

Synthesis of Dolph-Chebyshev Like Patterns from Non-Uniform, Non-Linear and Randomized Arrays

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Abstract—A pattern-synthesis approach previously described by the author is used here to develop Dolph-Chebyshev-like patterns for some unconventional array geometries. The approach is briefly summarized and then demonstrated for various non-uniformly spaced arrays and non-linear geometries. Geometries discussed include a V-shaped array, and x -aligned arrays having non-uniform element spacing as well as having elements located off the x -axis in the x - y plane. Arrays having randomly located elements are also explored. The goal is to show that patterns having equipple sidelobes can be produced by a variety of array geometries.

1. INTRODUCTION

Much attention in the electromagnetics literature has been devoted to the synthesis of linear arrays that produce a specified radiation pattern from either a continuous aperture distribution or a specified array itself, some classic examples of which are given in [1–6]. A commonly used approach involves the Schelkunoff polynomial as a representation of the desired pattern [7]. There are numerous, more-recent and widely varying approaches to pattern synthesis, some examples of which are described in [8–22]. The implementation details vary to a greater or lesser extent, but often involve manipulating the roots of Schelkunoff polynomial to achieve the desired result.

A different approach for synthesizing the radiation patterns of linear arrays [23–25] is described here, extending that approach beyond the initial non-uniform Dolph-Chebyshev arrays discussed there as well as to nonlinear arrays. It involves iteratively adjusting the maxima of the various lobes to produce a prescribed relationship among them. When used for patterns having uniform-amplitude side lobes, the source weights match those obtained from a Dolph-Chebyshev analysis. The procedure is outlined in the next section, with several different examples of its application presented in Section 3 and some concluding comments in Section 4. Various other approaches to synthesizing non-uniform, linear arrays, in some cases having equipple sidelobes, can be found in [26–31].

2. THE SYNTHESIS PROCEDURE

The pattern-synthesis approach described in [23] begins with the pattern of a linear, possibly non-symmetric, array of isotropic radiators lying on the x -axis of the coordinate system

$$P(\theta) = \sum_{n=1}^N S_n \exp^{i(kx_n \cos \theta + \beta_n)} \quad (1)$$

where x_n is the location along the x -axis of the n 'th of N isotropic radiators or elements having amplitudes S_n and relative phases β_n with θ the far-field observation angle relative to the positive array

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axis. The array length is set so as to produce N maxima over 180 degrees by nominally spacing the elements at intervals of 0.5 wavelengths for uniform arrays. The iterative process begins as follows:

1) A starting pattern $P^{(1)}(\theta)$ is computed typically using prescribed sources $S_n^{(1)} = 1$ with $\beta_n = 0$. The pattern $P^{(1)}(\theta)$ is sampled finely enough in θ to accurately locate its positive and negative maxima at the angles $\theta_n^{(1)}$, $n = 1, \dots, N$ with the corresponding pattern samples denoted by $P_n^{(1)}$. Most results that follow used a sampling interval of 0.2 degrees.

2) A matrix $[M^{(1)}]$ is then developed from the cosines of the angles where the maxima are found, since these multiply the source currents in Equation (1) to determine the pattern samples, as given by

$$[M^{(1)}] = \begin{bmatrix} \exp(ikx_1 \cos \theta_1^{(1)}) & \exp(ikx_2 \cos \theta_1^{(1)}) & \dots & \exp(ikx_N \cos \theta_1^{(1)}) \\ \exp(ikx_1 \cos \theta_2^{(1)}) & \exp(ikx_2 \cos \theta_2^{(1)}) & \dots & \exp(ikx_N \cos \theta_2^{(1)}) \\ \vdots & \vdots & \ddots & \vdots \\ \exp(ikx_1 \cos \theta_N^{(1)}) & \exp(ikx_2 \cos \theta_N^{(1)}) & \dots & \exp(ikx_N \cos \theta_N^{(1)}) \end{bmatrix} \quad (2)$$

with $M_{r,c}^{(1)} = \exp[ikx_c \cos \theta_r^{(1)}]$ being the r 'th row and c 'th column of $[M^{(1)}]$.

3) The matrix $[M^{(1)}]$ is inverted to develop an updated set of element currents $S_n^{(2)}$ using

$$\begin{bmatrix} S_1^{(2)} \\ S_2^{(2)} \\ \vdots \\ S_N^{(2)} \end{bmatrix} = [M^{(1)}]^{-1} \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_N \end{bmatrix} \quad (3)$$

where L_n are the levels of the desired maxima of the N pattern lobes found in step 1.

Were the locations in angle of the pattern maxima independent of the element currents, the synthesis procedure would end at this point. This will not usually be the case, however. Instead, the new element currents $S_n^{(2)}$ obtained from Equation (3) are used to develop a new pattern $P^{(2)}(\theta)$ and steps 1 to 3 are repeated. The matrix $[M^{(2)}]$ that results is used to compute the third set of element currents $S_n^{(3)}$. This process is continued until the pattern maxima converge to within 0.05 dB of their specified values. Note that the element currents are real for symmetric arrays and symmetric patterns but can be complex for more general geometries and patterns. All patterns presented below are shown in the X - Y plane with the array elements lying along the x -axis for linear arrays or in the X - Y plane for nonlinear arrays.

For a nonlinear array having elements in the X - Y plane the pattern is given by

$$P(\theta) = \sum_{n=1}^N S_n \exp^{ik(x_n \cos \theta + y_n \sin \theta + \beta_n)} \quad (4)$$

where x_n and y_n are the coordinates of the n 'th element. The synthesis procedure for the nonlinear array also follows the 3 steps just outlined.

The above approach is relatively straightforward. Its successful application requires that the iterated lobe maxima eventually converge to their desired values. Convergence has been consistently achieved for uniform arrays having uniform sidelobes, such as Dolph-Chebyshev patterns, and several other kinds of variations as shown in [23], usually occurring within 10 to 12 iterations.

