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Time-Frequency Analysis of Particle Beam Interactions with Resonant and Guiding Structures

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Abstract—This article describes numerical solutions for the electromagnetic interactions, known as 'wakefields', of a proton beam with an RF cavity and a beampipe. Using FDTD calculations, time-varying electromagnetic solutions are obtained. Unlike modal expansion methods, FDTD allows to compute transient wakefields due to proton beam passing through the structures. A popular time-frequency analysis approach, the short-time Fourier transform (STFT), is applied to the electromagnetic fields inside a resonant cavity and past an open-ended beampipe. STFT enables a more explicit interpretation of the transitions between the fields radiated by moving charges and the resonant modes. The described time-frequency analysis is useful to engineers and accelerator physicists who analyze proton beam dynamics. As an extension of electromagnetic simulations using an extended proton bunch, a numerical Green's function approach is proposed in order to account for the wakefields due to individual superparticles.

1. INTRODUCTION

Electromagnetic interactions of charged particle beams with guides (beampipes) and resonant structures, such as RF cavities, produce responses, known as 'wakefields'. Examples of systems that are affected by the electromagnetic interactions with the guiding structures include particle accelerators and vacuum tubes. In practical particle accelerators, beams are separated in time into multiple bunches, so electromagnetic wakefields can affect the particle dynamics both within the same bunch and in the following ones. For an accurate account of particle dynamics it is very important to have information about electromagnetic field evolution in time and space, at any position along the particle trajectories. Electromagnetic wakefields can be predicted either analytically [1–3] or numerically [2, 4, 5]. Simplified electromagnetic models from [2] can be used to calculate electromagnetic fields only due to ultra-relativistic charges, or those moving at speeds near the speed of light, for a few idealized geometries. Carron [6] considered several examples in which the electromagnetic fields due to a moving charge are influenced by discontinuities in material properties, such as a metal-vacuum boundary. Early analytical models for electromagnetic fields excited by beams in an RF cavity [2, 3] used modal expansions for the electric and magnetic potentials to represent the resonant modes. They can also include frequency-dependent surface impedances to accurately account for the loss in cavity walls. Numerical electromagnetic calculations using finite-difference time domain (FDTD) [7] and finite integration (FIT) [8] methods have been developed later. Some of them [9] include surface impedance boundary conditions (SIBC) that incorporate frequency-dependent surface resistance of the walls.

Accurate modeling of electromagnetic interactions can help one understand the dynamics of particles in multiple proton bunches in particle accelerators, allowing optimum beam control that greatly reduces or eliminates any instabilities [2]. As the proton beams used in accelerators become more intense, effects of the wakefields on particle dynamics become more pronounced. For instance,

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modern accelerator systems at Fermilab operate at intensities of 10^{12} and more protons per bunch. At such beam intensities, changes in particle momenta due to the wakefield-related Lorentz forces cannot be neglected.

RF cavities are very important components of synchrotrons that provide synchronized momentum kicks to the charged particles passing through them in short bunches. The stability of a beam depends on the match between the times of arrival of protons and the phase of the RF field in the cavity. The RF field is a superposition of the applied field and the wakefields. When the protons move in accelerating structures at velocities approaching the speed of light, their transit times are close to the time duration required for electromagnetic waves to traverse the cavity. The resulting short-range interactions between radiated fields and the structures prompt the need for combined time-frequency representation of the electromagnetic fields. Time-frequency analysis of transient oscillating fields in an RF cavity was first proposed by Moghaddar and Walton [10]. That paper focused on the electromagnetic wave scattering.

In this work, time-frequency analysis is applied to electromagnetic fields excited in an RF cavity by a moving proton bunch or a train of bunches. The same approach can be generalized to account for the wakefields due to a single moving charge.

2. THEORETICAL ANALYSIS

Electric field $\vec{E}$ and magnetic field $\vec{B}$ due to a point moving charge in free space can be found in the lab frame using the Liénard-Wiechert potentials [11], (10–72, 73):

$$\vec{E}(\vec{r},t) = \frac{qr'}{4\pi\varepsilon_0(\vec{r'}\cdot\vec{u})^3}\left[\left(c^2-v^2\right)\vec{u}+\vec{r'}\times(\vec{u}\times\vec{a})\right] \tag{1a}$$

$$\vec{B}(\vec{r},t) = \frac{1}{c}\vec{r}\times\vec{E}(\vec{r},t) \tag{1b}$$

$$\vec{u} = c\hat{r'}-\vec{v} \tag{1c}$$

where $\vec{v}$ is the particle velocity; $\vec{r'}$ is the particle position vector with respect to a fixed observer; $\vec{r}$ is the position vector from the origin; c is the speed of light; $\vec{a}$ is the particle acceleration; q is the amount of charge; ε_0 is the permittivity of free space.

