed in Table 1. Figure 5 shows the variation k according to Equations (10) and (11).

The structure of the phase inverter is shown in Figure 4(b), and the phase inverter can be simulated independently. The right line with defected ground structure is the phase inverter, and the left line is the reference line which is used to compare phase difference with the phase inverter. The phase inverter in this rat-race coupler consists of interdigital strip lines, five metallic via holes and defected ground structure. The 180° phase inverter is achieved by means of reversing electric field orientations with reference to the ground plane. The radial slot in the defected ground is the modified version of the conventional $\lambda_g/4$ short-stub structure and has the advantage of wider bandwidth. The butterfly-shape defected ground structure is chosen because the rat-race coupler with a limited top layer structure. Ansoft HFSS13.0 is used to simulate the proposed phase inverter.

The simulated results of the proposed phase inverter are shown in Figure 6. The simulated results show that the proposed phase inverter has a bandwidth of 22% (0.92 GHz–1.14 GHz) with better than 20 dB return loss and a phase deviation less than 7.5°.

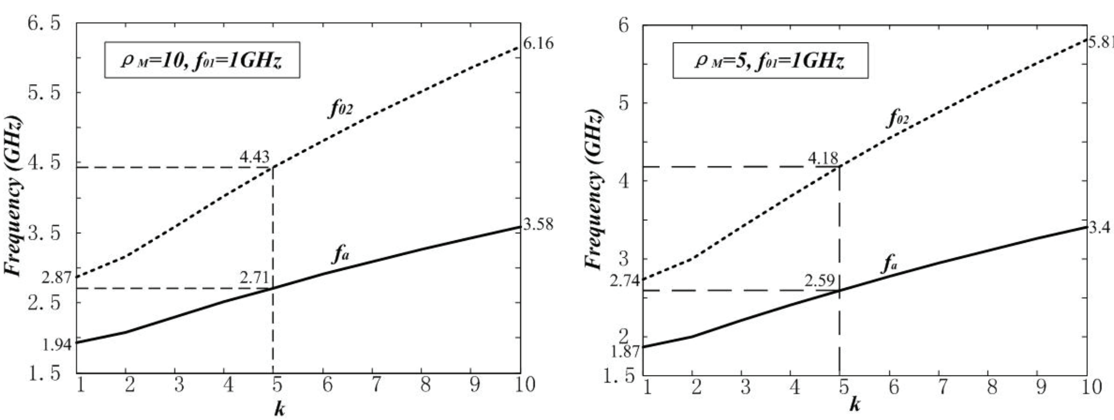


(a) (b)

Figure 4. The structure of the miniaturized rat-race coupler. (a) The dimensions of the rat-race coupler. (b) The configuration of the phase inverter.

Table 1. The design parameters of the rat-race couplers.

ρ_M	f_{01} (GHz)	k	f_a (GHz)	Z_1 (Ω)	Z_2 (Ω)	f_{02} (GHz)
10	1	10	3.58	15.89	158.9	6.16
		5	2.71	16.12	80.62	4.43
		1	1.94	22.36	22.36	2.87
5	1	10	3.4	22.47	224.72	5.81
		5	2.59	22.8	114.02	4.18
		1	1.87	31.62	31.62	2.74



(a) (b)

Figure 5. The variation of the power division ratio k. (a) $\rho_M = 10$, $f_{01} = 1\,\text{GHz}$. (b) $\rho_M = 5$, $f_{01} = 1\,\text{GHz}$.