However, convergence has sometimes failed for linear arrays with randomly assigned x -axis element locations, even after 20 or more iterations. Non-convergence has also been encountered for arrays having substantially nonzero y -axis locations, in excess of 0.2 wavelengths or so. There are at least two possible reasons for non-convergence. Perhaps most obvious is that the specified array geometry cannot produce the prescribed pattern using isotropic radiators, i.e., it is not realizable. Another reason could be that the iteration strategy employed here is inadequate as the iterations in such cases sometimes oscillates between two extremes. Nevertheless, as shown by the results below a variety of both non-uniform and nonlinear arrays that produce equiripple patterns can be successfully synthesized.

3. NUMERICAL RESULTS

The patterns of Figure 1, included as a reference for the other examples to follow, are for a 9-element array 4 wavelengths long whose side lobes are specified to be 10 to 60 dB below the main lobe in 10 dB steps. As noted in [23] for symmetric arrays, the element coefficients are real with their imaginary components, on the order of 10^{-6} , being negligible. The number of iterations required for sidelobe levels to converge to within 0.05 dB of the specified value varied from 3 to a maximum of 10 as the sidelobe level is decreased. As a standard of comparison for the arrays considered in the following, a sidelobe level of -20 dB is included for all geometries considered as well as a range of other sidelobe levels for some.

Patterns of 9, 13 and 17-element non-uniform arrays whose geometries are illustrated in Figure 2(a) are presented in Figure 2(b) for -20 dB sidelobes. The array lengths are 3.6, 5.6 and 7.6 wavelengths

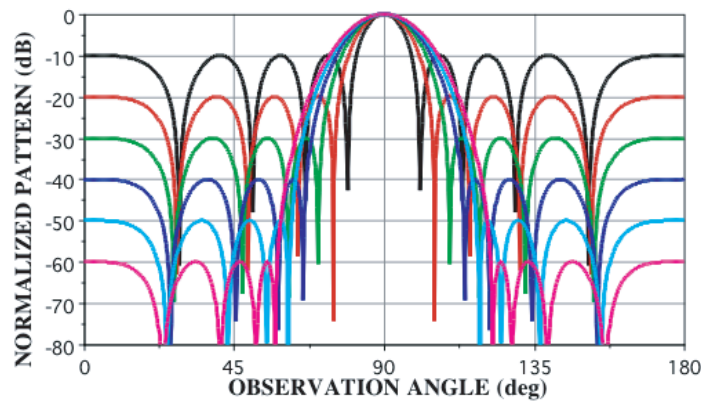
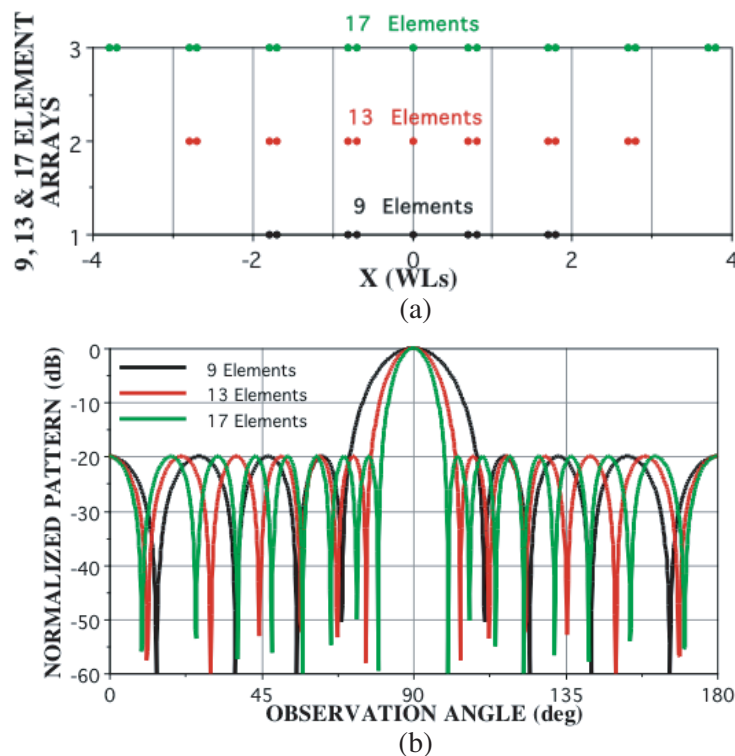


Figure 1. The synthesized pattern for a 9-element array 4 wavelengths long with specified side-lobe levels of -10 to -60 dB obtained using the method described above [23].



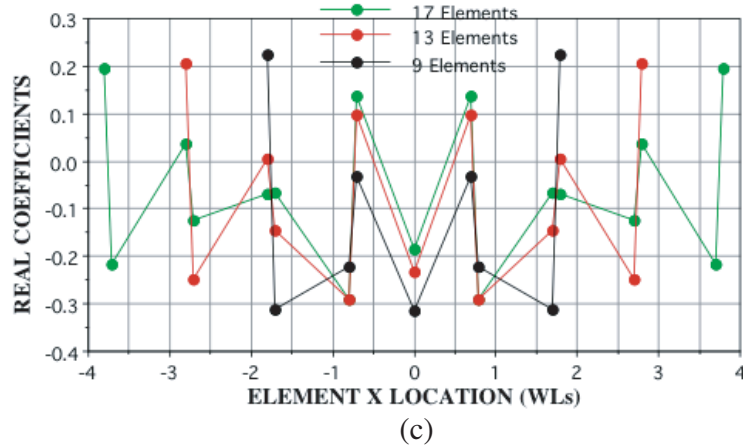


Figure 2. (a) The locations for 9, 13 and 17 element non-uniform arrays, (b) their -20 dB patterns and (c) their corresponding excitations.

