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Engineering Laser-Based Diagnostic in a Hot Wind Tunnel Jet: Measurement of the Temperature Structure Coefficient by Using an Optimization Technique

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Abstract—This paper is devoted to an engineering laser-based diagnostic technique which is able to extract the value of the temperature structure coefficient in a hot turbulent wind tunnel jet, by using a thin laser beam which is sent into the jet. Some experimental investigations are carried out to characterize the jet under study and the probabilities of the positions of the laser beam impact on a photocell are measured. The theoretical values of the same probabilities are computed by assuming that the laser beam direction is a Markov random process. By means of an optimization technique with constraints, based on the Golden Section algorithm, the temperature structure coefficient of the jet is determined. The validity of the result obtained is proved by a good agreement which is observed in the comparison between another parameter computed from that result and the previously published data.

1. INTRODUCTION

Because of their various applications in industry and in environment, turbulent flows continue to be of great interest and are increasingly studied. The fundamental study of turbulence [1] shows the great complexity of turbulent flows. Turbulence is a longstanding problem for which there is no analytical or numerical solution, except for few cases in which models are required [2] to be introduced in theoretical calculations or in numerical simulations for completed results. In many cases, experimentation is needed to confirm the results [2].

This paper is devoted to the measurement of a key parameter called temperature structure coefficient and denoted as C_T^2. The knowledge of C_T^2 enables the determination of the thermal turbulence intensity in any heated turbulent medium such as flames, hot jets, combustion chambers of engines, atmospheric boundary layers, and industrial boundary layers. To measure that coefficient, two types of experimental techniques are usually applied: the first type contains techniques for which probes are required to be placed inside turbulent flows [3]. The measurement technique presented in this paper belongs to the second type of techniques called diagnostic techniques or noninvasive techniques because no measuring sensor is introduced into the flow studied [2]. So, a thin laser beam is sent into the hot jet under study, perpendicularly to the flow direction. The temperature structure coefficient is measured from the examination of the luminous trace produced by that laser beam on a photocell placed outside the jet.

Experimental investigations carried out in the jet demonstrate that the laser beam path considered behaves as a direction of homogeneity for C_T^2. Also, this work represents a necessary preliminary step for a subsequent research in which the result obtained in this paper and the measurement technique applied in it will be extended to determine variable C_T^2 parameters. To validate the measurement technique, it

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is proved that another parameter, called structure coefficient of refractive index C_n^2 and computed from the result obtained, is close to the previously published data [4].

2. THEORETICAL FOUNDATIONS

For any heated turbulent medium, parameter C_T^2 is defined by Tatarskii [5] as:

$$C_T^2 = \alpha^2 (\bar{\sigma})^{-1/3} \bar{Q} \tag{1}$$

where α^2 is a positive dimensionless proportionality constant, $\bar{\sigma}$ the viscous dissipation, and $\bar{Q}$ the amount of temperature inhomogeneities disappeared per unit time because of the molecular diffusion in the medium. C_T^2 has a great importance in the study of any thermal turbulence. First, C_T^2 is needed to evaluate the well-known temperature structure function S_t [5] defined from the temperature fluctuations t by the following relation:

$$S_t = \overline{(t(\mathbf{x}+\mathbf{r}) - t(\mathbf{x}))^2} \tag{2}$$

for any positions vectors $(\mathbf{x})$ and $(\mathbf{x}+\mathbf{r})$. Second, C_T^2 is required for the determination of the Karman model of the temperature turbulence spectrum, which is well known to be the more realistic and complete model, and is written as [6]:

$$\psi_t(K) = 0.033 C_T^2 \left(K^2 + K_0^2\right)^{-11/6} \exp\left(-K^2/K_m^2\right) \tag{3}$$

where K_0 and K_m are the lower and upper limits of the inertial zone of turbulence defined as: $K_0 = 1/L_0$ and $K_m = 5.92/L_i$, L_0 and L_i being the outer and inner scales of turbulence.

For any heated medium in which the unique cause of the refractive index fluctuations is the temperature fluctuations, C_T^2 plays an important role and is connected [4–7] to the parameters C_n^2 and D_n characterizing the optical turbulence and respectively called the structure coefficient of the refractive index [5] and the diffusion coefficient of the jet [7]:

$$C_T^2 = \left(\frac{T_{\text{mean}}^2}{aP_0}\right)^2 C_n^2 \tag{4}$$

$$C_T^2 = A^2 \left(\frac{T_{\text{mean}}^2}{aP_0}\right)^2 K_m^{-1/3} D_n \tag{5}$$

The above relations hold if the air of the jet is considered as a perfect gas. $A^2 = 1.641$ is a positive constant whose value can be found in our previous works [4]; T_{mean} represents the mean temperature in the medium; $a = 79 \times 10^{-6}\ \mathrm{K \cdot mb^{-1}}$ is the Dale-Gladstone constant corresponding to the incident wavelength ($\eta = 6328\,\text{Å}$) of the laser beam radiation; P_0 is the mean pressure in the jet air assumed to be equal to the atmospheric pressure.

