Progress In Electromagnetics Research M, Vol. 65, 9–17, 2018

An ISAR Imaging Method for Search Radar Involving Nonuniform Angle Samples

Hyung Ju Kim[1, *], Kee Ung Bae[1], Won Young Song[1], Eun Jung Yang[2], and Noh Hoon Myung[1]

Abstract—This paper proposes a two-dimensional (2-D) inverse synthetic aperture radar (ISAR) imaging method with nonuniformly obtained angle samples. A one-dimensional (1-D) radar image, a range profile, is obtained using frequency samples within a given bandwidth. 2-D ISAR images are then obtained by acquiring the Doppler spectrum using range profiles obtained from multiple observation angles having a constant interval. However, when ISAR images are obtained by applying the range-Doppler imaging method for a target scattered signal with nonuniform angle samples, a clear image cannot be obtained. In this paper, we propose a method to generate a covariance matrix from a nonuniform angle sample and obtain an ISAR image based on the multiple signal characterization (MUSIC) technique. The proposed method can be applied to the target scattering signal using a search radar, which observes target with nonuniform aspect angles. We present a scattering signal model of a target for the search radar and provide ISAR images obtained by applying the proposed method to simulated and measured data, respectively. Results reveal that the proposed method improves image quality and reduces computation time compared to the conventional method.

1. INTRODUCTION

ISAR imaging is a target imaging technique based on estimating the Doppler spectrum of a partial structure of the target. It is one of the most important techniques in the area of noncooperative target recognition [1–3]. The range-Doppler imaging method is an efficient approach to obtain an ISAR image [4–7]. This method requires uniform observation angle samples in the back-scattered signals from the target. On the other hand, a search radar such as illustrated in Figure 1 observes target with nonuniform aspect angles. With nonuniform angle samples, ISAR images obtained with conventional range-Doppler method accompany significant blurring.

To obtain clear ISAR images from a signal with nonuniform angle samples, compressed sensing (CS) based approaches can be used [8–13]. However, CS-based approaches have two major drawbacks. First, a precise parametric model is required in order to achieve high quality ISAR images. Second, lot of computation time is required since the CS technique uses an optimization process such as linear programing.

In this paper, a MUSIC-based approach was proposed to improve image quality and reduce computation time. Also, a receiving target signal model with a search radar was introduced to deal with the nonuniform angle signal of a target. In order to apply the MUSIC technique, a covariance matrix acquisition for a given single snapshot type signal should be preceded [14]. However, the conventional way to form the covariance matrix with a single snapshot signal, so called spatial smoothing process

Received 18 January 2018, Accepted 14 February 2018, Scheduled 23 February 2018

* Corresponding author: Hyung Ju Kim (kimhj@kaist.ac.kr).

[1] Department of Electrical Engineering, Korea Advanced Institute of Science and Technology (KAIST), 291 Daehak-ro, Yuseong-gu, Daejon 305-701, Republic of Korea. [2] Agency for Defense Development (ADD), P. O. Box 35-3, Yuseong-gu, Daejeon, Republic of Korea.

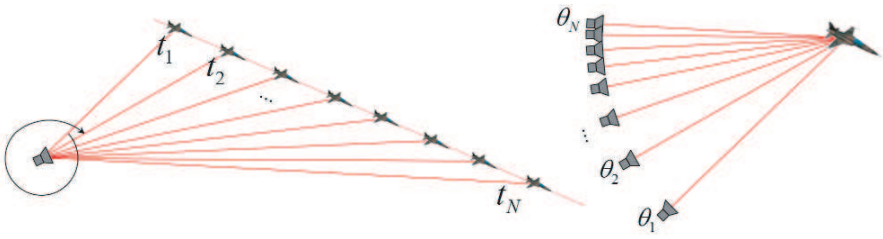

Figure 1. Nonuniform observation with search radar system.

(SSP), could not generate a meaningful covariance matrix from the nonuniform signal [15, 16]. Thus, estimated multiple snapshot signals, which were phase delayed versions of the given original single snapshot signals, were used to generate the covariance matrix. Experiments on simulated and real data were performed to validate the proposed method.

This paper is organized as follows. Section 2 introduces a signal model for search radar. Section 3 presents the proposed ISAR imaging method for the target back-scattered signal with a nonuniform observation angle. Section 4 presents experimental results to verify the proposed method. Finally, conclusions are given in Section 5.