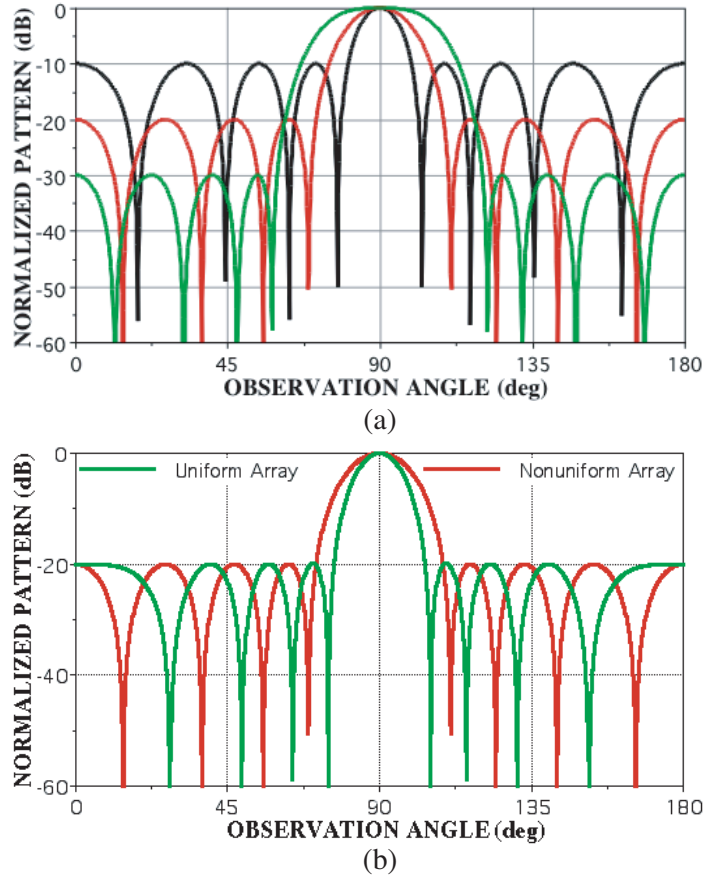


Figure 3. (a) Patterns for the 3.6-wavelength, 9-element non-uniform array of Figure 2(a) having -10 , -20 and -30 dB sidelobes and (b) a uniform Dolph-Chebyshev 4-wavelength, 9-element array compared with the 3.6 non-uniform array both with -20 dB sidelobes.

with off-center, 2-element pairs separated by 0.1 wavelength. The real element excitations included in Figure 2(c) reveal that the excitations for the two end elements are largest and of opposite sign, indicating the possibility of a super-gain phenomenon. These results do demonstrate that even extremely

non-uniform arrays can produce Dolph-Chebyshev-like, equiripple patterns.

The -10 , -20 , and -30 dB patterns of the 9-element array of Figure 2(a) are plotted in Figure 3(a), and the -20 dB patterns of a 9-element uniform Dolph-Chebyshev and non-uniform arrays are compared

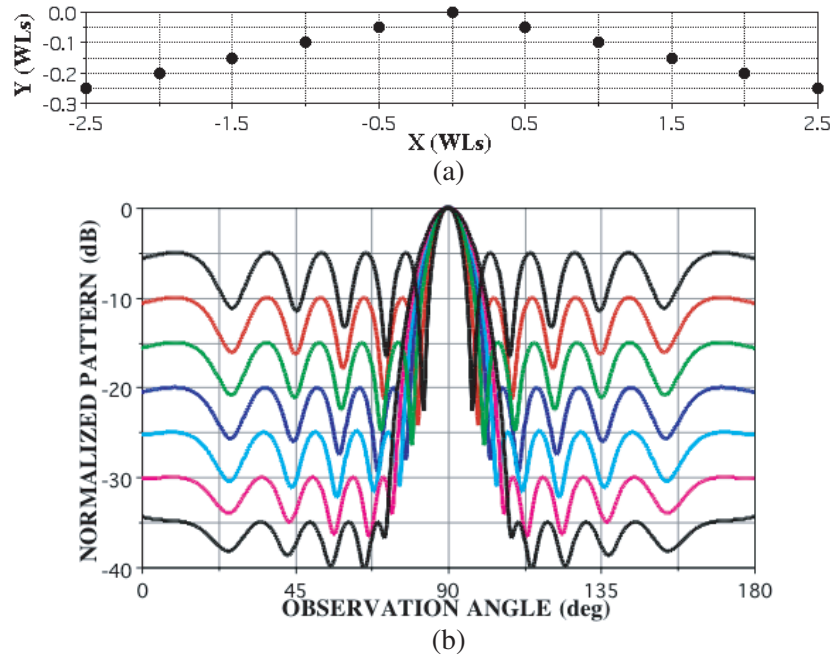


Figure 4. (a) Geometry of a 5-wavelength, 11-element Vee array and (b) its patterns having sidelobes varying from -5 to -35 dB in 5 dB steps.

