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Spatial Structure of Electromagnetic Field Diffracted by a Sub-Wavelength Slot in a Thick Conducting Screen

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Abstract—The eigen-mode technique of rigorous diffraction theory is employed for computation of spatial structure of electromagnetic field, arising under diffraction of a plane wave by a narrow slot of the width of the order of the wavelength or smaller in a perfectly conducting screen of finite thickness. The effects of little step change and of strong enhancement for relative averaged energy density are investigated in dependence of the slot width and depth. It is shown that the field in a space behind the slot represents the sum of a field, slowly and monotonically decreasing in the directions away from a slot, and a harmonic field with sinusoidal spatial inhomogeneities of the order of the wavelength. It is established that the comparative contributions of these two field constituents are unequal for various spatial components of the electric and magnetic fields, and also that the contribution of the first constituent decreases with increase of the slot width.

1. INTRODUCTION

Not so long ago, the authors of the work [1] have paid attention to the interesting effect of anomalous high concentration of electromagnetic energy under its transmission through narrow sub-wavelength holes in metallic screens. The subsequent investigations [2–5] have confirmed that such apertures and gaps in metal surfaces can tightly localize electromagnetic waves well below the diffraction limit and lead to strong field enhancements. These effects have been observed as for solitary apertures [2], as for their quasi-ordered or well-ordered set, forming a regular two-dimensional grating on a conducting surface [3, 4]. Great interest in such effects is generated by the opportunity of local field action on solitary nanoparticles and their applications to terahertz spectroscopy of separated molecules [4].

The standard approximate theory of optical transmission through apertures [6] is not applicable to modeling of the observed phenomenon, because the aperture dimension is of the order of the wavelength or smaller. Here, one should utilize the exact theory. For that, numerical methods (for example, the finite element method [3]) and various analytical and semi-analytical methods were used (see the review [5]). With the help of those one succeeded in explaining of basic anomalous effects of narrow slots transmission. However, in most cases, researchers restrict their consideration to theoretical computation of energetic parameters on the slot output immediately, and they have paid little attention to study of the field structure in the space behind that. In the present work, the main features of spatial structure of the narrow slot diffraction field are investigated in all regions of its propagation inside the slot and behind the screen. Our consideration uses a rigorous theoretical model of plane wave diffraction by a slot in a perfectly conducting screen of finite thickness [7], based on the eigen-mode technique.

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2. BASIC THEORY AND RESULTS OF COMPUTATIONS

We consider the problem of the stationary plane wave diffraction by a narrow slot in a thick perfectly conducting screen of finite thickness (Fig. 1), whose exact solution is presented in [7]. Here, the 2D diffraction picture is studied for two polarizations of incident wave, H (or TE) and E (or TM). The fields of the H polarization in all regions have the electric vector orthogonal to the plane of incidence xy (Fig. 1), being determined by the following equations (in the Gaussian system of units) [6, 8]

$$E_z = u \quad H_x = -\frac{i}{k}\frac{\partial u}{\partial y} \quad H_y = \frac{i}{k}\frac{\partial u}{\partial x} \tag{1a}$$

and for the E polarization the magnetic vector of fields is orthogonal to this plane

$$E_x = \frac{i}{k}\frac{\partial u}{\partial y} \quad E_y = -\frac{i}{k}\frac{\partial u}{\partial x} \quad H_z = u \tag{1b}$$

where u is the scalar field function, which should satisfy the Helmholtz equation [6], i the imaginary unite, $k = 2\pi/\lambda$ the wavenumber, and λ the wavelength of radiation (the exponential factor $\exp(-i\omega t)$, determining the temporal dependence of all fields, is omitted everywhere). The solutions for these polarizations separately proceed from representation of fields in various regions as superposition of their eigenmodes in the y-coordinate, satisfying the Maxwell equations and, partially, appropriate boundary conditions.

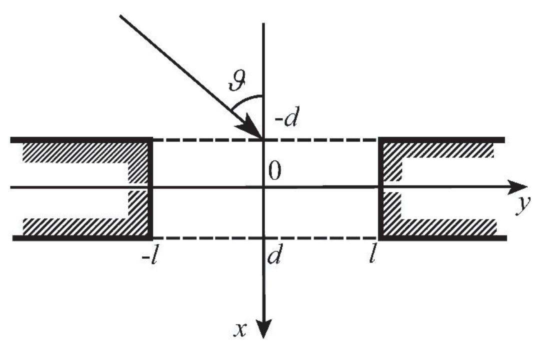

Figure 1. Geometry of the problem.

