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Experimental Verification of Quadrupole Model of the Electric Field of a Rotating Magnet

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Abstract—We performed an experiment to verify quadrupole model of the electric field of a rotating magnet. It is found that the rotating magnet insulated from the earth and enclosed in a conductive insulated screen induces the potential difference across an air capacitor arranged on the outside the screen. The field of an electric quadrupole cannot penetrate through the screen; therefore the electric field detected outside the screen has the source of another nature. The field observed in the experiment can be explained by arising of a fictitious electric charge upon rotating of the magnet in accordance with the transformations of the electromagnetic field in the theory of relativity.

1. INTRODUCTION

Magnetic field of an axially symmetric magnetic field source is stationary in the lab frame when it rotates about the axis parallel to its magnetic moment. The electric field produced by such a rotation will be potential. The first attempts to measure the potential electric field of a rotating solenoid or magnet were made in the beginning of the 20th century. In the experiments of Barnett, Kennard and Pegram electric fields of rotating solenoids and magnet were not detected [1–3]. In the experiment of Wilson and Wilson, the potential difference was measured across a magnetic dielectric cylinder. The cylinder was rotated in the external magnetic field of a static coaxial solenoid surrounding the magnetic cylinder [4]. The measured potential difference values correspond to the formula $U = n\Phi(1 - \frac{1}{\mu\varepsilon})$, derived from the special theory of relativity, where ε and μ are permittivity and permeability, and Φ is the magnetic flux through the magnetic dielectric cylinder, n the number of revolutions per second [4].

As will be shown below, it follows from the Wilson experiment that the rotating permanent magnet produces the electric field in the lab frame

$$E = \frac{1}{c} B \times (\omega \times r)\,, \tag{1}$$

where B and ω are the magnetic field and angular velocity of rotation of a magnet, and r is the radius vector drawn from the axis of rotation to the point of observation, c the speed of light.

Equation (1) is the electric field transformation from the frame comoving with the rotating magnet to the lab frame in the theory of relativity [12]. The fact that the Wilson experiment was repeated in our day points to the importance of the Wilson experiment for the theory [5]. In 1998 the radial components of the electric field of a rotating cylindrical magnet were measured by the authors (without an external magnetic field) in [6]. The results of the experiment were found to be in agreement with Eq. (1). Their next experiments also confirmed this equation [7, 8]. In 2014 Misiucenko performed an independent experiment on measurements of the electric field of a rotating ring magnet [9]. Misiucenko measured the potential electric field inside and outside the ring magnet and also confirmed Eq. (1). He

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also measured the electric field in the frame of reference rotating with the magnet and found that it is approximately hundred times less than that in the lab frame.

Current theoretical models of the potential electric field of a rotating magnet contradict Eq. (1) [10–12]. They all give the quadrupole electric field, when there are the density of electric charge of one sign on the magnet poles and one of the opposite sign — at the equator. Herewith, the total electric charge of an isolated rotating magnet is always equal to zero [12]. This fact gives a possibility to make another experiment on verification of the quadrupole model of the electric field of a rotating magnet. The electric field outside a conducting shield must be zero for the quadrupole if we enclose the magnet in the shield and bring the magnet into rotation. We made the first such an experiment in 2001 [13]. The experiment showed that the electric field of the rotating insulated magnet, enclosed in an insulated conductive shield, was non-zero, and its magnitude was about 1/3 of the field without the shield. It was found that the electric field is not detected when the shield is connected to earth. The experiment was a qualitative, and experimental error was not evaluated. It is of interest to repeat the experiment [13] and to carry out measurements to verify the quadrupole model.

2. EXPERIMENT

In this work we carry out an experiment on measurements of the potential difference across a ring air capacitor, located outside the shielded magnet (Fig. 1). The magnet can rotate around the axis of symmetry parallel to its magnetic moment. The ring air capacitor is arranged in the equatorial plane of the magnet coaxially with it. In our experiment neodymium ring magnet with a size of $80 \times 25 \times 8$ mm and magnetic moment of $(4397 \pm 11) \cdot 10^2$ Gs cm is enveloped in a closed cylindrical shield 85 mm diameter and 15 mm height made of metal-clad glass textolite (copper foil thickness is 0.1 mm). There is a hole 24 mm in diameter in the bottom of the shield for a bush. The magnet and shield are insulated from earth.

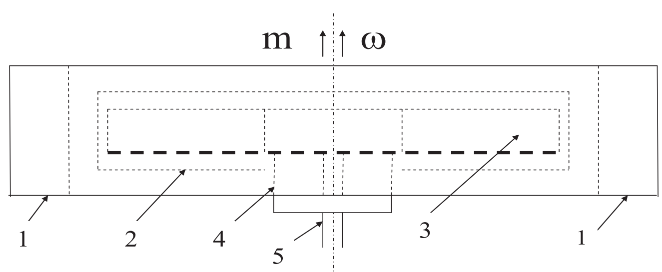

Figure 1. A schematic representation of the experimental apparatus: 1 — air ring capacitor; 2 — copper shield; 3 — neodymium magnet; 4 — bush; 5 — shaft of an electromotor.