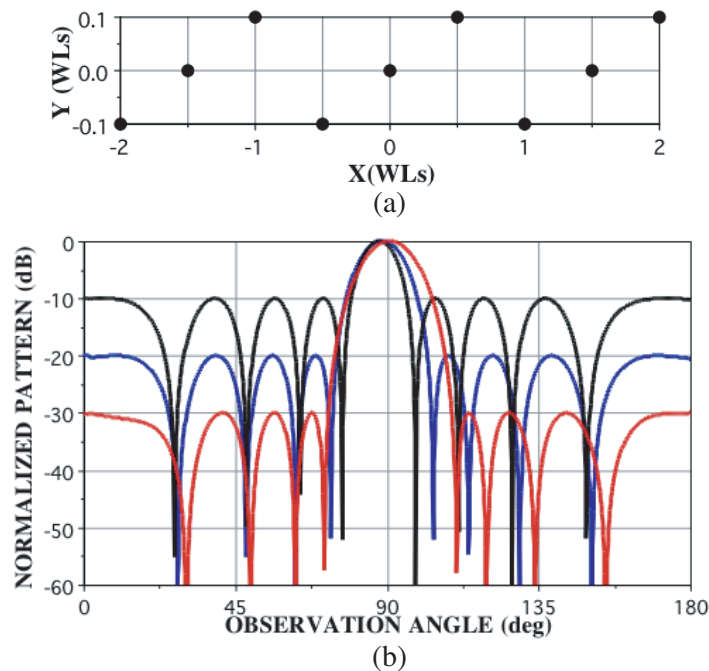


Figure 5. (a) Geometry of a 4-wavelength, 9-element array having elements systematically offset at ± 0.1 wavelengths from the array axis and (b) its patterns having -10 to -30 dB sidelobes.

in Figure 3(b). The non-uniform array exhibits a broader mainlobe than the uniform array but it retains the same number of sidelobes.

An eleven-element, 5-wavelength Vee array whose geometry is displayed in Figure 4(a) produces the series of patterns plotted in Figure 4(b) where the sidelobe levels vary from -5 to -35 dB. Comparison of Figures 1 and 4(b) reveals that the mainlobes of the Vee array are somewhat narrower than those for the 4-wavelength linear uniform array. This is evidently due to its greater length rather than its nonlinear geometry.

A nonlinear 9-element, 4-wavelength array having the geometry plotted in Figure 5(a) produces the -10 to -30 dB patterns of Figure 5(b). Not all of pattern main lobes point to broadside, with the difference from broadside decreasing with as the sidelobe depth decreases. It's not clear but seems reasonable to ascribe this result as being due to the nonsymmetrical geometry of this particular array.

The symmetric, nonlinear array shown in Figure 6(a) produces the symmetric -20 dB equiripple pattern of Figure 6(b). Similarly to the case of the Vee-array patterns of Figure 4, the depths of the sidelobe minima for this nonlinear array are less than 10 dB. This is much less than the 10s of dB minima exhibited by the sidelobes of the linear Dolph-Chebyshev array of Figure 1.

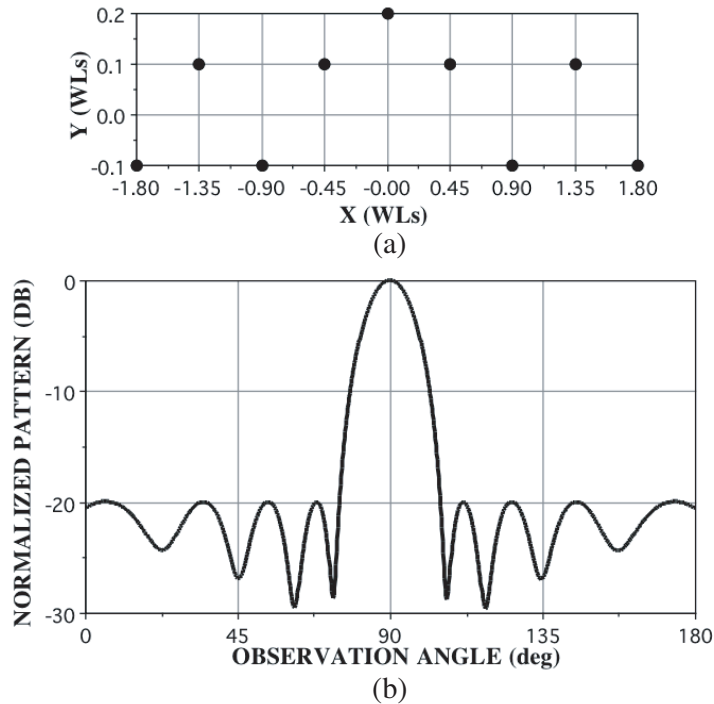


Figure 6. (a) Geometry of a 3.6-wavelength, 9-element array having its side elements displaced from the array axis at ± 0.1 wavelengths and (b) its center element at $+0.2$ wavelengths and its -20 dB pattern.

Patterns for 2 symmetric, nonlinear arrays having element locations alternating in their Y -axis locations between ± 0.05 and ± 0.1 wavelengths are shown in Figures 7(b) and 7(c) respectively, with the array geometry illustrated in Figure 7(a) for the ± 0.1 case. The depth of the sidelobe minima is generally smaller for the ± 0.1 -wavelength case compared with that for ± 0.05 wavelengths.

The patterns produced by offsetting varying numbers of array elements $+0.1$ wavelengths in the Y direction as demonstrated in Figure 8(a) for elements 3 to 7, are plotted in Figure 8(b). The major effect on the patterns is a slight variation in their minima as the number of offset elements changes. The results of Figure 8(c) demonstrate their respective patterns are numerically identical for elements 3 to 7 shifted to $+0.1$ or -0.1 wavelengths.

The non-uniform array of Figure 9(a) whose elements are offset by ± 0.05 wavelengths along the Y -axis results in the radiation pattern of Figure 9(b) where the pattern of a uniform Dolph-Chebyshev