For example, inside the slot, the scalar field function is presented in the form of infinite sum of discrete modes, because their field function (for the H polarization) or its normal derivative (for the E polarization) should vanish on the slot walls (at $y = \pm l$)

$$\begin{aligned} u \;=\; \sum_{n=1}^{+\infty} \Big\{ & \sigma_{(s)n}^{-\nu} \left[a_n^{(s)} \exp\left(ik\sigma_{(s)n}(d+x)\right) + (-1)^\nu b_n^{(s)} \exp\left(ik\sigma_{(s)n}(d-x)\right)\right] \cos\left(k\xi_{(s)n} y\right) \\ & + i\sigma_{(a)n}^{-\nu} \left[a_n^{(a)} \exp\left(ik\sigma_{(a)n}(d+x)\right) + (-1)^\nu b_n^{(a)} \exp\left(ik\sigma_{(a)n}(d-x)\right)\right] \sin\left(k\xi_{(a)n} y\right) \Big\} \end{aligned} \tag{2}$$

where $\nu = 0$,

$$\xi_{(s)n} = (\pi/kl)(n - 1/2) \quad \xi_{(a)n} = (\pi/kl)n \tag{3a}$$

for the H polarization, and $\nu = 1$,

$$\xi_{(s)n} = (\pi/kl)(n - 1) \quad \xi_{(a)n} = (\pi/kl)(n - 1/2) \tag{3b}$$

for the E polarization, being

$$\sigma_{(s,a)n} = \sqrt{1 - \xi_{(s,a)n}^2} \tag{4}$$

for both polarizations. Every slot mode is represented by the sum of two waves with the amplitudes $a_n^{(s,a)}$ and $b_n^{(s,a)}$, which propagate (at real value of $\sigma_{(s,a)n}$) or decay (at imaginary value of $\sigma_{(s,a)n}$) in two opposite directions along the slot depth (in the x coordinate), but in the direction of the y coordinate, the field of each of these modes is a standing wave, being symmetrical (index (s)) or antisymmetrical (index (a)) in this coordinate.

In contrast to [7], here we use dimensionless values of parameters of propagation $\xi_{(s,a)n}$ and $\sigma_{(s,a)n}$ for all modes, extracting the dimensional multiplier k from these parameters.

Because of the absence of any constraints in the y-direction outside the slot, the spectrum of eigenmodes is continuous and is represented by the Fourier integral. For example, in the region behind the screen ($x \geq d$), which represents a free half-space,

$$u = \int_{-\infty}^{+\infty} \alpha^{-\nu} B(\beta) \exp\{ik[\alpha(x-d) + \beta y]\}\, d\beta \tag{5}$$

where

$$\alpha = \sqrt{1-\beta^2} \tag{6}$$

$B(\beta)$ are the amplitudes of the continuous spectrum modes, being dependent on the propagation constant β. Determining the parameter of normal propagation α as Eq. (6) with nonnegative imaginary part, we provide satisfaction of the Helmholtz equation for the diffraction field in Eq. (5) and its decrease at great distances from a slot. In the region before the screen ($x \leq -d$) one should take into consideration the field of an incident plane wave with the unit amplitude

$$\begin{aligned} u \;=\; & \{\exp[ik\alpha_0(x+d)] - (-1)^\nu \exp[-ik\alpha_0(x+d)]\} \exp(ik\beta y) \\ & +(-1)^\nu \int_{-\infty}^{+\infty} \alpha^{-\nu} A(\beta) \exp\{ik[-\alpha(x+d) + \beta y]\}\, d\beta \end{aligned} \tag{7}$$

where $\alpha_0 = \cos\vartheta$, $\beta_0 = \sin\vartheta$, ϑ is the angle of incidence of a diffracting wave on the surface of a screen, showing angular deflection of its direction of propagation from the normal to this surface. Here, we extract a reflected wave (the second summand in the figured brackets) also in the explicit form. Amplitudes of all diffraction modes in various regions $a_n^{(s,a)}$, $b_n^{(s,a)}$, $A(\beta)$ and $B(\beta)$ are initially unknown, and they are determined in the process of diffraction problem solving from the boundary conditions on the planes $x = -d$ and $x = +d$, coinciding with the boundaries of a screen [7].