2. SIGNAL MODEL FOR SEARCH RADAR

It was assumed that the target was moving straightforward with constant velocity and without changing direction in the 2-D plane. When the target was observed with a rotating antenna which has constant angular velocity, the target appeared with a nonuniform time interval, as shown in Figure 1. In other words, a search radar with a rotating antenna observes the target with nonuniform aspect angles. Nonuniform aspect angles should be determined to model received signals in a search radar. When the antenna rotates with angular velocity ω, the target observation angle θ_n, for the nth angle sample, can be expressed as follows:

$$\theta_n = \omega t_n = \frac{2\pi}{T}\left(nT + t'_n\right) = 2\pi n + 2\pi\frac{t'_n}{T} \tag{1}$$

where T is the rotation period of antenna, t_n the target observation time, and t'_n the time difference between the antenna rotation period and the observation time at the nth angle sample. When the target moves within the x-y plane, as shown in Figure 2, $\tan\theta_n$ is expressed as follows:

$$\tan\theta_n = \frac{x_n}{(R + y_n)} = \frac{vt_n\cos\phi}{(R + vt_n\sin\phi)} \tag{2}$$

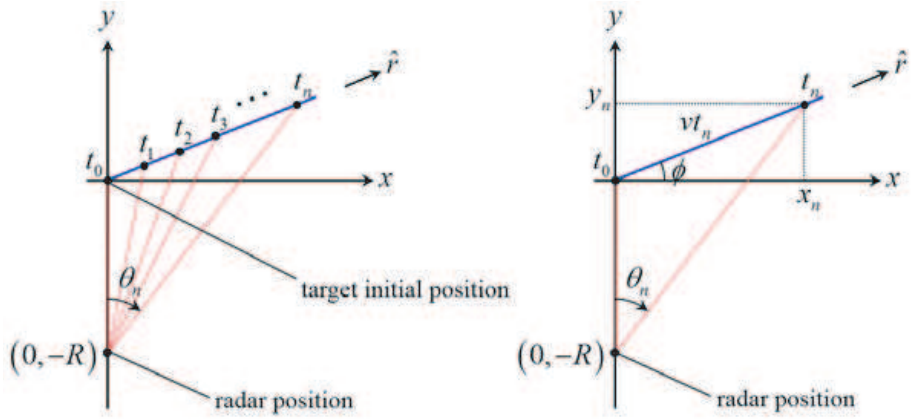

Figure 2. Geometry of nonuniform observation.

where R is the initial distance between the radar and the target, and v is the velocity of the target. Substituting Eq. (1) into Eq. (2), we have

$$\frac{1}{\tan(\omega t_n)} - \frac{R}{vt_n \cos\phi} - \tan\phi = 0. \tag{3}$$

In Eq. (3), t_n can be derived with known ω, R, ϕ. Then, θ_n can be derived by Eq. (1).

The receiving signal was assumed to be a stepped-frequency signal with M pulses in the frequency domain and N bursts in the angle domain. The target was modelled by K ideal scatterer points based on the geometric theory of diffraction (GTD) model [17]. The backscattered raw data $s(n,m)$ from the target at frequency f_m and aspect angle θ_n can be expressed as follows:

$$s(n,m) = \sum_{k=1}^{K} a_k e^{-j\left(\frac{4\pi}{c} f_m\right) R_{k,n}} \tag{4}$$

where a_k is the complex amplitude of the kth scatterer, c the speed of light, and $R_{k,n}$ the distance from the radar to the kth scatterer in the aspect angle θ_n. Equation (4) is the scattered signal with nonuniform angle samples. An ISAR image will be obtained from the signals in Section 3.

3. ISAR IMAGING WITH NONUNIFORM SIGNAL

3.1. ISAR Imaging Based on MUSIC

The process of producing an ISAR image from a backscattered signal with nonuniform angle samples is composed of the following two steps. The first step is to obtain N range profiles at N aspect angle. We assume that the range profiles are already aligned. The second step is to estimate the Doppler spectrum in each range bin by performing the 1-D MUSIC method. The procedure is addressed below.