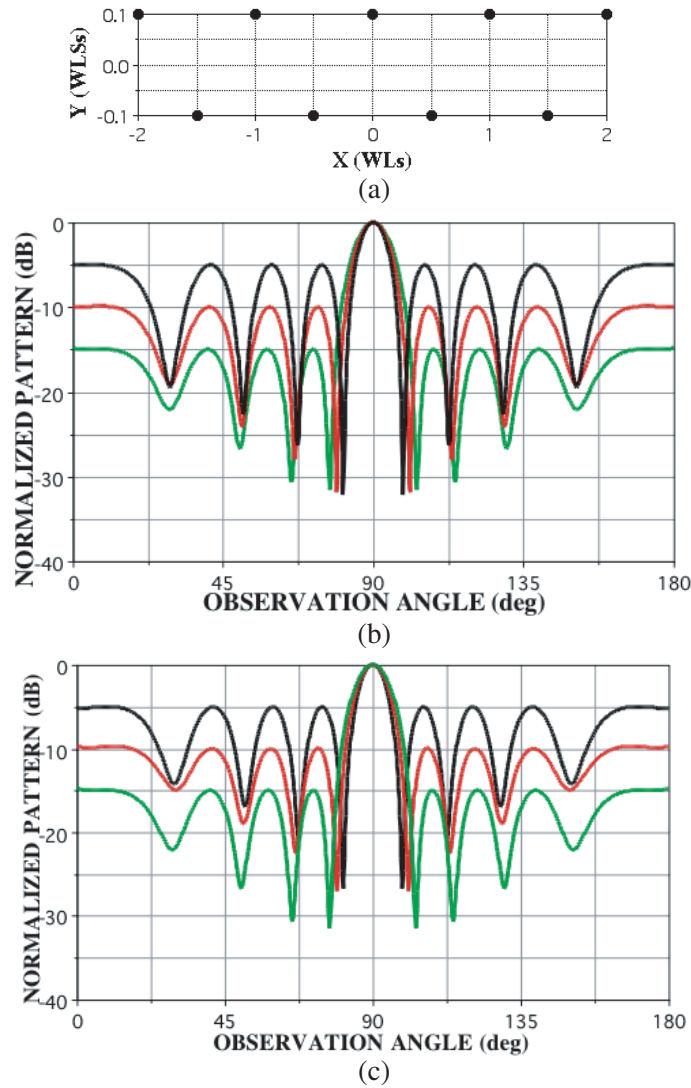
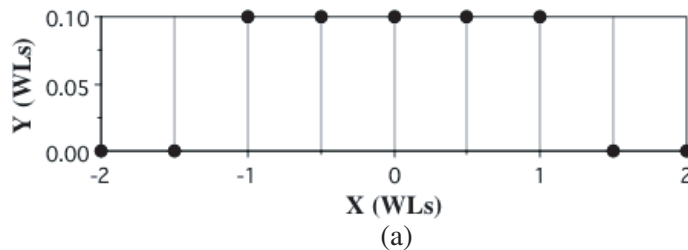


Figure 7. (a) Geometry of a 4-wavelength array having its 9 elements alternately displaced or offset from the x -axis and (b) its patterns for $y = \pm 0.05$ and (c) $y = \pm 0.1$ wavelengths.

array is included for comparison. The offset-array pattern has a mainlobe about twice that of the Dolph-Chebyshev array and has only 3 sidelobes. The real and imaginary coefficients or element strengths of this array is shown in Figure 9(c).

The radiation patterns of a 9-element, 3.62 wavelength array with random element spacings as displayed in Figure 10(a) are plotted in Figure 10(b) for sidelobe levels varying from -10 to -40 dB



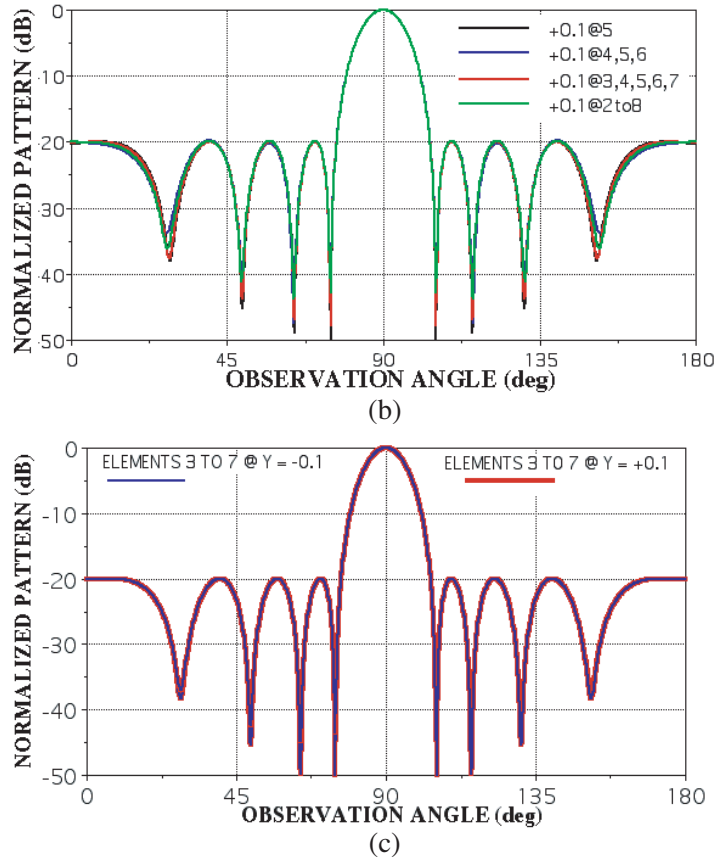
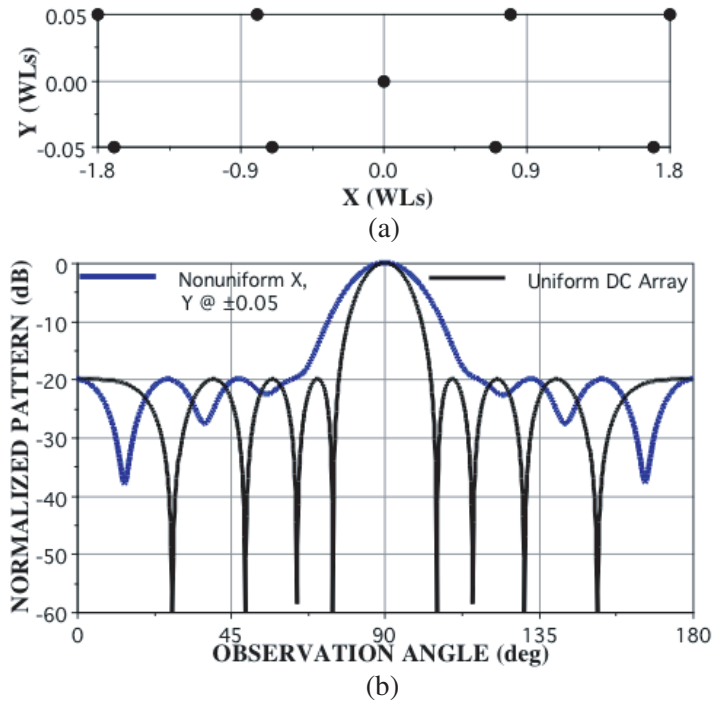