After determining of modes amplitudes in all regions with the help of Equations (2), (5) or (7), one can compute the spatial components of the electric and magnetic fields in every point of space, substituting these equations into Eq. (1) [7]. Figs. 2 and 3 show the results of such computation for the fields of the H and E polarizations in the same scale for two specific cases, which give a general insight about structure of field of diffraction by the slot in a thick screen. These results are computed for the case, when the angle of incidence $\vartheta = 30°$. For the case of normal incidence, when $\vartheta = 0°$, spatial structure of fields is analogous to the first one, but it becomes purely symmetrical or antisymmetrical in the y coordinate.

Using the spatial field components, one can determine local distribution of its energetic parameters, more particularly, calculate the averaged output density of electric field W for a slot (at $x = +d$, $-l \leq y \leq l$), which is proportional to the integral of squared absolute value of electric vector in the slot width.

$$W = (2l)^{-1} \int_{-l}^{+l} |E_z(d,y)|^2 dy \quad \text{or} \quad W = (2l)^{-1} \int_{-l}^{+l} \left(|E_x(d,y)|^2 + |E_y(d,y)|^2\right) dy$$

The results of such a calculation are presented in Fig. 4. They demonstrate the influence of the slot half-width l on the averaged output slot energy density W, normalized to the energy density of incident wave, for two different polarizations at two values of the slot half-depth d. For the H polarization (Fig. 4(a)), the graphs demonstrate nearly threshold characteristic, showing sharp increase near the values of the width, a multiple of a quarter of the wavelength, and fair decrease, inversely proportional to the width increase, up to the next abrupt change. This effect is caused by the situation when the effective transfer of energy along the depth of a slot is supplied by those slot modes, whose parameter of normal propagation $\sigma_{(s,a)n}$ (4) is real and provides propagation exactly, but not decay of the mode. For that, the corresponding parameter of lateral propagation $\xi_{(s,a)n}$ in Eq. (3), which is proportional to the value $\pi/kl = \lambda/2l$, should not be greater than the unite in magnitude. The gradual increase of