As suggested by Chernov [7], we assume that the direction of the laser beam is a Markov process in which the length path of the laser beam plays the role of time. So, the probability $P(\theta, \phi, z)$ for the laser beam, to have the direction (θ, ϕ) (θ, ϕ are azimuthal and polar angles) after having traversed a distance z is given by the following equation [7]:

$$\left(\frac{1}{\lambda K_m^{1/3}}\right) \frac{\partial P}{\partial z} = \frac{C_T^2}{\sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial P}{\partial \theta}\right) + \frac{C_T^2}{\sin^2\theta} \frac{\partial^2 P}{\partial \phi^2} \tag{6}$$

where λ is a constant defined as: $\lambda = \left(aP_0/T_{\text{mean}}^2\right)^2 \big/ A^2$. In the contribution by Chernov [7], the above equation is expressed in terms of D_n. It can be written in terms of C_T^2 by means of formula (5) found in our previous works [4].

3. MEASUREMENTS

3.1. Measurement of the Dynamical and Thermal Properties of the Jet

The hot turbulent jet of air is issued from a rectangular nozzle aperture (200 mm × 5 mm) of a wind tunnel shown in Fig. 1. Three perpendicular Cartesian axes (x, y, z) are defined on the nozzle aperture,

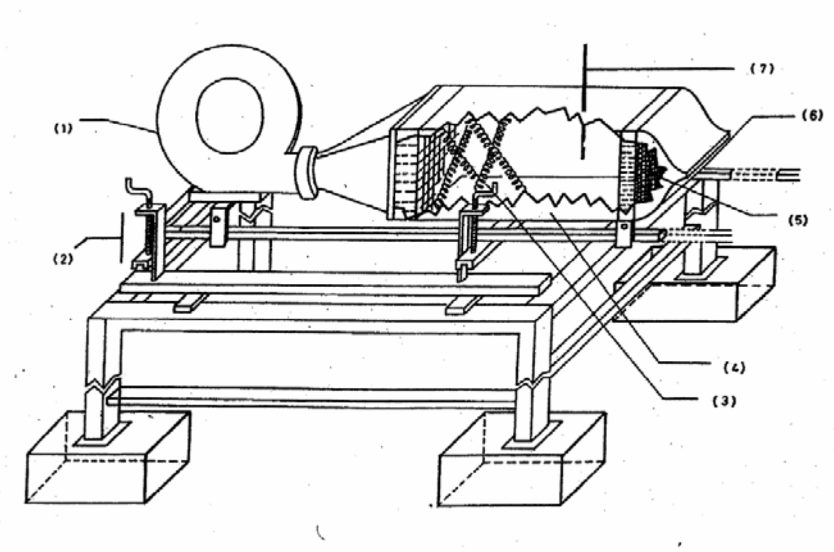

Figure 1. Wind tunnel: (1) Ventilating fan; (2) Vertical displacement; (3) Heating resistances; (4) Box for flow homogeneity; (5) Filter against turbulence; (6) Nozzle; (7) Thermocouple.

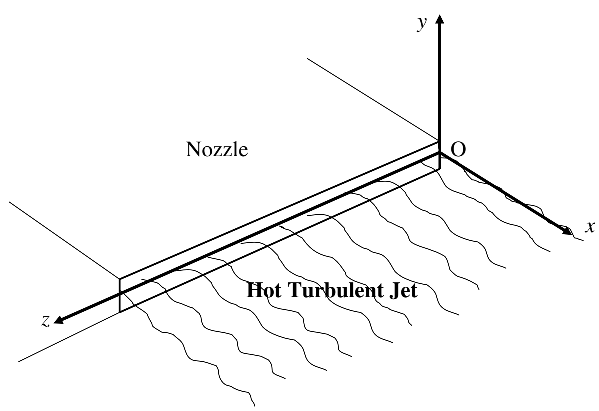

Figure 2. Cartesian coordinates defined on the nozzle aperture of the wind tunnel.

as shown in Fig. 2. To obtain stable experimental results, all measurements are done when the jet is fully developed. By means of a thermocouple, measurements of the mean temperature are performed. The rms of the temperature fluctuations are measured by applying the cold-wire anemometer technique [2, 3] in which we have used a wire whose diameter and length are respectively equal to 1 μm and 0.4 mm, with a current intensity $I_0 = 0.16\,\text{mA}$. To measure the mean velocity U, we use a moving graduated drum which rotates around a vertical axis and causes the vertical displacement of a cursor situated on a graduated ruler, such that 1 tower of the drum corresponds to 1 mm on the ruler. The drum helps to bring back a meniscus in order to measure the difference in level h between its position, which depends on U, and the initial level. That meniscus is illuminated, and its displacement can be exactly detected by using a sight objective. By means of a Pitot's tube, the mean velocity is then measured from the following relation [2]:

$$U = (2(P_d - P_0)/\rho)^{1/2} = 4\sqrt{h} \tag{7}$$

where ρ is the air specific mass, and P_d and P_0 are the dynamic and static pressure, respectively.

