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Stable Implicit Scheme for TM Transient Scattering from 2D Conducting Objects Using TD-EFIE

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Abstract—To improve stability of time-domain integral equation, a stable implicit scheme is proposed to solve the transverse-magnetic (TM) electromagnetic scattering from 2D conducting objects. The time-domain electric-field integral equation (TD-EFIE) was adopted and expressed using second-order derivative of the magnetic vector potential. To reduce numerical error, the magnetic vector potential was approximated by second-order central finite difference. TM transient scattering from 2D conducting objects was calculated by an implicit marching-on-in-time (MOT) scheme. To obtain stable numerical results, the TD-EFIE MOT implicit scheme was firstly combined with the time-averaging technique. The accuracy and stability of the scheme were demonstrated by comparison with the results from inverse discrete Fourier transform technique.

1. INTRODUCTION

It is well known that the marching-on-in-time (MOT) algorithm is one of the popular numerical methods to solve the time-domain integral equation (TDIE) [1]. Unfortunately, occurrence of late-time instability severely hinders their application in the analysis of transient electromagnetic scattering and radiation problems, especially in time-domain electric-field integral equation (TD-EFIE) [2]. Recently, the marching-on-in-degree (MOD) scheme has been proposed to obtain late-time stable results [3–5]. However, the MOT scheme is more excellent than the MOD in computational efficiency [4], and it is accelerated easily by fast methods such as the plane wave time domain (PWTD) [6] and the adaptive integral method (AIM) [7]. Hence, it is urgent to develop a stable MOT-based scheme.

To improve the numerical stability of MOT scheme, many approaches have been suggested during past several years, such as time-averaging scheme [8], implicit time-stepping scheme [9], and temporal basis functions [10–11]. Moreover, some TD-EFIEs with different forms have been presented to improve late-time stabilities. The TD-EFIE can be expressed using time derivative of the magnetic vector potential which can be approximated by finite differences such as forward [12], backward [13] and central finite-difference schemes [14]. By a time derivative, Rao et al. expressed TD-EFIE using the second derivative of the magnetic vector potential to calculate the transient response of 3D conducting objects [15]. These MOT methods mentioned above extend the span of the stable region, though they cannot completely resolve the late-time instability problem. Recently, the implicit MOT scheme in conjunction with the time-averaging technique was suggested and showed extreme stability for 3D conducting objects [16]. The method prompts further development of implicit MOT scheme and breaks the bottleneck that the time-averaging scheme was only applicable to the traditional explicit method. Compared with 3D objects, the late-time instability of 2D objects is presented more easily because of the infinite tail of the time domain Green's function.

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In this letter, a novel TD-EFIE with second derivative of the magnetic vector potential is proposed to analyze transient scattering from 2D conducting objects. The time-averaging technique was used for the proposed TD-EFIE MOT implicit scheme. Numerical results show stable transverse magnetic (TM) transient scattering responses for 2D objects, even for complex coupling structures.

2. THEORY AND FORMULATIONS

An infinitely long conducting cylinder with contour C was used as a scattering object. The cylinder was parallel to the z-axis and was illuminated by a transient TM wave. The induced current J is z-directed only because of the polarized electric-field E^i along the z-direction. TD-EFIE may be derived using the boundary conditions as follows:

$$\left[\frac{\partial A(\rho,t)}{\partial t}\right]_{\tan} = \left[E^i(\rho,t)\right]_{\tan} \tag{1}$$

where

$$A(\rho,t) = \frac{\mu}{4\pi}\int_C \int_{Z'=-\infty}^{+\infty} \frac{J\left(\rho', t-R/c\right)}{R} dz' dC' \tag{2}$$

In Eq. (2), R is the distance between the source point ρ' and observation point ρ. Equation (1) is differentiated with respect to time, obtaining

$$\left[\frac{\partial^2 A(\rho,t)}{\partial t^2}\right]_{\tan} = \left[\frac{\partial E^i(\rho,t)}{\partial t}\right]_{\tan} \tag{3}$$