An asynchronous electromotor can spin up the magnet to the speed of 2400 rev/min (40 Hz). Inner and outer capacitor's conductors are 96 ± 1 mm and 115 ± 1 mm diameters, 26.0 ± 0.5 mm height. The capacitor is made of a brass sheet 0.4 mm thick, and its electrical capacity is $C_c = 11.4 \pm 0.3$ pF. The bush 20 mm diameter and 10 mm height for mounting the magnet on the motor shaft is made of duralumin. The magnet is glued to the glass textolite, which is bolted to the bush with three screws.

The magnet with the shield, air ring capacitor and transistors of the input cascade of the electrometer are placed in the shielded space. Time constants of discharge of the electrostatic voltmeter through magnet-earth, shield-earth insulations were measured in order to estimate the quality of insulation upon rotating the magnet. The electrostatic voltmeter C5023 with the input capacitance of 30 pF was charged to 145–150 V by a source of direct voltage. After that the electrostatic voltmeter was discharged to the ground through the rotating magnet or the shield inside that the magnet was rotating. Discharge time constant was in the range between 40 and 60 minutes, depending on air humidity and voltmeter's voltage polarity. The magnet spin up time to the maximum speed is about 3 seconds, so the influence of conduction currents in air and currents on insulator surfaces on the measurement results can be neglected. The intensity of the vortex component of the electric field was measured by a coil located in place of the air capacitor. The intensity of the vortex component was about 0.001 mV/cm, and it did not affect the measurement results. Measurements were performed by the method of double deflection [1], when the magnet is rotated alternately in opposite directions. This allows subtracting the electrical noise caused by electrification of the shield by air streams (triboelectric effect).

Results of double deflection measurements are presented in Table 1 in scale (divisions) of the electrometer. The sensitivity of the electrometer was about 76 div/mV.

Table 1.

Magnet without a shield (div.)	Magnet with an insulated shield (div.)	Magnet with an earthed shield (div.)
947 ± 21	335 ± 15	-3.4 ± 5.8

3. DISCUSSION

According to electrostatics, the scalar potential of the electric field can be represented as the solution of the Poisson equation

$$\phi = \int\limits_S \frac{\sigma}{r} dS + \int\limits_V \frac{\rho}{r} dV, \tag{2}$$

where σ, ρ — surface-charge and volume-charge densities, V — volume bounded by the surface S. For the charge sources inside and on the surface of closed shield, expression (2) can be transformed using the mean value theorem to the expression

$$\phi = \frac{\sigma(r_1)}{r_1} \int\limits_S dS + \frac{\rho(r_2)}{r_2} \int\limits_V dV = \frac{q_1}{C_1} + \frac{q_2}{C_2}, \tag{3}$$

where r_1, r_2 — coordinates of mid-points, q_1, q_2, C_1, C_2 — values of charge and capacitance of the shield and magnet, respectively.

As a number of real electric charges — electrons and ions in the insulated magnet and shield — remains unchanged upon rotation of the magnet, the electric potential also does not change. However, under a rotation of the magnet we observe an occurrence of the potential difference across the air capacitor outside the shield. This result contradicts the quadrupole model as the charge of quadrupole is zero, and field of quadrupole does not penetrate through a closed conducting envelope.

Random charges on the shield and magnet can appear in results of air friction (triboelectric effect), and their potential is independent of the direction of rotation of the magnet and can be eliminated by the double deflection method.

The Lorentz force acting on the free electric charges rotating with the magnet redistributes the charges in the conductor between the volume and the surface, forming a quadrupole [15]. Such an event as a redistribution of free charge in the conductor or displacement of the bound charge in the dielectric under action of the Lorentz force is independent of an observer and takes place in all reference frames. Therefore, if this event is observed in the lab frame, then it is to be observed in the reference system rotating with the magnet. The latter contradicts the results of Wilson's experiments [4, 5] and Misiucenko's experiment [9].

Formula for the potential difference on the rotating magnetic dielectric in the experiment of Wilsons can be derived from Minkowski equations for moving media [11]

$$D = \varepsilon E + \frac{\varepsilon}{c} [VB] - \frac{1}{c} [VH], \tag{4}$$

where $V = \omega r = 2\pi n r$ is the linear speed of rotation of the magnetic dielectric ($V \ll c$).