Since the direction of propagation of the unperturbed laser beam is along the z-axis, it is necessary to describe the jet behaviour by measuring the dynamical and thermal properties of the jet along that direction. So, along the jet straight line ($x = 300\,\text{mm}$, $y = 0$, z variable) parallel to the z

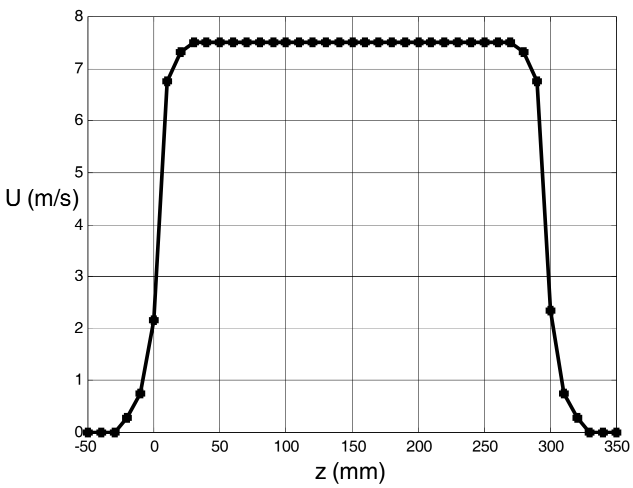

Figure 3. Mean velocity as function of z along the z axis, at the distance $x = 300\,\text{mm}$ from the plane of the nozzle aperture, that is along the mean path of the laser beam.

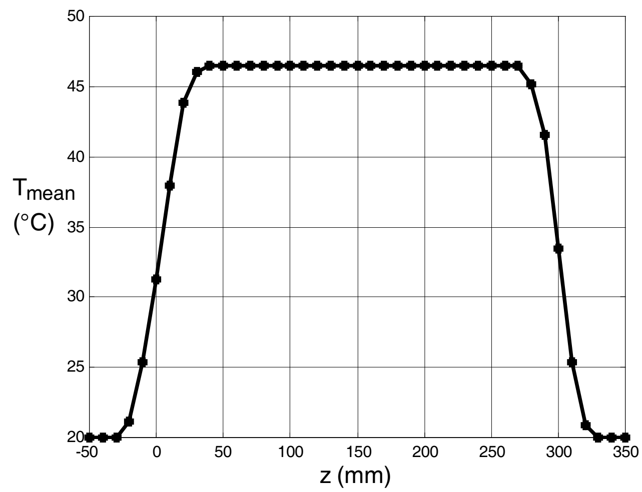

Figure 4. Mean temperature as function of z at the distance $x = 300\,\text{mm}$ from the plane of the nozzle aperture, that is, along the mean path of the laser beam.

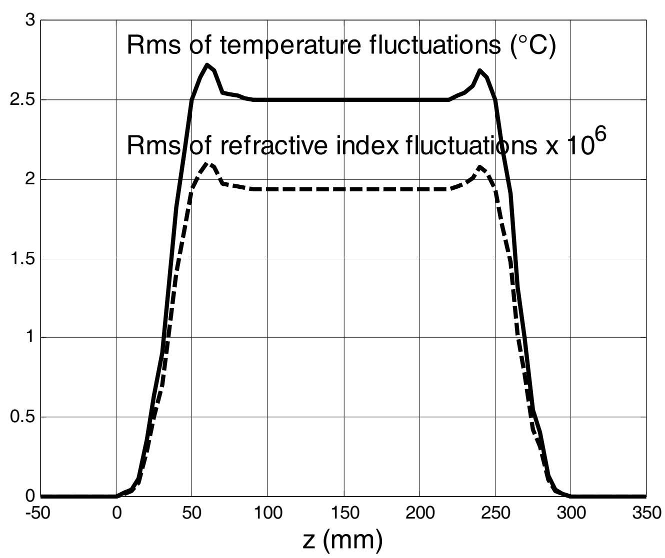

Figure 5. Rms of temperature fluctuation and as function of z along the z axis, at the distance $x = 300\,\text{mm}$ from the plane of the nozzle aperture, that is along the mean path of the laser beam.

axis, measurements of mean velocity, mean temperature, rms of temperature fluctuations, and rms of refractive index fluctuations are carried out. The measured values plotted as function of z in Figs. 3, 4, 5 remain constant and are respectively equal to $7.5\,\text{m/sec}$, $46.5°\text{C}$, $2.50°\text{C}$, and 2.0×10^{-6}, except in a short area at both borders of the jet. This demonstrates that the jet under study is plane; in addition, the mean-velocity curve shows that the jet width is $Z \approx 300\,\text{mm}$, at the distance $x = 300\,\text{mm}$, along the z axis.