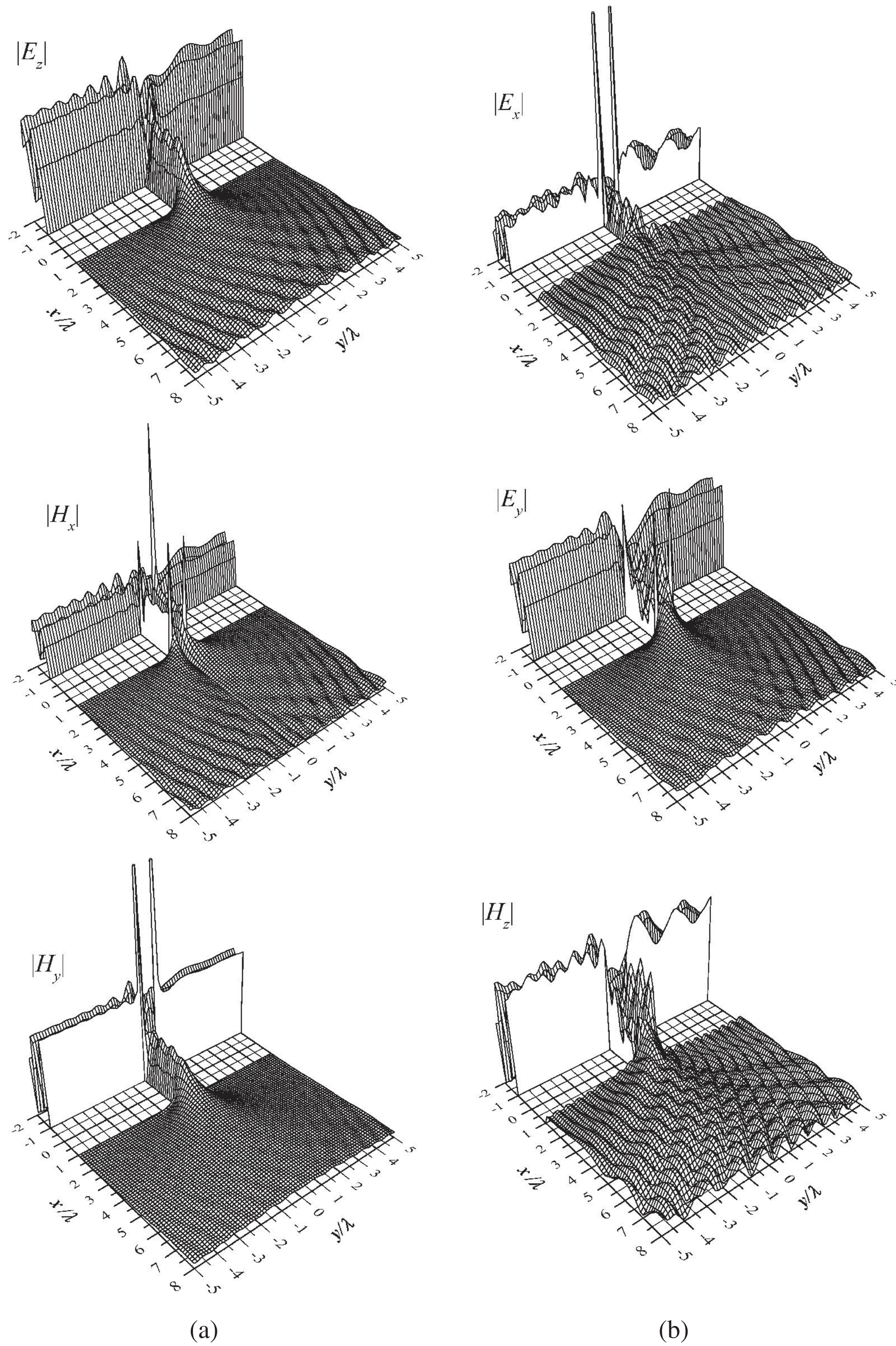

Figure 2. Spatial distribution of various components of the electric and magnetic fields in magnitude under diffraction of a plane electromagnetic wave of (a) the H polarization and (b) the E polarization by the slot with the half-width $l = 0.375\lambda$ in the perfectly conducting screen of the half-thickness $d = \lambda$, when the angle of incidence $\vartheta = 30°$.