It must be noted that the electric scalar potential is excluded in the differential version of TD-EFIE in Eq. (3), which is obviously different from the TD-EFIE of 3D conducting object [15]. Moreover, by approximating the second-order derivative of magnetic vector potential at t_{i-1}, Equation (3) can be expressed as follows:

$$\frac{A(\rho,t_i) - 2A(\rho,t_{i-1}) + A(\rho,t_{i-2})}{(\Delta t)^2} = \frac{\partial E^i(\rho,t_{i-1})}{\partial t} \tag{4}$$

where Δt is the time step size and $t_i = i\Delta t$. To obtain the induced currents, the surface of conducting cylinder is divided into many square patches. The current J is expanded using the pulse basis functions $f_n(\rho)$ as

$$J(\rho,t) = \hat{z}\sum_{n=1}^{N} I_n(t) f_n(\rho) \tag{5}$$

where N is the number of linear segments along the contour C. The pulse basis function associated with the nth segment is defined as

$$f_n(\rho) = \begin{cases} 1 & \rho \in n\text{th segment} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Subsequently, the standard definition of inner product was used, and the differential version of TD-EFIE can be expressed as follows:

$$A\left(\rho_{m-1/2}, t_i\right) = (\Delta t)^2 \frac{\partial E^i\left(\rho_{m-1/2}, t_{i-1}\right)}{\partial t} + 2A\left(\rho_{m-1/2}, t_{i-1}\right) - A\left(\rho_{m-1/2}, t_{i-2}\right) \tag{7}$$

where

$$A\left(\rho_{m-1/2}, t_n\right) \approx \sum_{k=1}^{N} \sum_{l=-\infty}^{\infty} I_k\left(t_n - \frac{R_{mkl}}{c}\right)\kappa_{mkl} \tag{8}$$

and

$$R_{mkl} = \sqrt{\left|\rho_{m-1/2} - \rho_{k-1/2}\right|^2 + (l\Delta\tau_k)^2} \tag{9}$$

$$\kappa_{mkl} = \frac{\mu}{4\pi}\int\int_{k,l\ \text{patch}} \frac{ds'}{R_m} \tag{10}$$

$$R_m = \sqrt{\left|\rho_{m-1/2} - \rho'\right|^2 + z'^2} \tag{11}$$

where l is defined as the ordinal number of the square patch along the z-axis. For the implicit MOT scheme, we define $t_R = t_i - R_{mkl}/c$. The current value $I_n(t_R)$ was extrapolated using a linear interpolation in time. When $t_{j-1} < t_R \leq t_j$ and $t_j < t_{i-1}$, $I_n(t_R)$ can be written as

$$I_n(t_R) = (1-\delta)I_n(t_{j-1}) + \delta I_n(t_j) \tag{12}$$

where

$$\delta = (t_R - t_{j-1})/\Delta t \tag{13}$$

When $t_{i-1} < t_R \leq t_i$, the current value $I_n(t_R)$ can be written as

$$I_n(t_R) = I_n(t_{i-1})R_{mkl}/c\Delta t + I_n(t_i)\left(1 - R_{mkl}/c\Delta t\right) \tag{14}$$

Consequently, Equation (7) was written in a matrix equation as follows:

$$[\alpha_{mn}]\,[I_n(t_i)] = [\beta_m(t_R)] \tag{15}$$

Thus, the induced current at each time step is obtained iteratively using matrix equation (15). In the following, spurious oscillation was observed in the presented implicit algorithm because of numerical error caused by the approximation of finite difference. In the traditional explicit algorithm, a time-averaging technique was adopted to eliminate the late-time instability. Thus, the averaged value $\tilde{I}_m(t_i)$ is approximated as [8]:

$$\tilde{I}_m(t_i) = 0.25 \times \left[I_m(t_{i+1}) + 2I_m(t_i) + \tilde{I}_m(t_{i-1})\right] \tag{16}$$