3.2. Measurement of the Probabilities of the Positions of the Laser Beam Impact on a Photocell Placed outside the Jet

3.2.1. Experimental Setup

The experimental setup is shown schematically in Fig. 6. A red light beam (wavelength = 6328 Å), having initial diameter $e = 0.8\,\text{mm}$, is created from a 1 mW He-Ne laser. The laser beam is passed through the path ($x = 300\,\text{mm}$, $y = 0$, z variable) already considered in Section 3.1. After having traversed the jet, the laser beam reaches a photoelectric cell placed outside the jet on the plane (x, y), at a distance $D = 500\,\text{mm}$ from the outlet jet border. To keep the position of the laser source unchanged when seeking the desired incident direction of the laser beam, we need to use a plane mirror which is placed between the turbulent jet and the laser source. The electrical signal transmitted by the photocell is displayed by a storage oscilloscope connected to an interface which provides information to a computer for statistical calculations. The photocell transmits two voltages whose amplitudes are proportional to the coordinates (x, y) of the laser beam impact position on the photocell plane. Since these amplitudes are very weak, it is necessary to use an amplifier which gives information to the interface. The refractive index fluctuations created at least by the thermal fluctuations in the jet cause directional fluctuations of the laser beam, amplitude fluctuations of the laser wave, and phase fluctuations of the laser wave front which induce intensity fluctuations of light wave by diffraction process. The laser beam then produces a luminous trace on the photocell.

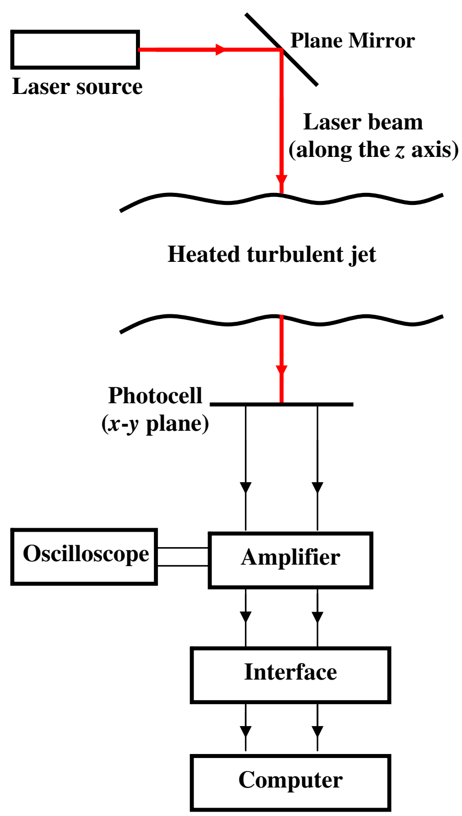

Figure 6. Experimental set-up with all devices used.

3.2.2. Preliminary Investigations Before Measuring the Probabilities

From the physical point of view, the concept of probability of the direction of the laser beam defined in a turbulent jet has no sense if the laser beam undergoes large diffraction effects due to turbulence. So, it is mandatory to verify the applicability of the geometrical optics approximation in the case of the problem studied. We then need to evaluate the importance of the diffraction effects undergone by the laser beam in the jet. For that, we have measured the diameter of the laser beam footprint on the photocell plane, and the value we have obtained is 0.85 mm, which corresponds to an increasing equals to 6.25% of the initial diameter of the laser beam. This leads to a conclusion that the laser beam studied remains sufficiently narrow such that diffraction effects may be negligible compared to refraction effects, along the whole laser beam path, i.e., the geometrical optics approximation may be applied. This occurs if the following conditions are met [2, 4].

- The incident wavelength η of the unperturbed laser beam radiation must be very small, compared to the inner scale L_i of the turbulent inhomogeneities in the hot jet ($L_i = 1\,\text{mm}$ [2]).
- The whole path distance Z traversed by the laser beam must be very great, compared to the outer scale L_0 of the turbulent inhomogeneities ($L_0 = 10\,\text{mm}$ [8]).
- The size of the first Fresnel zone $\sqrt{\eta Z}$ must be smaller than the inner scale L_i.
- The laser beam intensity fluctuations are neglected.