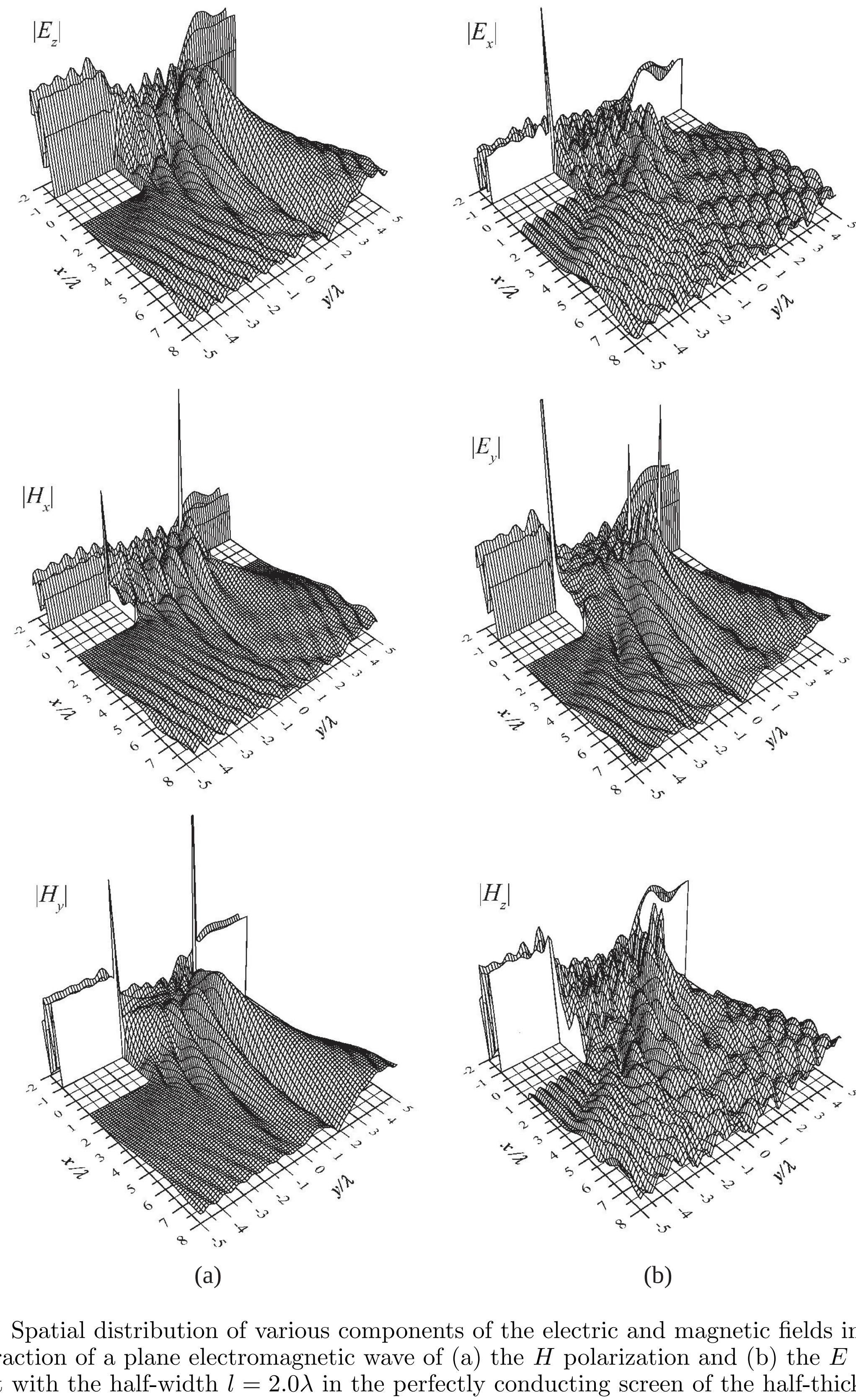

Figure 3. Spatial distribution of various components of the electric and magnetic fields in magnitude under diffraction of a plane electromagnetic wave of (a) the H polarization and (b) the E polarization by the slot with the half-width $l = 2.0\lambda$ in the perfectly conducting screen of the half-thickness $d = \lambda$, when the angle of incidence $\vartheta = 30^\circ$.

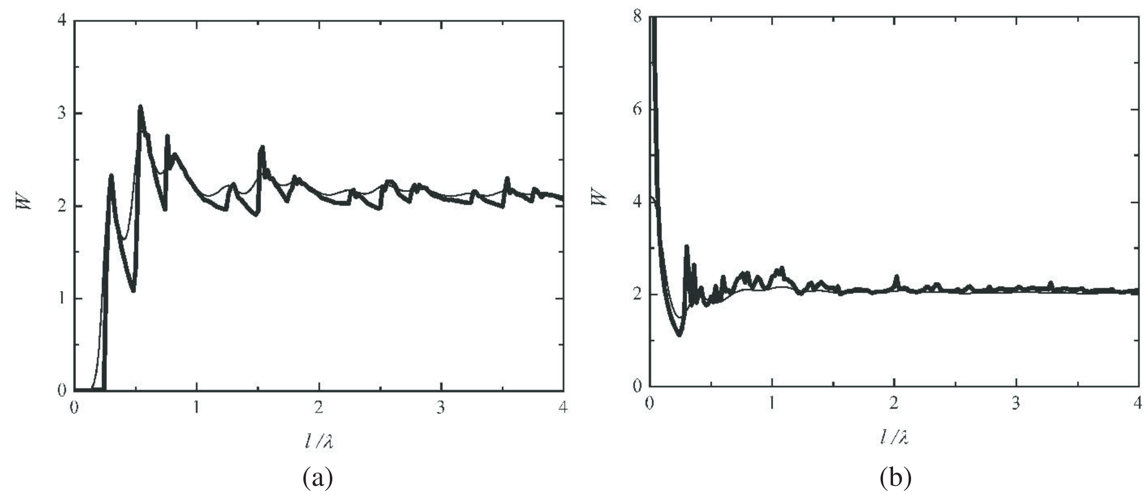

Figure 4. Relative averaged energy density W on the exit of a slot aperture as a function of its half-width l at two different values of the screen half-thickness (slot half-depth) $d = 0.1\lambda$ (thin lines) and $d = 3.0\lambda$ (thick lines) for (a) the H polarization and for (b) the E polarization of incident wave; the angle of incidence $\vartheta = 30°$.

the slot width provides slow decrease of the tangential propagation parameters $\xi_{(s,a)n}$ for slot modes and connection of a regular mode to the process of effective energy transmission through a slot, as its normal propagation parameter $\sigma_{(s,a)n}$ (4) becomes real. Such connection displays as a sharp increase of transmitted energy. However after that for some time, the number of modes, being effective transmitters, remains unchanged, and the energy density decreases with increase of the slot width. The described effect appears more explicitly in the cases of thick screens, where the presence of decay in slot modes provides small value of their fields on the slot output, and at the same time for slots of small depth, decaying modes can make appreciable contribution here to the total field.