As shown in Figure 3, $R_{k,n}$ can be expressed as follows:

$$R_{k,n} = r + x_k \cos\theta_n + y_k \sin\theta_n \tag{5}$$

where r is the reference range from the radar to imaging center, x_k the down-range of k-th scatterer point, and y_k the cross-range of the k-th scatterer point. Distance $R_{k,n}$ contain rotational components of the target, which cause a blurring effect in images with long-observation time. After range compression and translational motion compensation, the receiving signal in one range cell can be expressed as

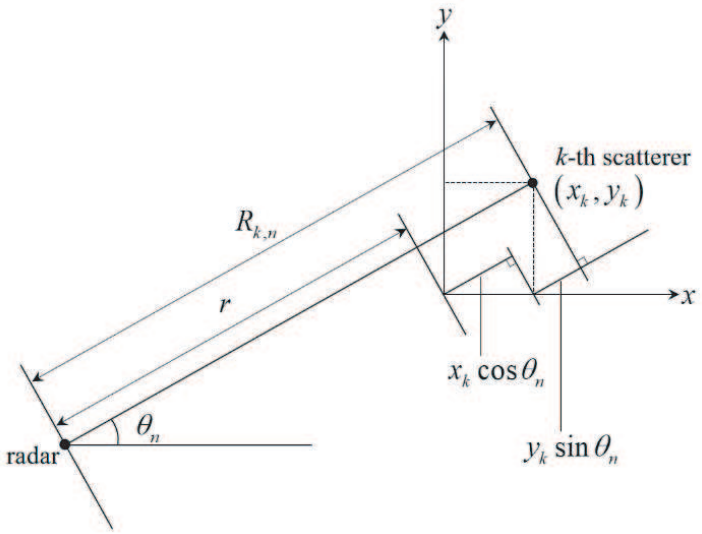

Figure 3. Geometry of ISAR signal model.

follows [10]:

$$s(n) = \sum_{k=1}^{K'} a_k e^{-j\left(\frac{4\pi}{c} f_c\right)(r + x_k + y_k \theta_n)} \tag{6}$$

where f_c is the centre frequency of the radar signal bandwidth, and K' is the total number of scatterers in selected range bin. Now, we consider the noise term, and the angle domain signal at the mth range bin is expressed as follows:

$$\mathbf{x}_m = \sum_{k=1}^{K'_m} \mathbf{a}\left(y_{k,m}\right) s_{k,m} + \mathbf{n}_m \tag{7}$$

where K'_m is the total number of scatterers in the m-th range bin, $s_{k,m}$ the amplitude of the kth scatterer at the mth range bin, $\mathbf{n}_m$ an additive white Gaussian noise with zero mean at the mth range bin, and $\mathbf{a}$ a nonuniform steering vector that contains the cross-range of each scatterer. The steering vector is expressed as follows:

$$\mathbf{a}(y) = \left[e^{-j2\pi\left(\frac{2f_c}{c}\right)y\theta_1},\ e^{-j2\pi\left(\frac{2f_c}{c}\right)y\theta_2},\ \ldots,\ e^{-j2\pi\left(\frac{2f_c}{c}\right)y\theta_N}\right]^T \tag{8}$$

where superscript T indicates the transpose operation.

In each range bin, SSP should be used to generate the covariance matrix since the angle domain signal is a single snapshot signal. However, because of the nonuniform angle samples, the rank of the covariance matrix generated by conventional SSP does not match the number of scatterer points in the selected range bin. Hence, it causes significant burring in image results.

In this paper, multiple phase delayed angle domain signals were used to generate the covariance matrix instead of a single angle domain signal. Total L phase delayed angle domain signals were estimated from the given angle domain signal.

The total cross-range interval can be determined from given observation angular interval of the target. Then, a set in a cross-range interval $[y_{\min},\, y_{\max}]$ can be defined as follows [18, 19]:

$$Y = [y_{\min},\, y_{\min} + \Delta y,\, y_{\min} + 2\Delta y,\, \ldots,\, y_{\max}] \tag{9}$$

where $y_{\min}$ is the minimum value of cross-range, $y_{\max}$ the maximum value of cross-range, and Δy the small arbitrary interval of cross-range. The small arbitrary interval forms a tradeoff relationship between computation time and accuracy of estimation. Thus, in this paper, Δy is determined and used as an empirical way.

Substituting each element of Eq. (9) into Eq. (8) and arranging each steering vector into a matrix form give

$$A_m = \left[\mathbf{a}\left(y_{\min}\right),\, \mathbf{a}\left(y_{\min} + \Delta y\right),\, \ldots,\, \mathbf{a}\left(y_{\max}\right)\right] \tag{10}$$

Matrix A_m is the array manifold of the real array. Meanwhile, the lth vector of the total L phase delayed steering vectors is expressed as follows:

$$\mathbf{a}_l(y) = \left[e^{-j2\pi\left(\frac{2f_c}{c}\right)y(\theta_1 + l\Delta\phi)},\ \ldots,\ e^{-j2\pi\left(\frac{2f_c}{c}\right)y(\theta_N + l\Delta\phi)}\right]^T \tag{11}$$