Under the above conditions, the applicability of the geometrical optics approximation is allowed. So, the random propagation of the laser beam may be approximated as a random walk process in which the laser beam is regarded as a laser ray, as in Refs. [9] and [10].

The room medium is at rest before experiments, and we observe that the laser impact on the photocell remains nearly unchanged before heating the jet. Hence, the effects of pressure fluctuations may be negligible in the jet, and the temperature fluctuations are the unique cause of the refractive index fluctuations, as expected in theory predictions explained in Section 2.

3.2.3. Method of Measurement of the Probabilities

To measure the probabilities of the positions of the laser beam impacts on the photocell, an initial measuring square is built in the photocell plane after some samples, and this square is the zone defined as: $[-0.40\,\text{cm}, 0.40\,\text{cm}] \times [-0.40\,\text{cm}, 0.40\,\text{cm}]$. After having eliminated the points which do not correspond to any impact of the laser beam on the photocell, we suitably reduce the surface of this initial square, and the final measuring area needed for the measurements of the probabilities of the laser beam impact positions is the square $[-0.20\,\text{cm}, 0.20\,\text{cm}] \times [-0.20\,\text{cm}, 0.20\,\text{cm}]$. Hence, the diameter (0.85 mm) of the laser beam footprint represents 21.25% of the size of the final measuring square, and the footprint surface represents 3.5% of the surface of this square. The final measuring square is cross-ruled in 1600

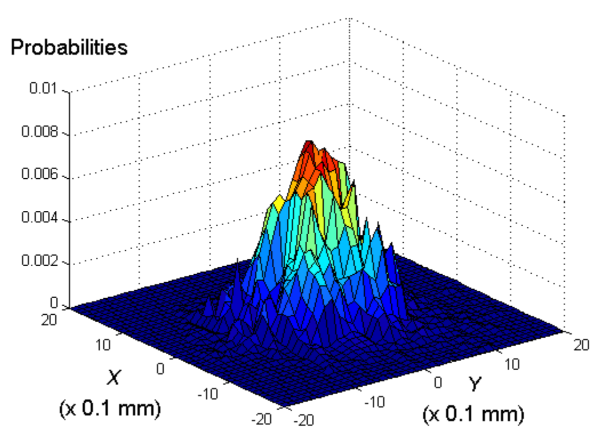

Figure 7. The measured experimental probabilities of the position of the laser beam impacts on the photocell plane.

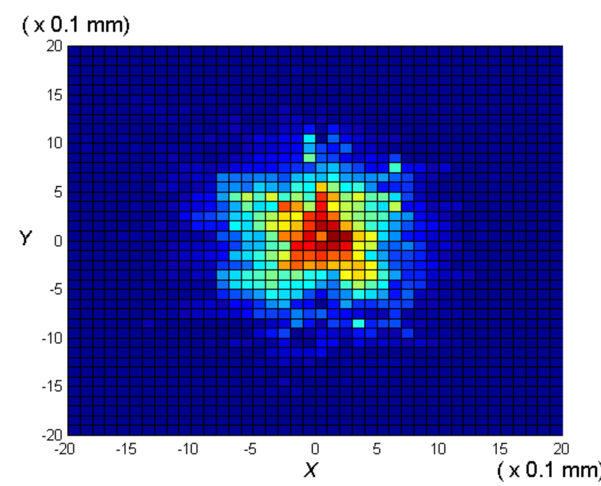

Figure 8. The experimental luminous trace produced by the laser beam in the photocell cell plane.

small squares of the same size $c = 0.01$ cm, defined by the following discretized points:

$$x(l) = -0.20 + l \cdot c, \quad l = 0, 1, \ldots, l_{\max}, \quad (l_{\max} = 40) \tag{8a}$$

$$y(m) = -0.20 + m \cdot c, \quad m = 0, 1, \ldots, m_{\max}, \quad (m_{\max} = 40) \tag{8b}$$

The quantity $Q(l, m)$ denotes the probability for the centre of the laser beam impact to be situated within the square $[x(l), x(l+1)[\times[y(m), y(m+1)[$. The interface is able to store $2 \times (2^5 \times 2^5) = 2048$ impacts of the laser beam in approximately 20 seconds. The measured values $Q(l, m)$ are plotted in Fig. 7, and the corresponding luminous trace produced by the laser beam on the photocell is presented in Fig. 8. One observes a regular image of the luminous trace which shows that probabilities are maximal in the central zone of the final measuring square and are nearly equal to zero at the boundaries of that square.

