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Study of Load Characteristics in Wireless Power Transfer System with Ferrite Core

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Abstract—For wireless power transfer via magnetic resonant coupling (MRC-WPT), magnetic coupling between resonant coils can be greatly enhanced when a ferrite core is introduced inside the coils. Based on the equivalent circuit model of wireless power transfer system, transfer characteristics of the MRC-WPT system with air resonant coils and with a ferrite core are respectively analyzed in this paper. The influence mechanism of the load on the power transfer efficiency is investigated. Also, the requirement of load for improving transfer efficiency is derived when adding the ferrite core to the system. The numerical simulation and experiment result indicate that the transmission efficiency in the MRC-WPT system with ferrite core is higher than that in the counterpart with air resonant coils in the whole transfer region when the load is larger than the maximal critical load. In addition, for different transfer distances, the system efficiency for the system using the ferrite core tends to become lower than that in the air coil system when the load is smaller than the critical load.

1. INTRODUCTION

Wireless power transfer (WPT) technology makes it possible to deliver electromagnetic energy to the load without direct wire connection. Generally, wireless power transfer in the near-field can be categorized into inductive WPT (IWPT), magnetic resonant coupling WPT (MRC-WPT) and radio wave [1]. Compared with the other two types of technology, inductive WPT has been maturely developed and utilized in many real applications [2, 3]. Since the inspiring work of Massachusetts Institute of Technology has been published in 2007, an increasing amount of research has been conducted on wireless power transfer via magnetic resonant coupling (MRC-WPT) due to its characteristics of long transfer distance, high transmission efficiency and great convenience [4–8]. For a better transfer performance, there have been extensive reports on the analysis for MRC-WPT system [9–11].

To extend the transfer distance, resonant coils with large size are used [12]. However, its application is generally restricted because of the large volume. An alternative method is the utilization of intermediate coils [13–15]. By positioning the repeater between the transmitting coil (Tx) and receiving coil (Rx), the transfer distance is increased. Nevertheless, the transmission efficiency drops rapidly when the repeater is deviated from a specific position, and it is inconvenient for practical applications to place the additional resonator in the free space between coils. High Q-factor of coils is a possible solution to increase the transfer efficiency of MRC-WPT system [16]. However, too high operating frequency will result in the instability of the system and increase the cost.

High permeability materials, such as ferrite cores, can be used to enhance the magnetic coupling between coils, and hence the transmission efficiency is increased in long distance [17–22]. In [17], a novel configuration of a magnetic resonant structure is proposed. Compared with the traditional cylindrical ferrite core, the proposed ferrite core has a smaller demagnetizing factor and an increased

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mutual inductance. With the proposed ferrite core, the average efficiency of the MRC-WPT system is improved by 80%, and the system is less sensitive to the variation of the pitch and load. In [18], the coil structure and the configuration of the ferrite core have been designed and optimized for efficient power transfer. The result indicates that the power can be efficiently delivered to the load even in case of misalignment. To reduce the magnetic flux leakage, a coupler with assistive coils is proposed in [19]. Each side of the coupler is composed of a major coil, four small assistive coils and ferrite bars. By applying the proposed method, the system is capable of transferring power with the efficiency of over 90%. In [20], an optimal power transmission of flat spiral coils is presented where flat spiral coils are connected with ferrite materials. The magnetic field of the flat spiral ferrite core materials and of the flat spiral air core is analyzed and compared. It is found that the efficiency can be improved by using ferrite materials. Compared with the traditional system using air core, the transfer efficiency is increased by 42% with the proposed system. To improve the misalignment tolerance of the WPT system with ferrite core and minimize the core loss, the configuration of the ferrite cores is optimized in [21]. The flux density maintains uniform, and the core loss is reduced by 39% in the optimized ferrite core compared with the uniform thickness core. In [22], the Q-factor of coils with ferrite core is calculated. It is revealed that the transfer efficiency is increased by using ferrite core with high permeability and low hysteresis loss. When the Mn-Zn ferrite core is inserted within coils, the efficiency is increased to 61.45%. These researches on the ferrite core structure, in cooperation with other technologies such as the optimization of the coil configuration technology, undoubtedly contribute to improving the power transfer efficiency of the MRC-WPT system. However, the transfer performance of the system with the ferrite core is also dependent on the load which influences the estimation and comparison of the system efficiency.

In this paper, the transfer characteristic of an MRC-WPT system with ferrite core is analyzed in detail. At the beginning, the power transmission efficiencies of the system with air resonant coils and with a ferrite core are calculated respectively based on the equivalent circuit model of MRC-WPT system. The requirement of the load to increase the efficiency is analyzed when adding the ferrite core to the system. After that, the mutual inductance and the transmission efficiency under different loads for the two different systems are simulated. Then, experiments are carried out to validate the theoretical analysis. Finally, concluding remarks are drawn.