However, when a charge moves through a structure, such as a beampipe with conducting walls or emerges from an open-ended beampipe into free space, or traverses a cavity, these potentials are no longer applicable to the analysis. Analytical methods for solving problems of particle beam interaction with structures [1, 2] are based on expansions of the electric and magnetic potentials into either multipole harmonics, or cavity modes that are spatial eigenfunctions of the Helmholtz equation with appropriate boundary conditions. An advantage of full-wave methods over modal expansion ones is that the first allow transient field solutions during time intervals when steady resonant responses are not yet established. Such transitions from an excitation to a resonance are seen when electromagnetic waves travel some distance from the moving charges to reach cavity or waveguide walls.

Thus, in this paper, numerical calculations are carried out using the finite difference time domain method (FDTD) [7]. This method treats the electromagnetic field evolution naturally by solving discrete versions of the Maxwell's equations. According to the particle-in-a-cell approach [13], the space charge in a particle beam is represented by a large (10^5–10^6) set of superparticles. Their number is chosen several orders of magnitude less than the actual number of protons in a bunch, in order to keep the complexity of the problem at a manageable level. All calculations in this paper are carried out in the lab frame.

The electromagnetic interactions of a particle beam with the surroundings are accounted for by introducing current density distributions due to moving charges in Equation (2a) into the Ampere-Maxwell's Equation (2b) at every time step. The Faraday's equation is given by Equation (2c). Continuous Equations (2a)–(2c) are replaced in FDTD calculations with the corresponding approximations [7].

$$\vec{J} = \vec{v}\rho(\vec{r}) = \sum_{n=1}^{N}\vec{v}_n\rho(\vec{r}_n)\delta(\vec{r}-\vec{r}_n), \tag{2a}$$

where $\rho(\vec{r}_n)$ is the charge density at position $\vec{r}_n$, and $\vec{v}_n$ is the velocity of the n-th charge.

$$\nabla \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \vec{J} \tag{2b}$$

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}, \tag{2c}$$

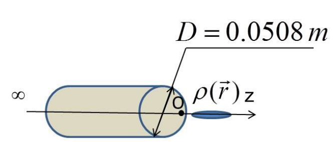
where $\vec{E}$ and $\vec{H}$ are electric and magnetic fields, and t is time. The current density $\vec{J}$ is spatially interpolated and imposed on the FDTD grid at every time step of the algorithm. A model of this nature can account for the moving charged particles that travel through a structure at any speed, up to the speed of light.

3. ELECTROMAGNETIC FIELDS DUE TO A MOVING PROTON BUNCH

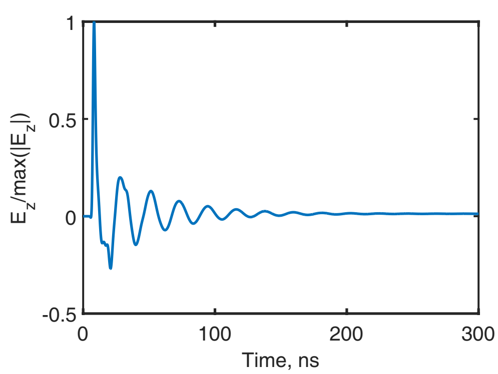
In the following examples, electromagnetic fields are computed using FDTD for moving proton bunches, obeying a Gaussian distribution of the charge density in the direction of motion (positive z-axis), with the standard deviation of the proton time of arrival $\tau = 4$ ns, corresponding to the dispersion of proton positions in the bunch of $\sigma_z = 1.2$ m. Along two other axes, the bunch is much thinner, so it is considered to be infinitely thin. The speed of the protons is assumed to be constant, $v = 2.99 \times 10^8$ m/s, corresponding to the kinetic energy of 12 GeV. FDTD simulations are carried out on a rectangular grid with the spatial resolution of $\Delta x = \Delta y = \Delta z = 2.54$ cm and the time step of $\Delta t = 4.23 \times 10^{-11}$ s. The simulation domain is terminated with a uniaxial perfectly-matched absorber [7] with the thickness of 8 Yee's cells.

3.1. An Open-Ended Beampipe

In order to illustrate the electromagnetic fields that are radiated when moving protons leave a beampipe for an unbounded free space, the Maxwell's equations were solved for an open-ended 5.08 cm diameter cylindrical pipe. It is assumed that the protons move along the axis of symmetry of this semi-infinite beampipe. For simplicity, the pipe walls are modeled as infinitely-conducting. Figure 1(a) shows the simulated structure, and Figure 1(b) shows the transient response of the normalized E_z component of the electric field at the distance 2.54 cm past the opening.



(a)



(b)

Figure 1. Simulated geometry (a) and E_z field response (b) for an open-ended beampipe.