Since the free electric charges and currents of conductivity are zero in a magnetic dielectric, and external electric fields are not available, it can be stated that $D = 0$, $H = H_0$ [4], where H_0 is the magnetic field strength of the external stationary solenoid. Taking account of the above, it follows from Eq. (4) that the expression for the electric field in a rotating magnetic dielectric in the lab frame is

$$E = -\left(1 - \frac{1}{\varepsilon\mu}\right)\frac{[VB]}{c}, \tag{5}$$

where $B = \mu H_0$ is the magnetic field in the magnetic dielectric. Integrating Eq. (5) along the radius of the magnetic dielectric cylinder, we deduce the potential difference, which has been confirmed in experiments [4, 5].

$$U = \frac{nB\pi\left(R_2^2 - R_1^2\right)}{c}\left(1 - \frac{1}{\varepsilon\mu}\right). \tag{6}$$

where $B \cdot \pi(R_2^2 - R_1^2)/c = \Phi$ is the magnetic flux penetrating the magnetic dielectric cylinder, and R_1 and R_2 are the inner and outer radii of the cylinder.

Equation (5) can be transformed to the form

$$E = -\left(1 - \frac{1}{\varepsilon}\right)\frac{[VH_0]}{c} - 4\pi\frac{[VM]}{c}, \tag{7}$$

where $M = (\mu - 1)H_0/4\pi = B/4\pi$ is the magnetization of the dielectric. The first term in Eq. (7) is the electric field of the polarized dielectric. The second term in Eq. (7) is equivalent to Eq. (1) and the electric field of the rotating permanent magnet. If the dielectric is replaced by a conductor ($\varepsilon = \infty$), then the first term in Eq. (7) will be the field of redistributed free charges. However, H_0 is the magnetic field of the fixed solenoid. Therefore, polarization of the dielectric and redistribution of the free charge occurs only for a relative rotation of the dielectric and magnetic field source. Self-magnetization of the rotating magnetic dielectric does not cause any polarization of it (ε is not contained in the second term of Eq. (7)). Thus, the theoretical quadrupole model, based on polarization and redistribution of free electric charges in a magnetized body in self-magnetic field, are refuted by Wilson's experiment. This means that in a reference system co-moving with the rotating magnetized body, the electric field and electric polarization are equal to zero ($E' = 0$, $P' = 0$), when the external magnetic field produced by the fixed solenoid is zero.

In the experiment of Misiucenko where an air capacitor and an electrometer rotated together with the magnet, the signal amplitude decreased to about one hundred times with respect to one detected when only one magnet rotates. Consequently, the electric field of the redistributed charge was not found in the reference frame rotating with the magnet. Electromagnetic interaction of a rotating magnet and the charge depends on their relative rotation. This interaction is zero, when the charge is rotated together with the magnet.

Existent theoretical calculations of the electric field of the rotating magnetized sphere use special relativity transformations only inside the sphere and the Laplace equation $\Delta\varphi = 0$ outside the sphere [11, 12].

Application of the electric field transformation inside the sphere

$$E\left(r < R\right) = -\frac{V}{c} \times B = B \times (\omega \times r)/c \tag{8}$$

allows us to calculate the volume-charge density and to write the Poisson equation. The solution of this boundary value problem leads to the electric quadrupole field [11, 12]. However, this transformation can also be applied outside the sphere, where B is not equal to zero. Application of Eq. (8) in the whole space inside the light cylinder ($r \sin\theta \ll c/\omega$, r, θ — spherical coordinates) leads to the expression

$$divE = 4\pi\left(\frac{\left(\omega \times r\right)j}{c^2} - \frac{\omega B}{2\pi c}\right), \tag{9}$$

where $j = cM_0 \sin\theta$ — density of molecular currents on the surface of an uniformly magnetized sphere, M_0 — residual magnetization of the sphere. It follows from Eq. (9) that there is electric charge density not only at the surface and inside the sphere, where $B = 8\pi M_0/3$, but outside the sphere in vacuum, where $M_0 = 0$, $B \neq 0$. Thus, there is fictitious electric volume-charge density outside the sphere in vacuum

$$\rho = -\frac{\omega B}{2\pi c} \tag{10}$$

and outside the magnet the Poisson equation is valid, but not the Laplace equation.

The decision of this new boundary value problem leads to the scalar potential outside the sphere ($r > R$):

$$\varphi = \frac{m\omega}{cr}\sin^2\theta \tag{11}$$

This potential in Eq. (11) is similar to the Coulomb potential. Therefore, the electric field of the rotated magnet penetrates through the conductive shield surrounding the magnet.

Since the density of real electric charges in the magnet (electrons and ions) does not depend on its rotation, the expression in parentheses of Eq. (9) is the fictitious electric charge density. The real source of the observed electric field is molecular current rotating together with the magnet. The fictitious charge can be applied as a possible way to describe the detected electric field by analogy with a fictitious current successfully applied in [14] to describe and resolve the paradox of rotating charged spheres. It can be shown that the introduction of fictitious electric charges does not violate the law of conservation of charge.

4. CONCLUSIONS

In this experiment effect of the penetration of the electric field of the rotating magnet through the insulated copper shield surrounding the magnet is observed. The quadrupole electric field of a rotating magnet cannot explain the observed effect. The experiment with the shielded magnet demonstrates that there is electric field outside the shield in the lab frame. Results of the experiments with the shielded magnet can be explained by occurrence of a fictitious electric charge in accordance with transformations of the electromagnetic field in the theory of relativity.

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