Now we denote $\bar{A}_{m,l}$ as an array manifold of a phase delayed array and compute for set Y as follows:

$$\bar{A}_{m,l} = \left[\mathbf{a}_l\left(y_{\min}\right),\, \mathbf{a}_l\left(y_{\min} + \Delta y\right),\, \ldots,\, \mathbf{a}_l\left(y_{\max}\right)\right] \tag{12}$$

We use the least square method (LSM) to estimate a constant transformation matrix $B_{m,l}$ that satisfies the following relationship:

$$B_{m\cdot l} \cdot A_m = \bar{A}_{m,l} \tag{13}$$

With transform matrix $B_{m,l}$, we can estimate a signal that is phase delayed by $l\Delta\phi$ from a given single snapshot signal, expressed as follows:

$$\bar{\mathbf{x}}_{m,l} = B_{m\cdot l} \cdot \mathbf{x}_m \tag{14}$$

The transformation matrix means changing given single snapshot signal to a phase delayed signal by $l\Delta\phi$. By repeating this process L times, L phase delayed signals can be obtained. Then, a sampled covariance matrix can be generated by using the estimated multi-phase delay signals expressed as follows:

$$\mathbf{R} = \frac{1}{L}\sum_{l=1}^{L}\left(\bar{\mathbf{x}}_{m,l}\cdot\bar{\mathbf{x}}_{m,l}^{H}\right) \tag{15}$$

Using this covariance matrix, the MUSIC technique can be used to attain a proper Doppler spectrum in each range bin. The detailed procedure can be found in [6, 20].

3.2. Improving Computation Efficiency Using a Threshold

Applying the proposed 1-D MUSIC method to estimate the Doppler spectrum in every range bin may cause computation inefficiencies. This is because it operates in noise regions that do not contain the scatterers of the target. To distinguish signal regions from noise regions, the energy sum of envelopes was calculated in each range bin as follows:

$$E(m) = \frac{1}{N}\sum_{n'=1}^{N}\left|\mathbf{IFFT}_m\left(s\left(n',m\right)_{1\times M}\right)\right|^2 \geq \text{threshold} \tag{16}$$

where $\mathbf{IFFT}_m$ denotes the inverse fast Fourier transform operation with respect to the variable frequency index m. The range bins whose energy sum of envelopes exceeded a threshold were selected as signal regions. The proposed method was implemented only to the signal regions

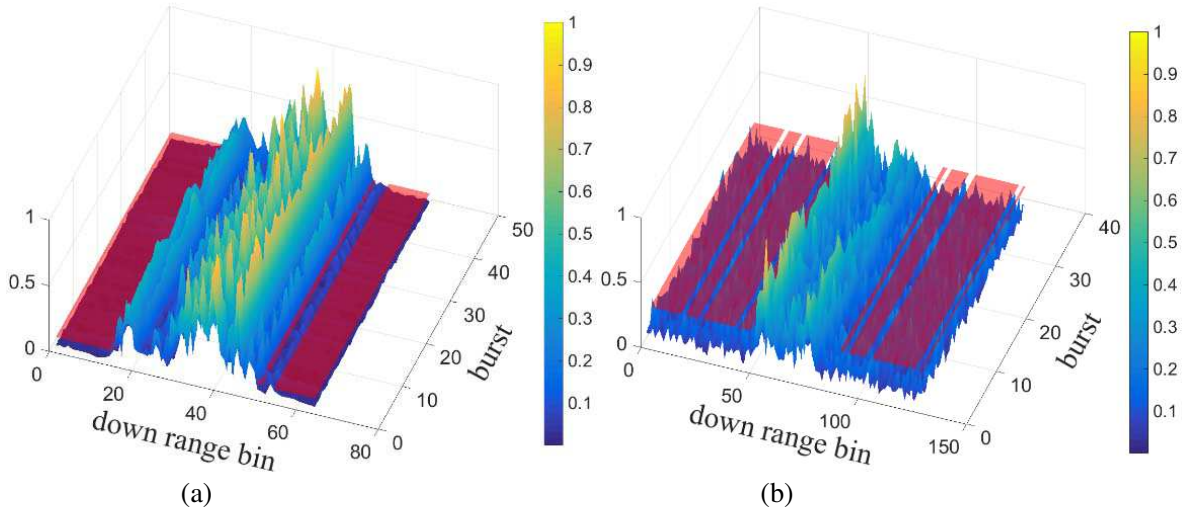

Figure 4. (a) Range profile history of the simulated Boeing 727 model with threshold. (b) Range profile history of the measured Boeing 727 model with threshold.