As one can see from Figure 1(b), the electric fields radiated due to a beampipe discontinuity represent decaying oscillations.

3.2. An RF Cavity

A cylindrical RF cavity of the diameter $D = 0.813$ m and length $L = 1.397$ m, used in this example, is assumed to have perfectly-conducting walls. The protons move in and out of the cavity via cylindrical

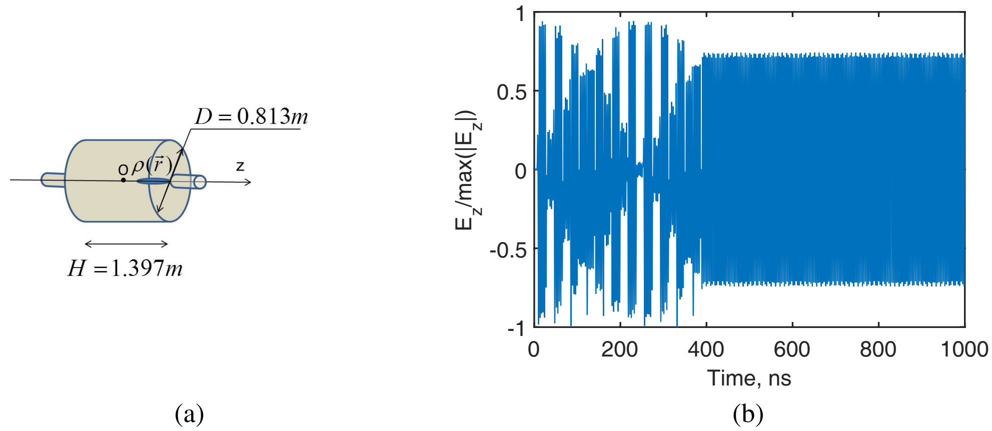


Figure 2. Simulated geometry (a) and transient E_z response (b) for the RF cavity.

beampipes. The theoretical fundamental resonant frequencies of the cavity are: for TM_{010} mode, $f_{010} = 302.3\,\mathrm{MHz}$, and for the TE_{111} mode, $f_{111} = 241.6\,\mathrm{MHz}$. In practical accelerators and storage rings, protons are sent into the system as trains of periodically repeated bunches. Therefore, a train of 21 proton bunches, following with the repetition period of $T = 18.94\,\mathrm{ns}$ is used as an excitation. Figure 2(a) shows the geometry of the RF cavity used in the analysis, and Figure 2(b) shows the normalized electric field E_z component in the center of the cavity.

It follows from the results presented in Figure 2(b) that electromagnetic fields radiated by the moving charges interacting with the cavity walls transition into steady-state oscillatory modes that are supported by the cylindrical cavity after the multibunch excitation ceases around 400 ns time. These results are consistent with previously reported general findings about field evolution in similar structures [3].

4. TIME-FREQUENCY ANALYSIS USING STFT

Time-frequency analysis of transient electromagnetic fields in an RF cavity can be carried out using a windowed short-time Fourier transform (STFT) [13], given by Equation (3).

$$\mathrm{STFT}(E(\vec{r}))\{m,\omega\} = \sum_{n=-N}^{N} E(\vec{r},n)w[n-m]e^{-j\omega n}, \tag{3}$$

where $E[m]\{\vec{r},\omega\}$ is the electric field at time t_m; $\vec{r}$ is the position vector; w is the window, ω is the frequency; n is an integer shift index; $2N+1$ is the total length of the transform. An STFT of the magnetic field has an identical functional form. In practical computations, fast Fourier transforms are used along with the index hopping. A sliding Hamming window has an adjustable width. This width defines the resolution of the algorithm in frequency.

4.1. An Open-Ended Beampipe

The computed electric field sampled at the symmetry axis, 5.08 cm past the beampipe end, was processed through STFT with the following parameters: FFT length: 40,000, Hamming window length: 10,000, and the hopping parameter: 100 time steps. The resulting time-frequency distribution of E_z component is presented in Figure 3.

The time-frequency map in Figure 3 shows that interactions of moving protons with the open-ended beampipe can result in a relatively broadband radiated field with decaying amplitude.

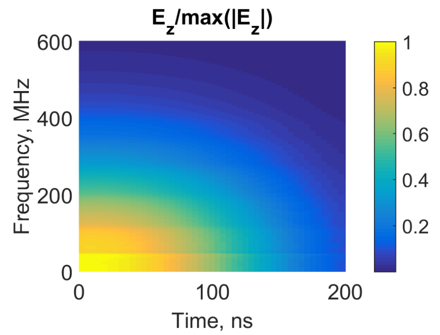


Figure 3. The time-frequency distribution of E_z field for a proton bunch leaving a beampipe.