Figure 8. (a) Geometry of 9-element, 4-wavelength arrays having various elements displaced from the x -axis, and (b) patterns for elements 5, 4 to 6, 3 to 7, and 2 to 8 at $Y = +0.1$ wavelengths, and (c) patterns for elements 3 to 7 at $y = +0.1$ and -0.1 .



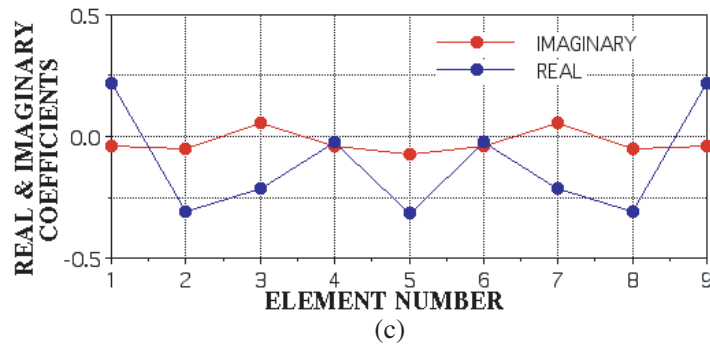


Figure 9. (a) Geometry of a non-uniform, 9-element, 3.6-wavelength having off-center elements at $Y = \pm 0.05$ wavelengths, patterns of the offset array and (b) a uniform Dolph-Chebyshev array and the real and (c) imaginary element excitation coefficients of the offset array.

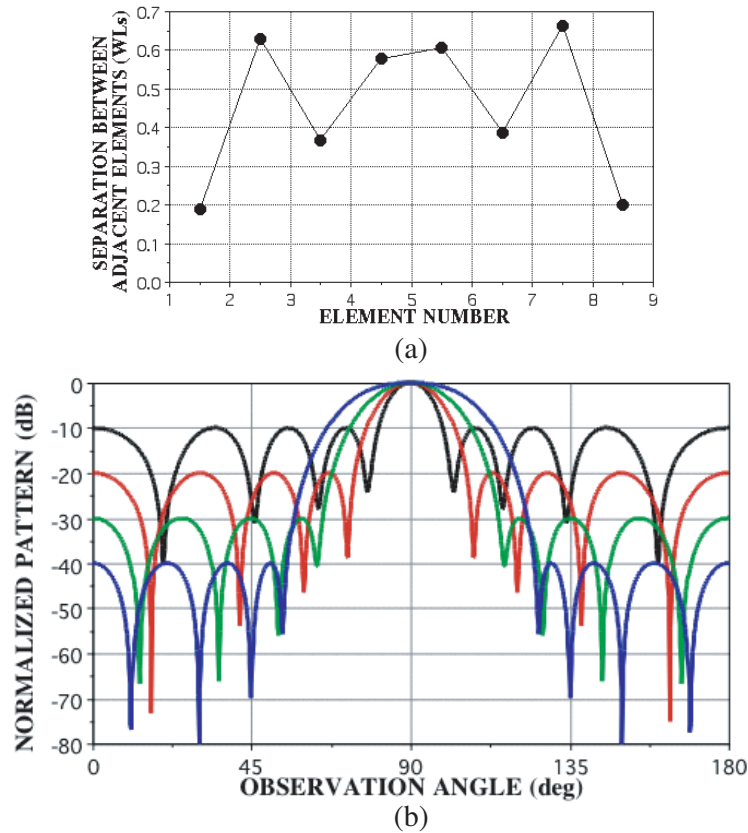


Figure 10. (a) Geometry of a 9-element, 3.62 wavelength, non-uniform array having random element separations, and (b) their corresponding patterns with sidelobe levels varying from -10 to -40 dB in 10 dB steps.

in 10 dB steps. These patterns for the randomized array are similar to those for the non-uniform 3.6-wavelength array of Figure 3(a) even though their element spacings are quite different.

The element separations for 3, 9-element, randomized arrays of varying widths in Figure 11(a), one of which is that of Figure 10(a), are compared in Figure 11(b) with a uniform 9-element Dolph-Chebyshev array 4 wavelengths long for the case of -20 dB sidelobes. All arrays have similar mainlobes in spite of their widths varying from 3.62 to 4.065 wavelengths, but they do exhibit noticeable variations

in their sidelobe structure.

Four arrays having the randomized Y locations shown in Figure 12(a) produce the -20 dB sidelobe patterns of Figure 12(b). These various patterns are seen to have similar mainlobes but do vary in the depths of their various sidelobes. In these respects these patterns exhibit similarities with the randomized x -location patterns of Figure 11,

The result of randomizing both the X and Y element locations is shown for two arrays in Figure 13.