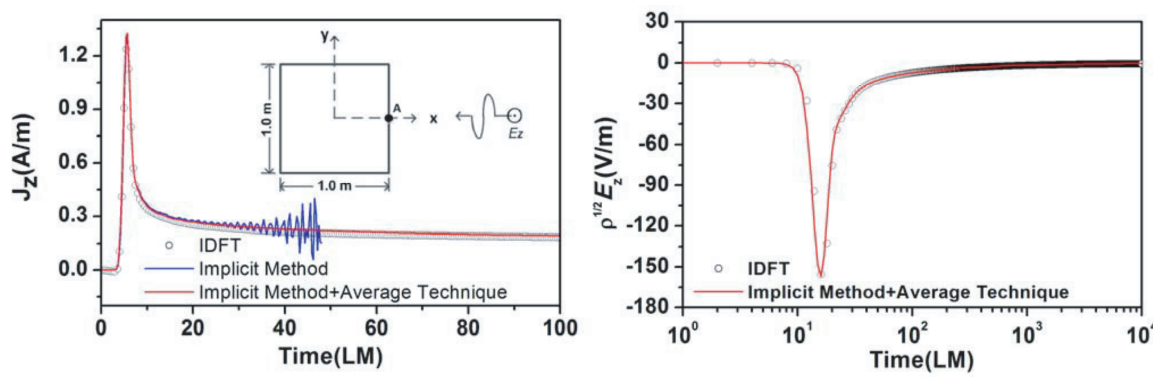
However, in this letter the time-averaging scheme is applied to the proposed TD-EFIE MOT implicit scheme to obtain stable transient response. It is worth mentioning that the time-averaging scheme is not appropriate for the implicit MOT algorithm to solve traditional TD-EFIE [9].

3. NUMERICAL RESULTS AND DISCUSSIONS

The proposed TDIE-MOT implicit scheme was employed to analyze the transient scattering from different 2D objects. The incident waves are TM-polarized Gaussian impulse given by the following:

$$E^i(\rho,t) = E_0 \frac{4}{\sqrt{\pi}T} e^{-\left[\frac{4}{T}\left(ct - ct_0 - \rho\cdot\hat{k}^i\right)\right]^2} \tag{17}$$

where $E_0 = 120\pi\hat{z}$, $T = 4LM$, $ct_0 = 6LM$, and $\hat{k}^i = -\hat{x}$. To demonstrate the accuracy and stability of the proposed TD-EFIE MOT implicit method, the first example was considered a square cylinder with 1.0 m side length, as shown in the inset of Fig. 1. Each side of the square cylinder was divided uniformly



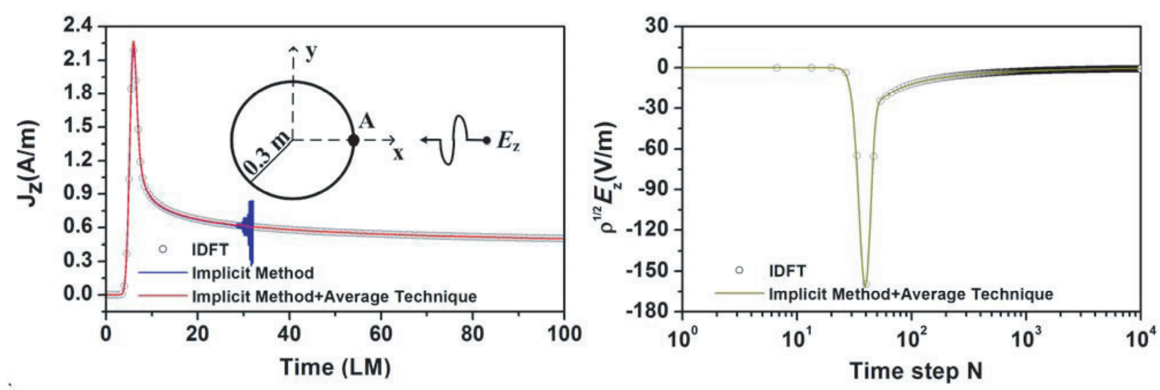
(a) (b)

Figure 1. Transient scattering response of a square cylinder illuminated by a TM-polarized Gaussian plane wave. (a) Transient current response; (b) Far backscattered electric field response.

into 9 linear segments. The time step was $0.3LM$. The current at point A was shown in Fig. 1(a). A spurious oscillation was observed if the implicit method is used alone. Combining the implicit method with the time-averaging scheme, the spurious oscillation was suppressed obviously and stable numerical results were obtained. It is obvious that the results based on our method are agreement well with those from the inverse discrete Fourier transform (IDFT) technique. In addition, the far backscattered electric field responses obtained by presented method are perfectly in accordance with those by IDFT in Fig. 1(b), which verifies the validity of our proposed method.