The range profile histories of simulated and measured Boeing 727 plane models is shown in Figure 4(a) and Figure 4(b), respectively. The SNRs of the simulated and measured models were 30 dB and unknown, respectively. The empirical threshold levels are displayed in Figure 5(a) and Figure 5(b). The simulated and measured Boeing 727 plane models have 56.3% and 35.9% signal regions, respectively. Hence, the computation efficiency can be improved by selectively applying the proposed 1-D MUSIC method to signal regions.

4. EXPERIMENTAL RESULTS AND ANALYSIS

To verify the proposed approach, a simulated Boeing 727 plane model with a size of 256×64 cells is used. The center frequency of the radar was 9 GHz, and the bandwidth was 150 MHz. The radar image

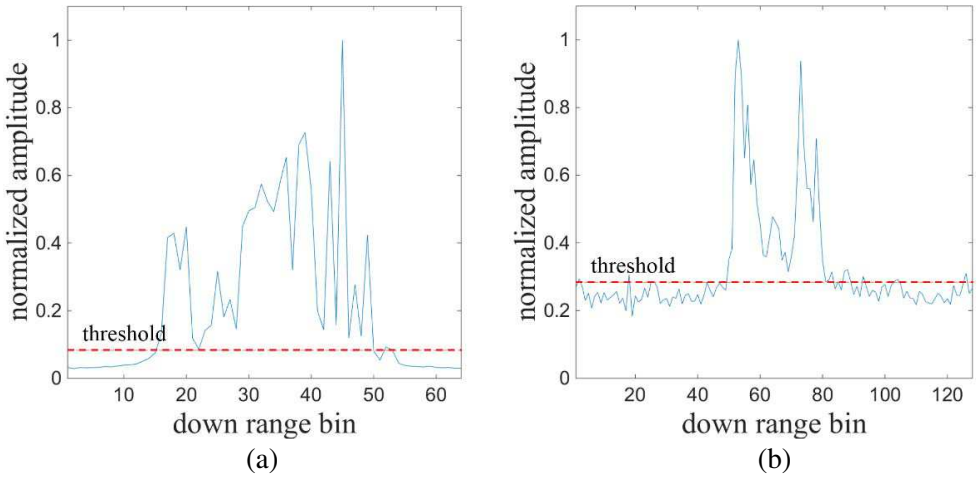

Figure 5. (a) The energy sum of envelopes for the simulated Boeing 727 model. (b) The energy sum of envelopes for the measured Boeing 727 model.