3. EXPERIMENTAL RESULTS

For experimental demonstration, a substrate with dielectric constant 2.2, loss tangent $\delta = 0.0029$, and thickness $h = 0.8\,\text{mm}$ is chosen. The fabricated board has an area of $52.29 \times 53.38\,\text{mm}^2$. The dimensions of the rat-race coupler are shown in Figure 4, and l_1 and l_2 are quarter wavelengths at f_a, respectively. The final dimensions are $\theta = 80°$, $w_0 = 2.46\,\text{mm}$, $w_1 = 14\,\text{mm}$, $w_2 = 0.2\,\text{mm}$, $w_s = 0.2\,\text{mm}$, $w_g = 0.2\,\text{mm}$, $l_1 = 13\,\text{mm}$, $l_2 = 16.2\,\text{mm}$, $l_3 = 20.6\,\text{mm}$, $l_g = 8\,\text{mm}$. The diameter of the metallic vias

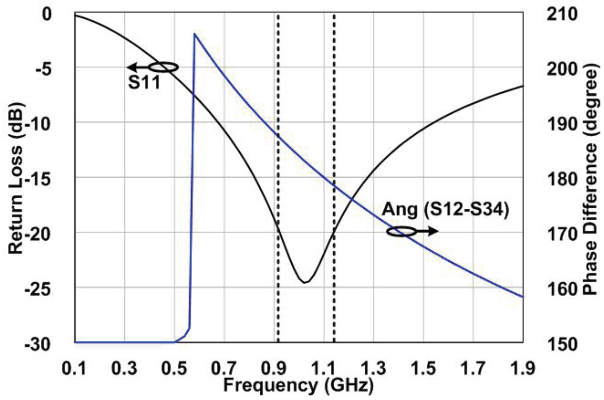


Figure 6. The simulated results of the phase inverter.

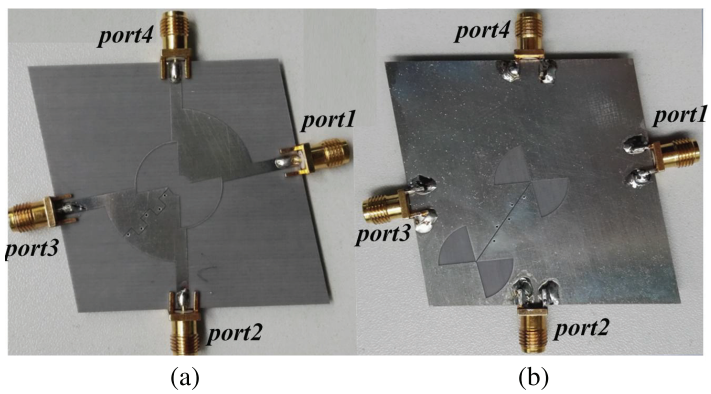


Figure 7. The picture of the miniaturized rat-race coupler. (a) Top layer. (b) Bottom layer.

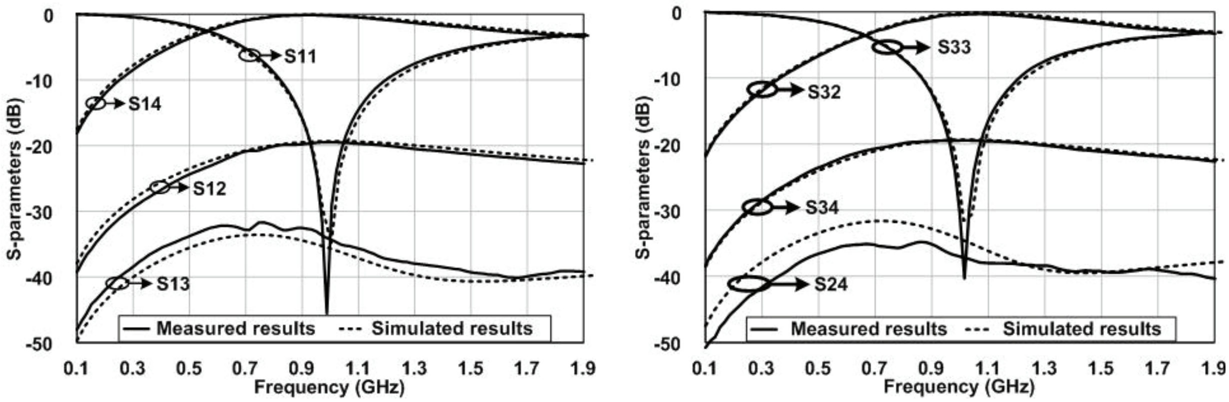
is 0.3 mm. Ansoft HFSS13.0 is used to simulate the proposed rat-race coupler. The measurements are made by an AV3672 vector network analyser. Figure 7 shows pictures of the fabricated miniaturized rat-race coupler.