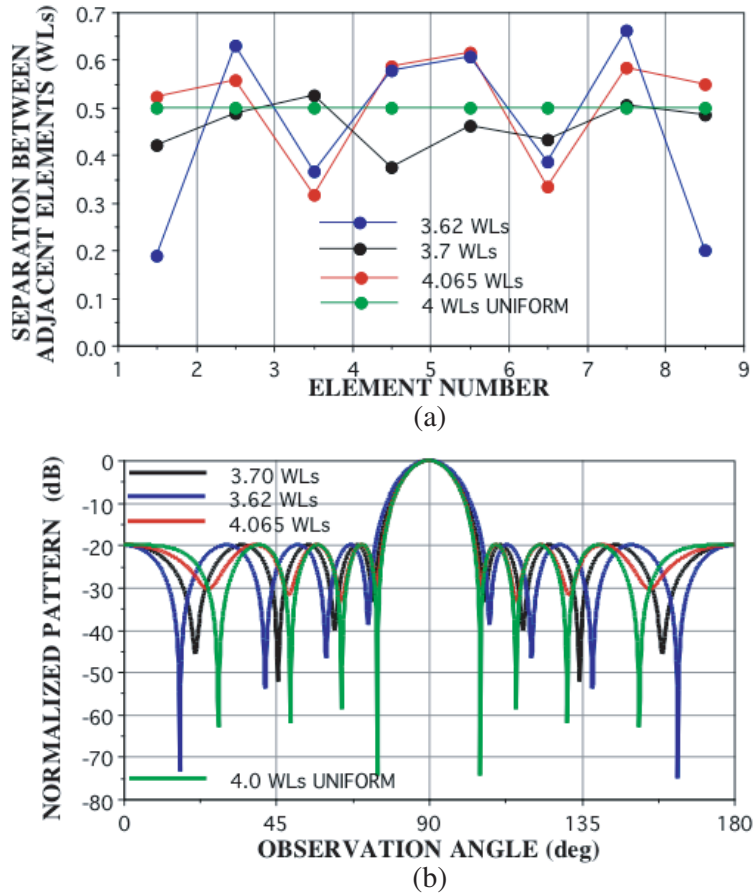
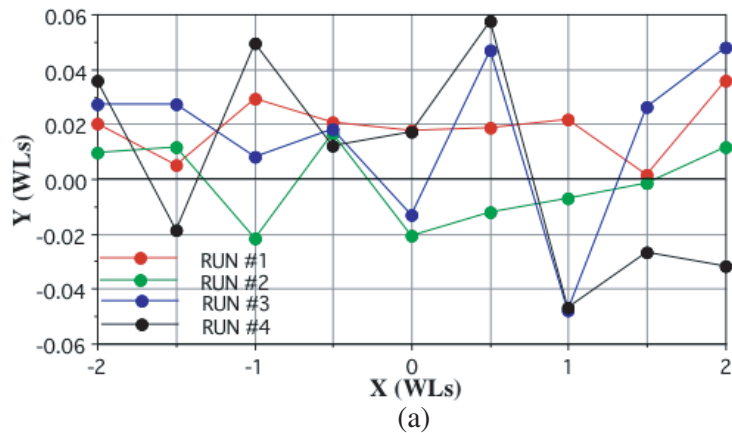


Figure 11. (a) Element separations for 3 randomized arrays and a uniform Dolphy-Chebyshev array and (b) their -20 dB patterns.



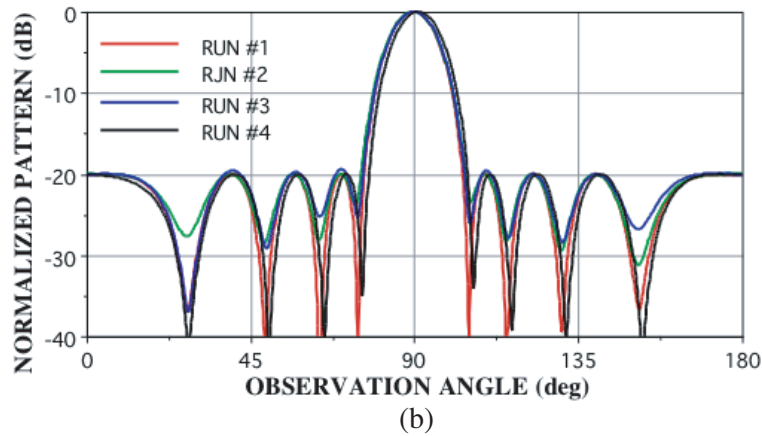


Figure 12. (a) The 4 randomized 4-wavelength arrays produce (b) the various -20 dB patterns.

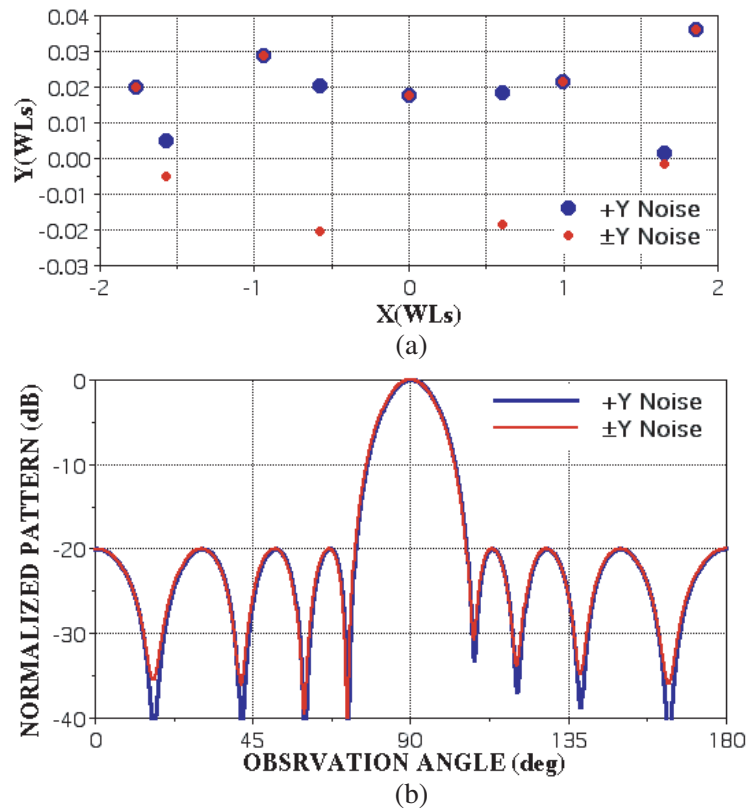


Figure 13. (a) The two 9-element, 3.6 wavelength arrays having $+$ and \pm randomized offsets produce (b) nearly identical -20 dB patterns.