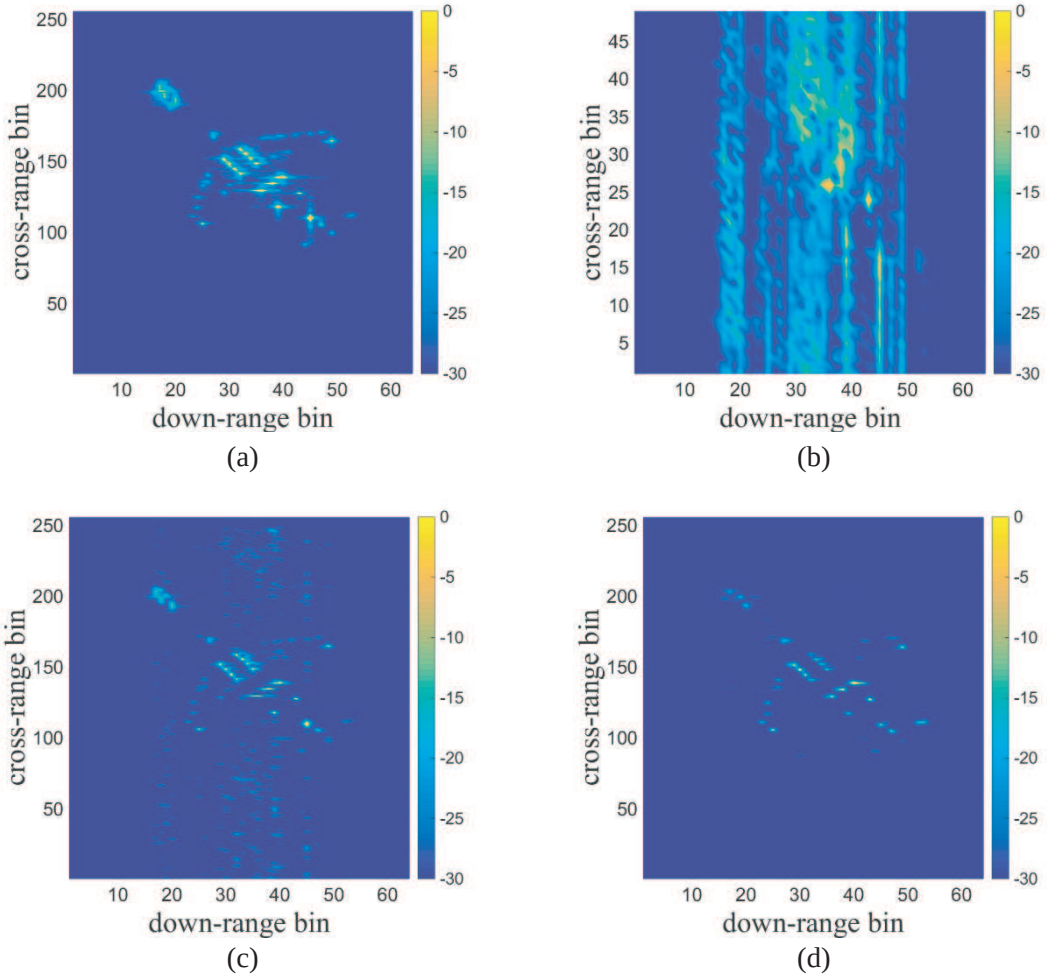

Figure 6. ISAR imaging results of simulated Boeing 727 plane model, (a) with conventional range-Doppler method for 256 angle sample, (b) with conventional range-Doppler method for nonuniform 49 angle sample, (c) with CS method for nonuniform 49 angle sample, (d) with proposed method for nonuniform 49 angle sample.

consisted of 64 range cells and 256 uniform angle cells. The ISAR imaging result of these raw data by adopting conventional range-Doppler processing is presented in Figure 6(a).

Preprocessing for nonuniform angle sampling is needed to consider a search radar. The following parameters were used to generate a nonuniformly sampled model from a uniformly sampled Boeing 727 plane model such as that obtained from the search radar. The rotation per minute (RPM) was 30, the distance between the radar and the target was 20 km, the velocity of the target was 340 m/s, and the target moving vector was (1, 5). The signal is reconstructed as a search radar signal with 49 nonuniform angle samples out of 256 uniform angle samples given. Since the search radar does not continuously observe the target, we set the number of angle samples to be reduced in this way. An ISAR image for a 49×64 size of search radar signal with the conventional range-Doppler method is shown in Figure 6(b). Due to the reduction in the number of angle samples and the nonuniformity, the ISAR image accompanied resolution degradation and significant blurring. ISAR images for a 49×64 size of search radar signal with the CS method and the proposed nonuniform MUSIC method are shown in Figure 6(c) and Figure 6(d), respectively. Using the CS method enabled reconstruction of the original image from a limited number of samples. Usually, the CS method shows the best performance when random samples are used. In this paper, the performance is limited because nonuniform samples are used instead of random samples. On the other hand, the proposed method is suitable technique for