4.2. An RF Cavity

To illustrate this approach, the computed electric field is sampled in the center of the cylindrical cavity. In both simulations, the same cavity dimensions as in Subsection 3.2 are assumed. In the first computation, a single proton bunch is sent through the cavity, and in the second one, a train consisting of 21 bunches, as in Subsection 3.2 is used. The STFTs computed with the same parameters as in Subsection 4.1 are shown in Figures 4(a) and 4(b), corresponding to single and multiple bunch excitations, respectively.

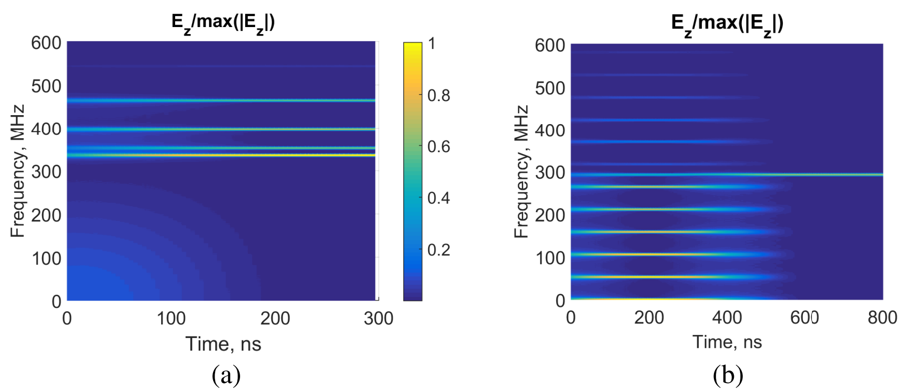


Figure 4. The time-frequency distribution for the E_z magnitude at the center of the cavity, computed for (a) a single proton bunch, (b) a train of 21 proton bunches, moving through an RF cavity.

As one can see from Figure 4(a), a single proton bunch radiates fields with a relatively broad spectrum that couple to several cylindrical cavity modes at once. From Figure 4(b), it can be noticed that a train of proton bunches produces the excitation that whose spectrum is close to a line spectrum, but only one cavity mode, namely, TM_{010}, is selected after all protons leave the cavity. This example illustrates how the nature of the excitation may affect the spectral evolution of wakefields in a cavity.

5. SIMULATING FIELDS DUE TO SUPERPARTICLES

In modern particle-based simulation codes for proton beams in accelerator structures, such as Synergia 2 [1], beam dynamics is modeled by computing Coulombic interactions among superparticles within the particle-in-a-cell framework [13]. On the other hand, time-domain numerical methods require that the excitation term in (2a) be described by a spectrally-bounded function, due to possible numerical instabilities. To resolve this controversy, an approach, also known in electromagnetic simulations as 'numerical Green's function' [14], can be used. Analytically, such a Green's function can be described by Equation (4):

$$G_E(\vec{r}, \vec{r}', t) = FT^{-1}\left\{\frac{FT(E(\vec{r}, t))}{FT(J(\vec{r}', t))}\right\}, \tag{4}$$

where FT and FT^{-1} are the Fourier transform and its inverse, respectively; E is the electric field at position $\vec{r}$; J is the current density produced by a moving charge at the position $\vec{r}'$. A similar numerical Green's function can also be computed for the magnetic field. Although this approach is computationally intense, the obtained Green's function can then be used to predict the Lorentz forces on particles at any position within a guiding or a resonant structure. This capability is very important when one wants to model particle beam dynamics in a self-consistent manner, i.e., not making restrictive assumptions about the charge density.

6. CONCLUSIONS

Interactions of proton beams with beampipes and RF cavities can be successfully modeled with full-wave numerical methods, such as FDTD or FIT. Using such methods, discontinuous boundary conditions are naturally applied to the Maxwell's equations, and time- and position-dependent fields are obtained.

However, interpreting the results of electromagnetic simulations of proton beams moving through structures may not be easy. Particularly, this is the case when the transit time of a proton beam is comparable to the time that it takes for the electromagnetic fields inside the cavity to develop into steady oscillations or when the excitation is complex. The use of time-frequency methods, such as STFT, enables a better analysis of the electromagnetic field evolution than either purely frequency-domain or time-domain ones.

As the analysis shows, both electric and magnetic fields have transitioned over a short period of time from a broadband radiation field into steady cavity modes that are narrowband. Multiple proton bunches traversing a cavity excite a time-varying current density that possesses a frequency spectrum close to a line spectrum. The coupling of these modes to the resonant cavity modes depends on their mutual alignment. Time-frequency analysis is a powerful analytical tool for studying such electromagnetic problems. Numerical electromagnetic analysis done for the moving bunches of protons can be used to compute numerical Green's functions for specific boundaries, enabling self-consistent analysis of particle dynamics.

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