For one the Y -location offsets from a linear array are all in the $+Y$ direction while for the second the same offsets alternate between $+$ and $-$ as illustrated in Figure 13(a). The resulting patterns are graphically indistinguishable except for the depths of their sidelobes. This particular result suggests that equiripple patterns can be produced for fairly general non-uniform and nonlinear array geometries.

Two last examples of noise effects on equiripple patterns are presented in Figure 14 for 9-element arrays as one illustration of how sensitive a Dolph-Chebyshev array is to errors in its geometry and excitation. The effect of randomizing a nominal 4-wavelength array geometry is presented in Figure 14(a)

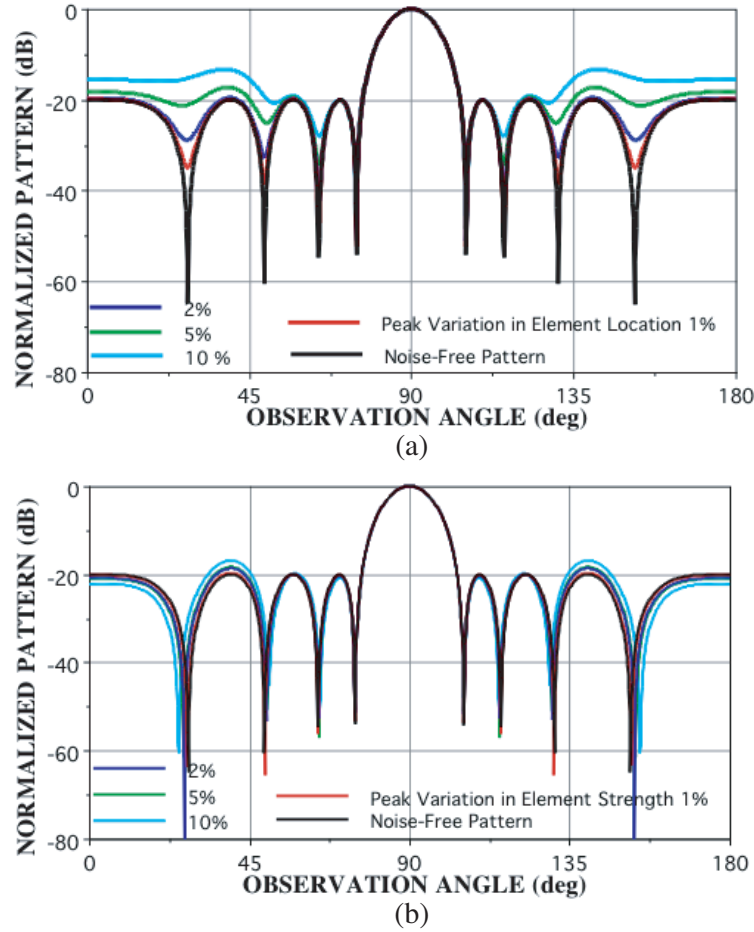


Figure 14. (a) Patterns of 9-element arrays having various peak variations in their nominal 4-wavelength wide and (b) similar variations in their excitations.

and the effect of randomizing element excitations is presented in Figure 14(b). Noise-free patterns are included as a reference along with the effects of randomizing locations and excitations over peak variations ranging from 1% to 10%.

The patterns of Figure 14 demonstrate that larger distortions of the nominal pattern are due to errors in element location than they are to their excitation. But the previous results have shown that it is possible to overcome such errors during the synthesis procedure to nevertheless produce a desired pattern.

4. CONCLUDING COMMENTS

An approach [23] for linear-array pattern synthesis that differs from earlier methods such as those that incorporate Schelkunoff polynomial has been summarized. It involves finding the angles of the pattern-lobe maxima produced by an initial set of arbitrary element currents for a prescribed array. A second set of element currents is then computed from a linear system of equations whose coefficients incorporate these initial angles and whose right-hand side is comprised of the lobe maxima desired in the synthesized pattern. A second pattern is computed whose lobe maxima are then compared with the desired values. If the desired and computed lobe maxima do not agree within a specified limit, a third set of element currents is computed using the updated lobe locations from the second pattern to obtain a third pattern. This iterative process is terminated when the most-recent lobe maxima agree to the specified degree of accuracy, typically 0.05 dB or better. Convergence to the desired pattern usually

occurs on the order of 12 iterations or fewer.

The original applications of this synthesis approach has been extended here to a variety of array geometries. The goal was to demonstrate the feasibility of synthesizing systematic and/or randomized, non-uniform linear arrays as well as nonlinear arrays to produce equipple, or Dolph-Chebyshev-like, patterns. This possibility has been achieved for a variety of such arrays. The results obtained show that equipple patterns might be realized in situations where uniform arrays are impractical.

Further work is needed to establish whatever limits might be associated with these kinds of geometries. Whether the convergence of such arrays sometimes fails is due to the synthesis procedure itself or fundamental limitations of such geometries needs further study. It would be useful to determine the self-impedance and mutual interactions of the array elements using a computer model such as NEC (Numerical Electromagnetics Code) [32]. Evaluating possible cross polarization produced by these more general geometries [33] should also be investigated. These various issues will be addressed in a following article.

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