The measured results of the proposed rat-race coupler are shown in Figure 8 along with the simulated results (only show the necessary results at working frequency $f_{01} = 1\,\text{GHz}$). As shown in Figure 8(a), when the power input is from port 1, the measured return loss is better than 20 dB, the isolation better than 30 dB, and the power division ratio is 19.53 dB at the operation frequency 1 GHz, which is very close to the expected one.

As shown in Figure 8(b), when the power input is from port 3, the measured return loss is better than 20 dB, the isolation better than 30 dB, and the power division ratio is 19.42 dB at the operation frequency 1 GHz. As shown in Figure 8, the isolation is perfect owing to using the phase inverter. The measured results are in good agreement with the simulated ones.

The phase differences of the rat-race coupler are shown in Figure 9. When the power input is from the port 1, the coupler has a phase deviation less than 6° at operation frequency 1 GHz. When the power input is from port 3, the coupler has a phase deviation less than 7° at operation frequency 1 GHz.

The circumference of the proposed large power division ratio rat-race coupler is $0.2799\lambda_g$ (λ_g is the wavelength relative to the frequency 1 GHz, and the circumference of the conventional coupler is $1.5\lambda_g$). The proposed coupler demonstrates a 81.34% size reduction as compared to the conventional coupler. For further demonstration of the proposed design, the comparison between this work and other publications is listed in Table 2 (the structure proposed by [5] as standard of comparison because its working frequency is the highest, and the length of microstrip will be shorter when the frequency is higher). It shows that the total size of this work is smaller than other works, while keeping large power division ratio.



(a) (b)

Figure 8. The S-parameters of the coupler. (a) $S11$, $S12$, $S13$, $S14$. (b) $S33$, $S32$, $S34$, $S24$.

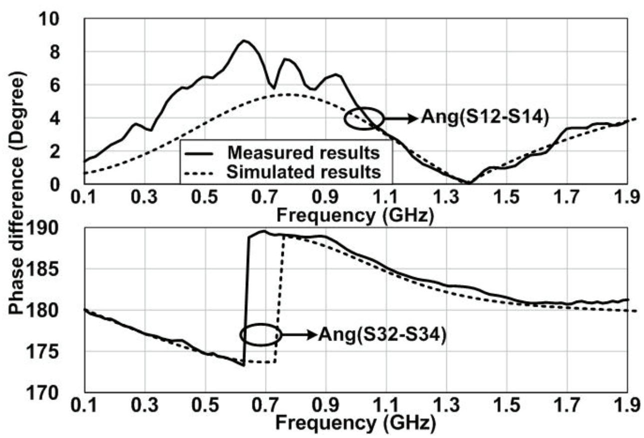


Figure 9. The phase difference of the coupler.

Table 2. Comparisons of published arbitrary power division rat-race couplers and this work.

	Miniaturized technique	Power division ratio	relative circuit size	operating frequency	fabrication process
[1]	Conventional microstrip line	9.54 dB	49%	1 GHz	single-layered PCB
[2]	Composite right/left-handed transmission line	6 dB	44%	2.5 GHz	single-layered PCB
[3]	artificial transmission-line	12 dB	29%	3.5 GHz	single-layered PCB
[4]	π-type stepped-impedance section	6 dB	21%	2.45/5.2 GHz	single-layered PCB
[5]	Conventional microstrip line	3.98 dB	100%	3.76 GHz	single-layered PCB
This work	Phase inverter	20 dB	18.66%	1 GHz	double-layered PCB

4. CONCLUSION

A novel design of miniaturized rat-race coupler with arbitrary power division ratio has been proposed. A 20 dB rat-race coupler with 81.34% size reduction is designed and fabricated. The structure of the rat-race coupler is miniaturized and simple. The realization of large power division ratio using microstrip line with high impedance has been solved easily. There is good agreement between the measured results and those of the theoretically designed, justifying the circuit configuration and the design theory.

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