For the E polarization (Fig. 4(b)), this effect is concealed by the other phenomenon — singular behavior of the fields at the edges of a slot. It is known that spatial components of electric field, which is normal to the edges, increase in magnitude with decrease of distance from those as an inverse cube root of this distance [9]. The spatial region of this anomaly near edges is very small, so for a wide slot its influence on the energy density is not substantial. However for a narrow slot, the anomalous region can cover its width wholly, causing here very great magnitude of energy density. Fig. 4 demonstrates that at $l < 0.1\lambda$ the averaged energy density of transmitted field on a slot is 4 and greater than that of the incident wave in free space by the factor of 4 and more. This effect provides the opportunity for devices with narrow sub-wavelength slots to stand duty as local concentrators of electromagnetic energy and to play the part of nanoantennae.

The dependences look like those observed under change of the wavelength of transmitting radiation with unchanged slot width [5], since here the absolute value of the slot width $2l$ does not matter, but its relative value in comparison with the wavelength $2l/\lambda$ is of importance. However, the appearance of curves for both cases is distinguished one from the other because the wavelength change causes the change of the relative slot depth $2d/\lambda$ (the screen thickness).

Considering the field in space behind the narrow slot, one should take into account its great divergence at subsequent propagation in free space (at $x > d$) (see Figs. 2 and 3), which is caused by scattering on the edges [7, 10]. Such divergence can be evaluated by a scalar parameter C_{div} [10], which is determined as a ratio of effective width of field energy spatial distribution at a given distance from a screen $x = \text{const}$ to the slot width, i.e., as increase of the width of a slot diffraction image at a given distance x with respect to the width of an aperture, creating this image. Fig. 5 demonstrates the computation results for the parameter of slot image divergence in dependence of its width, which have been obtained according to the method described in [10], for the distance $x = 4.0\lambda$. Here, one can see very great value of divergence, which is proper to very narrow slot. However, the faint characteristic of its dependence on screen thickness (depth of a slot) is slightly unexpected. An additional interesting

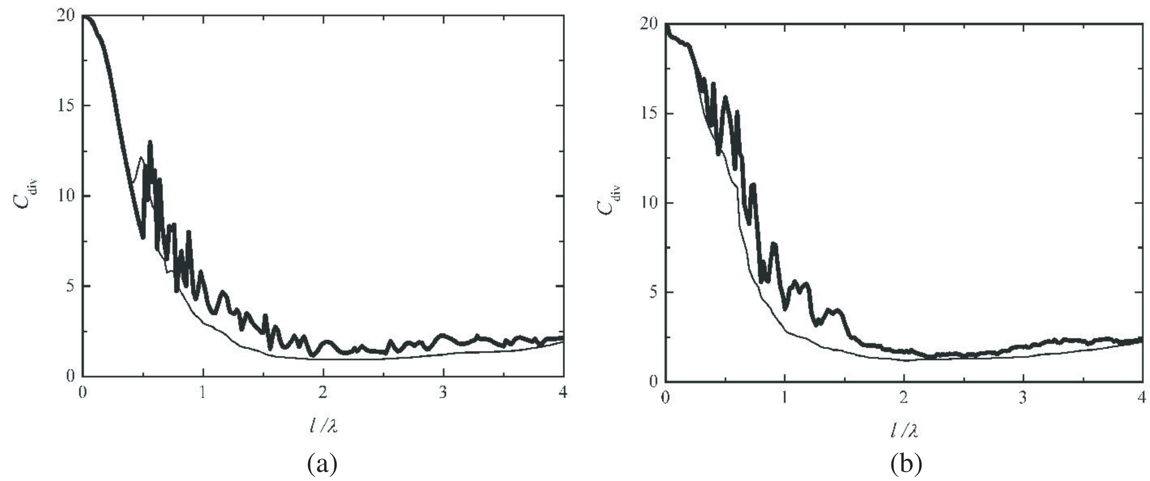

Figure 5. Integral divergence of diffraction field C_{div} on the distance of 4.0λ from a slot as a function of its half-width l at two values of the screen half-thickness (slot half-depth) $d = 0.1\lambda$ (thin lines) and $d = 3.0\lambda$ (thick curves) for (a) the H polarization and for (b) the E polarization of field; the angle of incidence $\vartheta = 30°$.

effect is the presence of the minimum of divergence in the region where the slot half-width $l \approx 2.0\lambda$, when the effective image width is about twice the slot width. At subsequent increase of the half-width l, divergence demonstrates slow rise, although it would seem that increase of a transmitting aperture should cause only decrease of transverse diffraction divergence.