4. NUMERICAL STRATEGY TO COMPUTE THE SAME PROBABILITIES

The computed values of the same probabilities are obtained by solving Eq. (6). The z distances are approximated as a set of discretized values numbered by the integer n as:

$$z \approx z(n) = z_n \tag{9a}$$

$$z' \approx z(n+1) = z_{n+1} \tag{9b}$$

$$z(0) = 0 \text{ and } z(n+1) - z(n) = \Delta z, \text{ for } n = 0, 1, \ldots, n_{\max} - 1 \tag{9c}$$

$$z(n_{\max}) = z_{\max} = Z \tag{9d}$$

The suitable value of the distance $z' - z = \Delta z$ is dictated by the convergence requirements of the numerical procedure which compute the probabilities and can be regarded as the dimension of the turbulent structures in which the propagation of light in the jet is considered rectilinear.

To discretize the angles (θ, ϕ), we notice that the laser beam deviates very slightly from its initial incident direction Oz. So, for any point M of its trajectory corresponding to the straight distance $r = OM$ (O is the entry point of the laser beam), we are allowed to make the approximation which considers that the direction of the position vector $\mathbf{OM}$ and the direction of laser beam are nearly identical. Therefore, the coordinates (x, y, z) of M can be connected to the angles (θ, ϕ) of the laser beam direction at the same point, by the following relations:

$$z = r \sin\theta \cos\phi \tag{10a}$$

$$x = r \sin\theta \sin\phi \tag{10b}$$

$$y = r \cos\theta \tag{10c}$$

The laser beam does not undergo change in direction from the outlet jet-border to the photocell plane. Also, if the photocell is placed at a distance D from the outlet jet-border, and if (x, y) are the coordinates of the laser beam impact on the photocell corresponding to the direction-of-arrival (θ, ϕ) of the laser beam, we found from Eq. (10):

$$x = (Z + D) \tan\phi \tag{11a}$$

$$y = (Z + D) \frac{\cot\theta}{\cos\phi} \tag{11b}$$

Since the cell plane (x, y) is cross-ruled as shown in Eq. (8), the experimental values $(x(l), y(m))$, which then characterize the predicted possible positions of the laser beam impact, define the discretized values $(\phi(l), \theta(m))$ of the angles (ϕ, θ) according to Eq. (11). This gives:

$$\phi(l) = \tan^{-1}\left(\frac{x(l)}{Z + D}\right) \quad l = 0, 1, 2, \ldots, l_{\max} \tag{12a}$$

$$\theta(l, m) = \cot^{-1}\left(\frac{y(m)}{\sqrt{x(l)^2 + (Z + D)^2}}\right) \quad l = 0, 1, 2, \ldots, l_{\max} \text{ and } m = 0, 1, 2, \ldots, m_{\max} \tag{12b}$$

Hence, the probabilities $P(\phi,\theta,z)$ are approximated as the quantities $P(\phi(l),\theta(m),z(n))$ written as: $P^n(l,m)$. We then apply a two-steps explicit discretization scheme with alternating directions, which is able to compute $P^n(l,m)$ according to the successive formulas:

$$P^{n+1/2}(l,m) = RP^n(l,m-1) + SP^n(l,m) + TP^n(l,m+1) \tag{13a}$$

$$P^{n+1}(l,m) = UP^{n+1/2}(l-1,m) + VP^{n+1/2}(l,m) + WP^{n+1/2}(l+1,m) \tag{13b}$$

with:

$$R_{l,m} = C_T^2\beta(\Delta z)\left(2 - h_m \cot\theta(l,m)\right)/\left(h_{m-1}^2 + h_{m-1}h_m\right) \tag{14a}$$

$$S_{l,m} = 1 + C_T^2\beta(\Delta z)((h_m - h_{m-1})\cot\theta(l,m) - 2)/h_m h_{m-1} \tag{14b}$$

$$T_{l,m} = C_T^2\beta(\Delta z)(2 + h_{m-1}\cot\theta(l,m))/(h_{m-1}h_m + h_m^2) \tag{14c}$$

$$U_{l,m} = 2C_T^2\beta(\Delta z)(1 + \cot^2\theta(l,m))/(g_{l-1}^2 + g_{l-1}g_l) \tag{14d}$$

$$V_{l,m} = 1 - 2C_T^2\beta(\Delta z)(1 + \cot^2\theta(l,m))/g_l g_{l-1} \tag{14e}$$

$$W_{l,m} = 2C_T^2\beta(\Delta z)(1 + \cot^2\theta(l,m))/(g_{l-1}g_l + g_l^2) \tag{14f}$$

$$g_l = \phi(l+1) - \phi(l) \text{ for } l = 0, 1, \ldots, l_{\max} - 1 \tag{14g}$$

$$h_m = \theta(l,m+1) - \theta(l,m) \text{ for } m = 0, 1, \ldots, m_{\max} - 1 \tag{14h}$$

$$\beta = \lambda K_m^{1/3} \tag{14i}$$

The initial condition is stated for $z = 0$ and is given in terms of Dirac δ as follows:

$$P(\theta,\phi,z=0) = \delta(\theta - \pi/2)\delta(\phi) \tag{15}$$

The values ($\theta_0 = \pi/2,\ \phi_0 = 0$) used in the above condition characterize the direction of the z axis, that is, the direction of the laser beam before entering the jet. This direction corresponds to the impact of the laser beam which is constructed as the centre of the final measuring square in the photocell plane and is then identified by the integers ($l = 20,\ m = 20$). So, the numerical form of the initial condition can be written in terms of Kronecker delta as:

$$P^{n=0}(l,m) = \delta_{l,20}\delta_{m,20} \tag{16}$$

About the boundary conditions, we notice that the probability for the laser beam to have an impact situated on the boundaries of the final measuring square or on the boundaries of any observation plane is equal to zero. Hence, we assume the zero Dirichlet boundary condition:

$$P^n(l,0) = P^n(l,m_{\max}) = 0 \text{ for } l = 0, 1, \ldots, l_{\max} \tag{17a}$$

$$P^{n+1/2}(0,m) = P^{n+1/2}(l_{\max},m) = 0 \text{ for } m = 0, 1, \ldots, m_{\max} \tag{17b}$$

5. CONSTRAINED OPTIMIZATION TECHNIQUE TO COMPUTE C_T^2

To determine the probabilities from Eq. (6), the value of C_T^2 is required. To compute it, we apply an optimization procedure which compares the experimental results $Q(l,m) = Q_{l,m}$ to the corresponding numerical results $P^n(l,m) = P^n_{l,m}(C_T^2)$. We then define a cost function $J(C_T^2)$ which measures the quadratic difference between the two sets of results:

$$J(C_T^2) = \sum_{l=0}^{l_{\max}} \sum_{m=0}^{m_{\max}} \left(P_{l,m}^{n=n_{\max}}(C_T^2) - Q_{l,m}\right)^2 \tag{18}$$

In the definition given in Eq. (18), C_T^2 becomes a parameter which permits one to adjust the numerical solution so that its difference with respect to the experimental results can be as small as possible. Therefore, the temperature structure coefficient of the jet under study can be regarded as the value of the parameter C_T^2 for which the J function is reduced to the minimum. With a view to minimizing J, we have applied the well-known Golden Section (GS) algorithm [11] whose schematization is given in

Fig. 9. This algorithm is strongly recommended to minimize any cost function which depends on a single variable, even if differentiation cannot be assumed in the connection link. In this work, we encounter an additional difficulty coming from the fact that the computation of the cost function considered requires solving a partial differential equation with constraints. Since the solutions are probabilities, those constraints impose that the values obtained must be situated between 0 and 1. The optimization problem solved in this work is well known as a constrained inverse problem of the parameter estimation type [12]. The result that we have obtained is:

$$C_T^2 = (3.75 \pm 0.01) \times 10^2\,\mathrm{K}^2\mathrm{m}^{-2/3} \tag{19}$$

From Eq. (4), one can compute the corresponding value for the structure coefficient of the refractive index of the jet studied; we then obtain: $C_n^2 = 2.30 \times 10^{-10}\,\mathrm{m}^{-2/3}$. This value is close to the result

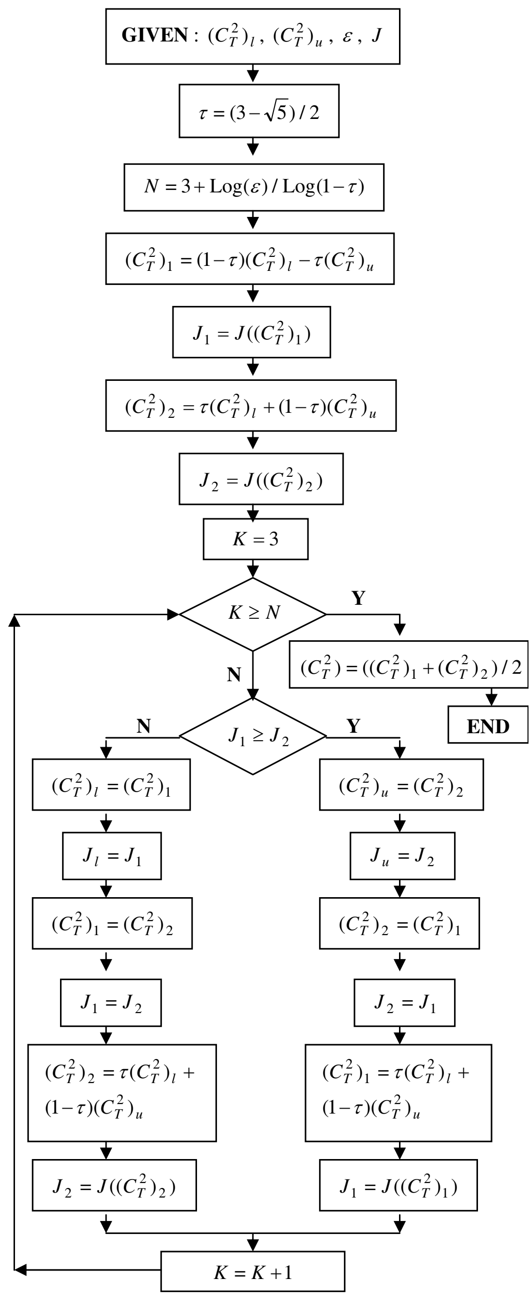

Figure 9. Schematization of the GS Algorithm applied.