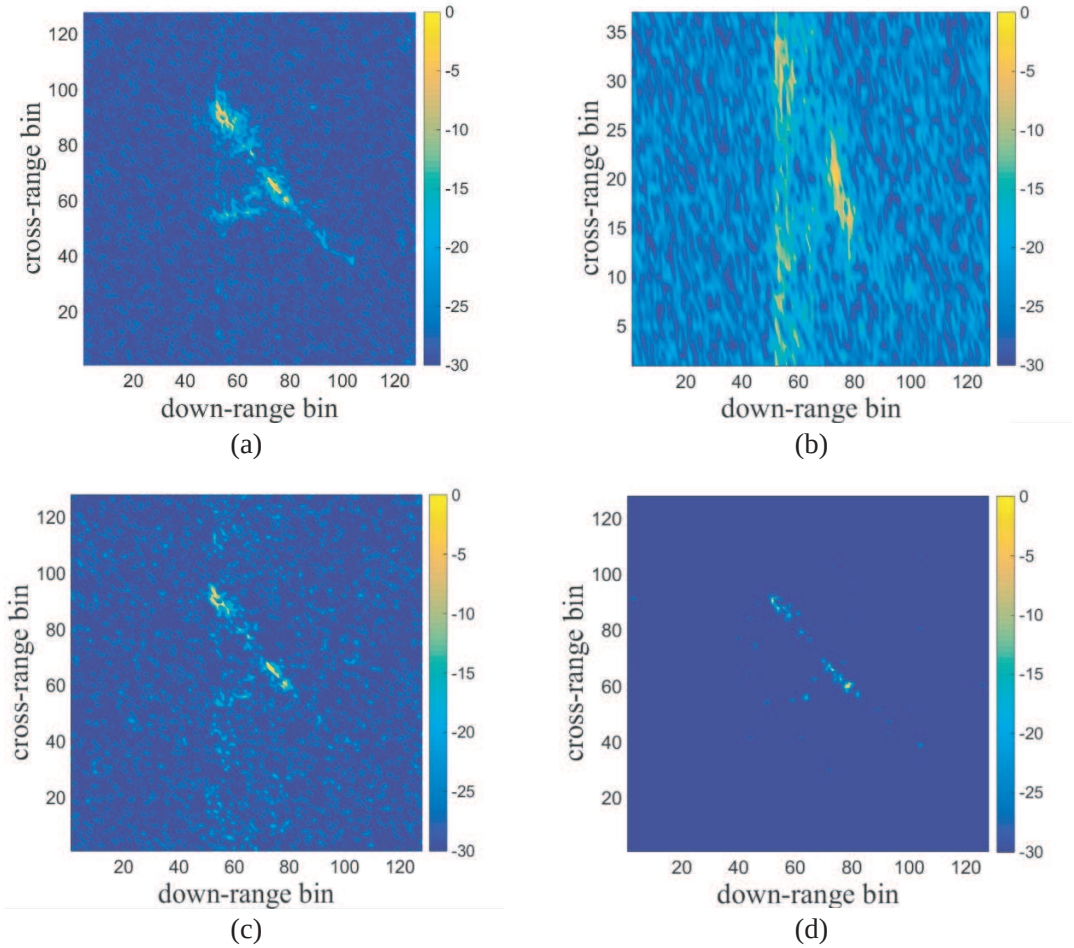

Figure 7. ISAR imaging results of measured Boeing 727 plane model, (a) with conventional range-Doppler method for 128 angle sample, (b) with conventional range-Doppler method for nonuniform 37 angle sample, (c) with CS method for nonuniform 37 angle sample, (d) with proposed method for nonuniform 37 angle sample.

nonuniform samples and as a result shows clear image with high resolution.

The proposed method was implemented in Matlab code and run on an Intel Core i7-3770K 3.50 GHz-CPU. In order to acquire the ISAR image, it took 10.1 seconds for the CS method and 6.2 seconds for the proposed method. Since the proposed method is not based on an optimization method like the CS method, the computation time was shorter.

In this paper, we performed experiment not only for the simulated plane model but also for a measured plane model. Measured Boeing 727 plane model with a size of 128×128 cells was used. The center frequency of the radar was 9 GHz, and the bandwidth was 150 MHz. The radar image consisted of 128 range cells and 128 uniform angle cells. The ISAR imaging result of this raw data by adopting conventional range-Doppler processing is presented in Figure 7(a). Similar to the simulated Boeing 727 experiment, the following parameters were used to generate a nonuniformly sampled Boeing 727 plane model from the uniformly sampled Boeing 727 plane model. The RPM was 22, the distance between the radar and the target was 20 km. We assumed the velocity of the target was 272 m/s, and the target moving vector was (1, 5). The signal was reconstructed as a search radar signal with 37 nonuniform angle samples out of 128 uniform angle samples given. The ISAR image for a 37×128 size of search radar signal with the conventional range-Doppler method is shown in Figure 7(b). Due to the reduction in the number of angle samples and the nonuniformity, the ISAR image accompanied resolution degradation and significant blurring. ISAR images for a 37×128 size of search radar signal with the CS method and the proposed nonuniform MUSIC method are shown in Figure 7(c) and Figure 7(d), respectively. It is clearly seen that the proposed method produced a superiorly clear image compared to the CS method. The proposed method achieved high noise suppression and high resolution images. With the same CPU that obtained the simulated Boeing 727 results, it took 10.7 seconds for the CS method and 5.5 seconds for the proposed method to obtain an ISAR image.

5. CONCLUSION

In this paper, a method to obtain a 2-D ISAR image from a scattering signal of a target having nonuniformly observed angle samples was proposed. From the nonuniform snapshot signal, LSM-based multi-phase delay signals were estimated to form the covariance matrix. Then, using the covariance matrix, ISAR images were obtained by estimating the Doppler spectrum based on MUSIC. The proposed method was applied to both simulated and measured plane models, and promising results are presented. This method can be used for ISAR image generation using a search radar.