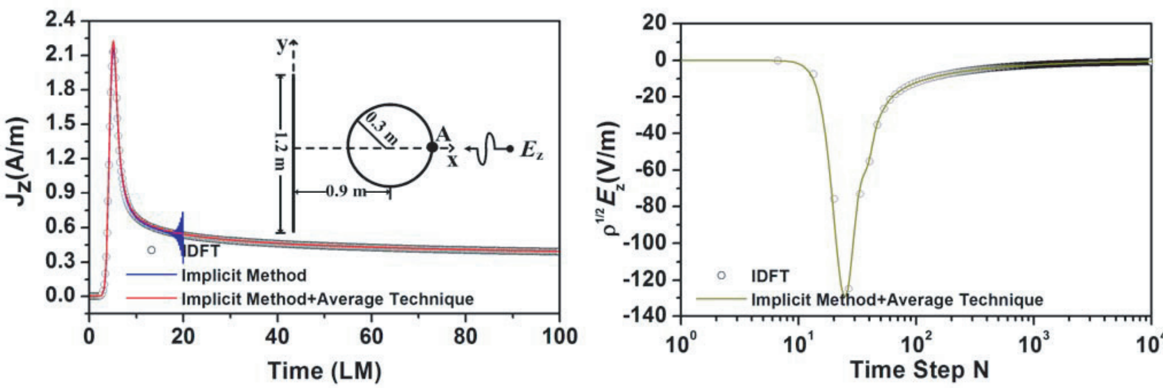
Transient response from an infinite circular cylinder with a radius of 0.3 m was analyzed to further demonstrate the stability of the proposed scheme. The contour of the cylinder was divided into 20 linear segments. The time step size was $c\Delta t = 0.15LM$. The induced current at point A is shown in Fig. 2(a). The results show that spurious oscillation is suppressed using the time-averaging technique. As expected, the current response is in good agreement with those obtained by IDFT technique. In addition, Fig. 2(b) shows the comparison of our scheme and the IDFT technique for backscattered electric field in far field zone. It is clear that the backscattered results in our scheme are in accordance with that of the IDFT technique for 10,000 time steps.

For the traditional TDIE-MOT scheme, the currents of a complex scattering structure are prone to spurious oscillation because of its non-smooth characteristic. The accuracy and stability of our scheme were further verified by analyzing transient scattering from a combinative object that consists of a straight strip and a circular cylinder. The strip with 1.2 m width was divided into 12 uniform segments.



(a) (b)

Figure 2. Transient scattering response of a circular cylinder illuminated by a TM-polarized Gaussian plane wave. (a) Transient current response; (b) Far backscattered electric field response.



(a) (b)

Figure 3. Transient scattering response of a combinative object illuminated by a TM-polarized Gaussian plane wave. (a) Transient current response; (b) Far backscattered electric field response.

The circular cylinder with 0.3 m radius was divided into 20 uniform segments. The combinative object was symmetrically arranged along the x-axis, as shown in the inset of Fig. 3(a). The time step in this case was $0.15LM$. Fig. 3(a) shows the currents at point A. Without the time-averaging technique, spurious oscillation occurs at $18LM$, as shown in Fig. 3(a), which is earlier than that of Fig. 2(a). When time-averaging technique is used, stable results are yielded, and in good agreement with those obtained by the IDFT solution. Fig. 3(b) shows the results of backscattered electric field response in far-field zone. Our scheme exhibits excellent stability and accuracy for a large number of time steps.

4. CONCLUSIONS

To obtain stable TM transient response from 2D conducting objects, an implicit MOT scheme is suggested and validated in detail. In this scheme, the TD-EFIE is expressed by the second derivative of the magnetic vector potential. The implicit solution is adopted to solve the proposed TD-EFIE where the magnetic vector potential is approximated by second-order central finite differences. It is noted that the implicit scheme based on the differential TD-EFIE can be combined with the time-averaging technique to obtain stable transient scattering response, although the time-averaging scheme is not appropriate for the traditional MOT implicit algorithm. Numerical results exhibit excellent stability and accuracy for a large number of time steps, even though this approach is used to analyze the transient response of complex coupling structures. It is also worth mentioning that the proposed MOT scheme will be accelerated using the some fast algorithms, such as the PWTD and the AIM.