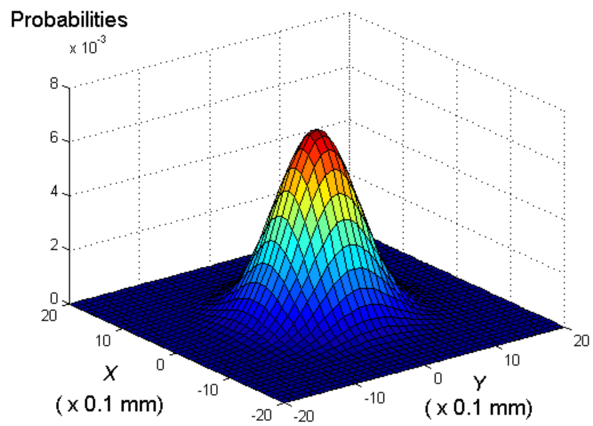

Figure 10. The computed probabilities of the position of the laser beam impact on the photocell plane.

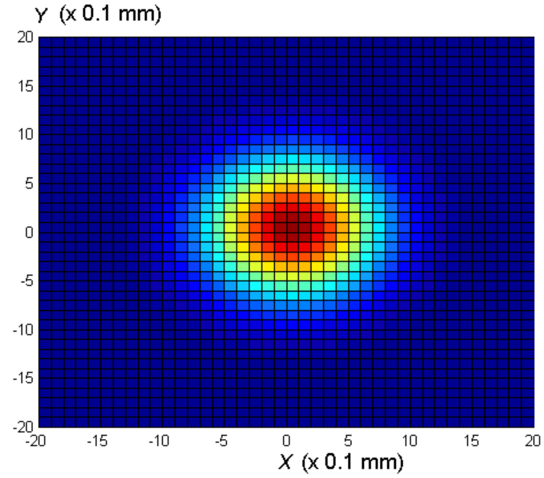

Figure 11. The computed luminous trace produced by the laser beam on the photocell plane.

$C_n^2 = 1.93 \times 10^{-10}\,\mathrm{m}^{-2/3}$ previously published by our research team for another hot turbulent wind tunnel jet [4]. For a strong indoor turbulence, Consortini et al. [13] have obtained: $C_n^2 = 2.96 \times 10^{-11}\,\mathrm{m}^{-2/3}$. In our previous paper [14], we have obtained the following value: $C_n^2 = 3.27 \times 10^{-8}\,\mathrm{m}^{-2/3}$ for a turbulent premixed butane — air flame by means of a laser based interferometer technique. For the measured temperature structure coefficient $C_T^2 = 3.75 \times 10^2\,\mathrm{K}^2\mathrm{m}^{-2/3}$, we have computed the probabilities $P_{l,m}^{n=n_{\max}}$ of the positions of the laser beam impact on the photocell plane. The obtained values are plotted in Fig. 10, and the corresponding predicted luminous trace produced by the laser beam on the photocell is presented in Fig. 11. Both figures must be compared to the experimental values plotted in Fig. 7 and to the experimental luminous trace presented in Fig. 8. A good agreement is observed from that comparison.

6. CONCLUSION

In this paper, we have proposed a technique to measure the temperature structure coefficient C_T^2 for a hot turbulent wind tunnel jet without introducing any probe in the jet. A laser ray is sent into the jet along a direction of homogeneity of C_T^2 and produces a luminous trace on a photocell placed outside the jet. The probabilities $Q(x, y)$ of the positions (x, y) of the laser ray impact on the photocell are measured. From the Markov process model for the laser beam direction, those probabilities are computed by means of a numerical procedure.

To extract the value of C_T^2, we have applied a numerical optimization technique in which a cost function $J(C_T^2)$ measuring the quadratic difference between results $P(x, y, C_T^2)$ and $Q(x, y)$ is reduced to the minimum by means of the Golden Section algorithm. Therefore, the temperature structure coefficient of the jet can be regarded as the value of the parameter C_T^2 for which the J function is reduced to the minimum.

In subsequent research, the result obtained in this paper and the measurement technique applied in it will be used to determine the variable temperature structure coefficient.

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