ACKNOWLEDGMENT

The authors wish to acknowledge the assistance and support of the Agency for Defense Development of Korea.

REFERENCES

1. Ausherman, D. A., A. Kozma, J. L. Walker, H. M. Jones, and E. C. Poggio, "Development in radar imaging," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 20, No. 4, 363–400, 1984.
2. Li, H. J. and S. H. Yang, "Using range profiles as feature vectors to identify aerospace objects," *IEEE Transactions on Antennas and Propagation*, Vol. 41, No. 3, 261–268, 1993.
3. Lee, J. H. and H. T. Kim, "Radar target discrimination using transient response reconstruction," *Journal of Electromagnetic Waves and Applications*, Vol. 19, No. 5, 655–669, 2005.
4. Chen, C. C. and H. C. Andrews, "Target-motion-induced radar imaging," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 16, 2–14, 1980.
5. Chen, V. C. and S. Qian, "Joint time-frequency transform for radar range-Doppler imaging," *IEEE Transactions on Aerospace and Electronic System*, Vol. 34, No. 2, 486–499, 1998.
6. Kim, K. T., D. K. Seo, and H. T. Kim, "Efficient radar target recognition using the MUSIC algorithm and invariant features," *IEEE Transactions on Antennas and Propagation*, Vol. 50, No. 3, 325–327, 2002.

7. Chen, V. C. and H. Ling, *Time-frequency Transforms for Radar Imaging and Signal Analysis*, Artech House, Norwood, MA, 2002.
8. Donoho, D. L., "Compressed sensing," *IEEE Transactions on Information Theory*, Vol. 52, No. 4, 1289–1306, 2006.
9. Zhang, L., M. Xing, C. W. Qiu, J. Li, and Z. Bao, "Achieving higher resolution ISAR imaging with limited pulses via compressed sampling," *IEEE Geoscience and Remoted Sensing Letters*, Vol. 6, No. 3, 567–571, 2009.
10. Wang, B., S. Zhang, and W. Q. Wang, "Bayesian inverse synthetic aperture radar imaging by exploiting sparse probing frequencies," *IEEE Antennas and Wireless Propagation Letters*, Vol. 14, 1698–1701, 2015.
11. Hou, Q., Y. Liu, and Z. Chen, "Reducing micro-Doppler effect in compressed sensing ISAR imaging for aircraft using limited pulses," *Electronics Letters*, Vol. 51, No. 12, 937–939, 2015.
12. Khwaja, A. S. and X. P. Zhang, "Compressed sensing ISAR reconstruction in the presence of rotational acceleration," *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, Vol. 7, No. 7, 2957–2970, 2014.
13. Karabayir, O., O. M. Yucedag, S. M. Yucedag, and S. Kent, "Performance analysis of compressive ISAR imaging for complex targets," *Journal of Electromagnetic Waves and Applications*, Vol. 28, No. 10, 1236–1245, 2014.
14. Thakre, A., M. Haardt, and K. Giridhar, "Single snapshot spatial smoothing with improved effective array aperture," *IEEE Signal Processing Letters*, Vol. 16, No. 6, 505–508, 2009.
15. Doron, M. A. and E. Doron, "Wavefield modeling and array processing, Part 1 — Spatial sampling," *IEEE Transactions on Signal Processing*, Vol. 42, No. 10, 2549–2559, 1994.
16. Belloni, F., A. Richter, and V. Koivunen, "DoA estimation via manifold separation for arbitrary array structures," *IEEE Transactions on Signal Processing*, Vol. 55, No. 10, 4800–4810, 2007.
17. Potter, L. C., D. M. Chiang, R. Carriere, and M. J. Gerry, "A GTD-based parametric model for radar scattering," *IEEE Transactions on Antennas and Propagation*, Vol. 43, No. 10, 1058–1067, 1995.
18. Friedlander, B. and A. J. Weiss, "Direction finding using spatial smoothing with interpolated arrays," *IEEE Transactions on Aerospace and Electronics System*, Vol. 28, No. 2, 574–587, 1992.
19. Rubsamen, M. and A. B. Gershman, "Direction-of-Arrival estimation for nonuniform sensor arrays: From manifold separation to Fourier domain MUSIC methods," *IEEE Transactions on Signal Processing*, Vol. 57, No. 2, 588–599, 2009.
20. Schmidt, R., "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, Vol. 34, No. 3, 276–280, 1986.