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REFERENCES

1. Rao, S. M., *Time Domain Electromagnetics*, Academic Press, New York, 1999.
2. Xiao, G., X. Tian, W. Luo, and J. Fang, "Impulse responses and the late time stability properties of time-domain integral equations," *IET Microw. Antennas Propag.*, Vol. 9, No. 7, 603–610, 2014.
3. Jung, B. H., T. K. Sarkar, Y. S. Chung, M. S. Palma, Z. Ji, S. M. Jang, and K. J. Kim, "Transient electromagnetic scattering from dielectric objects using the electric field integral equation with Laguerre polynomials as temporal basis functions," *IEEE Trans. Antennas Propag.*, Vol. 52, No. 9, 2329–2340, 2004.
4. Jung, B.-H., Z. Ji, T. K. Sarkar, M. Salazar-Palma, and M. Yuan, "A comparison of marching-on in time method with marching-on in degree method for the TDIE solver," *Progress In Electromagnetics Research*, Vol. 70, 281–296, 2007.
5. He, Z. and R. S. Chen, "A novel marching-on-in-degree solver of time domain parabolic equation for transient EM scattering analysis," *IEEE Trans. Antennas Propag.*, Vol. 64, No. 11, 4905–4910, 2016.
6. Ergin, A., B. Shanker, and E. Michielssen, "Fast evaluation of three-dimensional transient wave field using diagonal translation operators," *J. Comput. Phys.*, Vol. 146, No. 1, 157–180, 1998.
7. Yilmaz, E., D. S. Weile, J. M. Jin, and E. Michielssen, "A hierarchical FFT algorithm (HIL-FFT) for the fast analysis of transient electromagnetic scattering phenomena," *IEEE Trans. Antennas Propag.*, Vol. 50, No. 7, 971–982, 2002.
8. Vechinski, D. A. and S. M. Rao, "A stable procedure to calculate the transient scattering by conducting surfaces of arbitrary shape," *IEEE Trans. Antennas Propag.*, Vol. 40, No. 6, 661–665, 1993.

9. Rao, S. M., D. A. Vechinski, and T. K. Sarkar, "Transient scattering by conducting cylinders — Implicit solution for the transverse electric case," *Microw. Opt. Technol. Lett.*, Vol. 21, No. 2, 129–134, 1999.
10. Guo, X. Y., M. Y. Xia, and C. H. Chan, "Stable TDIE-MOT solver for transient scattering by two-dimensional conducting structures," *IEEE Trans. Antennas Propag.*, Vol. 62, No. 4, 2149–2455, 2014.
11. Tong, M. S. and P. C. Wang, "Stable solution of time-domain combined field integral equations for transient electromagnetic scattering by composite structures based on Nyström scheme and laguerre function," *IEEE Trans. Antennas Propag.*, Vol. 64, No. 7, 3239–3244, 2016.
12. Rao, S. M. and T. K. Sarkar, "An alternative version of the time-domain electric field integral equation for arbitrarily shaped conductors," *IEEE Trans. Antennas Propag.*, Vol. 41, No. 6, 831–834, 1993.
13. Davies, P. J., "On the stability of time-marching schemes for the general surface electric-field integral equation," *IEEE Trans. Antennas Propag.*, Vol. 44, No. 11, 1467–1473, 2002.
14. Jung, H. and T. K. Sarkar, "Time-domain CFIE for the analysis of transient scattering from arbitrarily shaped 3D conducting objects," *Microw. Opt. Technol. Lett.*, Vol. 34, No. 4, 289–296, 2002.
15. Rao, S. M. and D. R. Wilton, "Transient scattering by conducting surfaces of arbitrary shape," *IEEE Trans. Antennas Propag.*, Vol. 39, No. 1, 56–61, 1991.
16. Guan, X. P., Y. Su, and S. G. Wang, "Stable and accelerated procedure to calculate transient scattering using electric field integral equation," *IEE Electron. Lett.*, Vol. 43, No. 15, 794–795, 2007.