It should be borne in mind that diffraction convergence as a scalar parameter is a space-averaged characteristic of transmitting field and leaves out of account many features of its spatial structure. Figs. 2 and 3 display such a structure for two cases of various slot half-width $l = 0.375\lambda$ and $l = 2.0\lambda$, when the diffraction divergence is $C_{div} = 8.63$ and $C_{div} = 1.72$, respectively. Obviously, in the second case, the field accumulates greater in the region of geometrooptical image at $-l+(x-d)\tan\vartheta < y < l+(x-d)\tan\vartheta$ (ϑ is the angle of incidence), but for a wide slot as for a narrow one, the lateral amplitude distribution in the plane $y = \text{const}$ is characterized by the presence of spatial inhomogeneities of various scales.

It is known that the emission of a narrow aperture in the far zone is similar to the emission of a point source, and the smaller the aperture width is, the closer is the picture of its emission to the picture of divergent spherical waves. However, Figs. 2 and 3 show that the spatial field distribution in the near zone bears little resemblance to the picture of a spherical wave. One can notice that for a narrow slot (Fig. 2) the field behind that is represented by superposition of two fields: the first is a field which characterized by great-scale inhomogeneities and decreases monotonically in all directions away from the slot, and the second one is a harmonic field, which changes in space periodically in the sinusoidal fashion with the period of the order of the wavelength. For the tangential field components E_z, H_y (the H polarization) and E_y (the E polarization), which are parallel to the screen boundaries, the first constituent is dominant, but for the normal components E_x, H_x, and also for the tangential magnetic component H_z, the contribution of this part is small, and a dominant constituent is harmonic one. Therefore, behind the slot, the first group of electric and magnetic field components does not display strong dependence on the spatial coordinates and looks on amplitude like electrostatic field. Such a picture can be observed on rather great distances from a slot in two coordinates, of 4–5 wavelengths and more. However, under increase of the slot width $2l$, the region of "static" field existence decreases rapidly (Fig. 3), and at $l > 2\lambda$ one observes appreciable contribution of a harmonic constituent in all points of field. The amplitude value of harmonic oscillations for this constituent at small slot width (Fig. 2) demonstrates a small dependence on the spatial coordinates. This value increases slowly with increase of the normal x-coordinate and is slightly dependent on the tangential y-coordinate. However with growth of the slot width, the strong inhomogeneities displays in the spatial distribution of amplitudes value (Fig. 3): its magnitude becomes predominant in the region of the geometrooptical slot image ($-l + (x - d)\tan\vartheta < y < l + (x - d)\tan\vartheta$) and monotonically but rather rapidly decreases at moving away from this region.

3. CONCLUSION

The rigorous electromagnetic diffraction theory provides the opportunity to explain energetic effects of electromagnetic radiation transmission through sub-wavelength slots of small width, such as the effect of its great concentration near slot edges, and also the effect of saw-toothed increase of averaged energy density on the slot output. They are caused by a specific spatial structure of diffraction fields in the slot and near its boundaries. However, together with increase of electromagnetic energy density on the slot output with decrease of its width, one obtains also increase of divergence of output radiation. In contradiction to slots with a wide aperture, where fast field decay occurs on the boundaries of a transmitting beam, a narrow slot determines very smooth decrease in the propagation direction and in the transverse direction, what is well illustrated by Figs. 2 and 3. About possible applications of narrow slot energetic effects for local-pointwise excitation of nanoparticles or molecules, such excitation can be realized only in the cases, when energy excitation has a threshold character and when the value of threshold energy density is exceeded by radiation intensity only in a small region near a slot output.

The field behind the slot represents the superposition of the field with great-scale spatial inhomogeneities in amplitude and harmonic field, which changes in space in a sinusoidal fashion with the period of the order of the radiation wavelength. For various spatial field components, the comparative contributions of these two constituents are not equal: for the tangential electric components, which are parallel to a screen, and for the magnetic component, which is parallel to a screen and is normal to the slot edges, the first constituent is greater, but with increase of slot width the contribution of that into the total diffraction field decreases.

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