2. EQUIVALENT CIRCUIT MODEL FOR MRC-WPT

The equivalent circuit model of the MRC-WPT system with air resonant coils and with a ferrite core is illustrated in Fig. 1. Both transmitting coil and receiving coil can be described as the self-inductance (L_1, L_2) and the resistance of the resonant coils (R_1, R_2) connected in series. C_1, C_2 and C_2' are the compensated capacitance of the transmitting coil and receiving coil. $\mathbf{I}_1$, $\mathbf{I}_1'$, $\mathbf{I}_2$ and $\mathbf{I}_2'$ are the currents flowing in the transmitting loop and receiving loop, respectively. $\mathbf{V}_S$ and R_S indicate the voltage and internal resistance of the power source. R_L is the load. $\mathbf{V}_L$ and $\mathbf{V}_L'$ are the voltage across the load. M_{12} is the mutual inductance between L_1 and L_2, as shown in Fig. 1(a). The ferrite core provides a low reluctance path for the magnetic flux, which is denoted by C_{core}, L_{core} and R_{core} connected in series, as shown in Fig. 1(b). C_{core} can be ignored compared with C_2' [23]. M_{12}' is the mutual inductance between L_1 and $L_2 + L_{core}$.

When the operating frequency is ω, the equivalent circuit of the system with air resonant coils and with a ferrite core can be expressed as:

$$\begin{cases} \mathbf{V}_{in} = Z_{11}\mathbf{I}_1 - j\omega M_{12}\mathbf{I}_2 \\ Z_{22}\mathbf{I}_2 - j\omega M_{12}\mathbf{I}_1 = 0 \end{cases}, \tag{1}$$

$$\begin{cases} \mathbf{V}_{in}' = Z_{11}\mathbf{I}_1' - j\omega M_{12}'\mathbf{I}_2' \\ Z_{22}'\mathbf{I}_2' - j\omega M_{12}'\mathbf{I}_1' = 0 \end{cases}, \tag{2}$$

where $\mathbf{V}_{in}$ and $\mathbf{V}_{in}'$ are the voltage of the input port respectively, $Z_{11} = R_1 + j(\omega L_1 - 1/\omega C_1)$, $Z_{22} = R_L + R_2 + j(\omega L_2 - 1/\omega C_2)$, and $Z_{22}' = R_L + R_{core} + R_2 + j(\omega L_2 + \omega L_{core} - 1/\omega C_2')$.

The input power P_{in} and output power P_{out} of the system with air resonant coils can be calculated

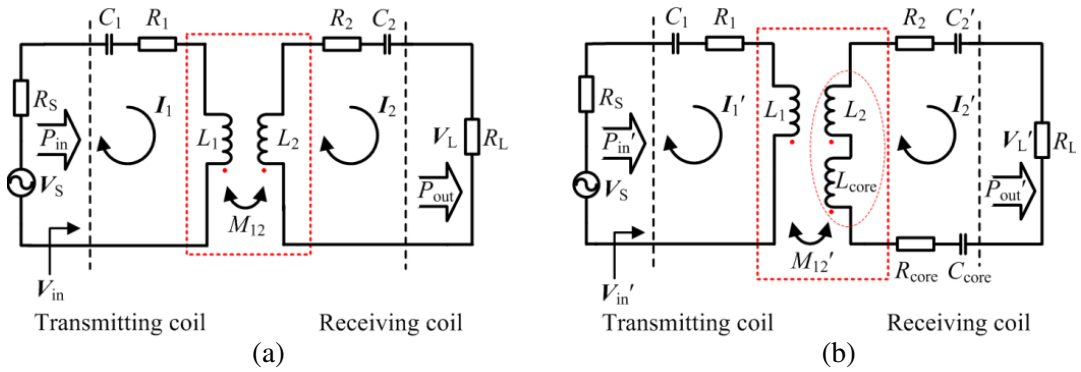

Figure 1. Equivalent circuit model of MRC-WPT system. (a) System with air resonant coils. (b) System with a ferrite core.

by using Eq. (1) as [24]:

$$\begin{cases} P_{in} = \mathrm{Re}\left[\dfrac{Z_{11}Z_{22} + (\omega M_{12})^2}{Z_{22}} I_1^2\right] \\ P_{out} = \mathrm{Re}\left[\dfrac{R_L (\omega M_{12})^2}{Z_{22}^2} I_1^2\right] \end{cases}, \tag{3}$$

Similar to the system with air resonant coils, the input power P'_{in} and output power P'_{out} of the system with the ferrite core can be written using Eq. (2) as:

$$\begin{cases} P'_{in} = \mathrm{Re}\left[\dfrac{Z_{11}Z'_{22} + (\omega M'_{12})^2}{Z'_{22}} I_1'^2\right] \\ P'_{out} = \mathrm{Re}\left[\dfrac{R_L (\omega M'_{12})^2}{Z'^2_{22}} I_1'^2\right] \end{cases}. \tag{4}$$

When the system with air core is tuned at the resonant frequency ω_0, the transfer efficiency of the MRC-WPT system η can be expressed by using Eq. (3) as:

$$\eta = \frac{R_L (\omega_0 M_{12})^2}{R_1 (R_2 + R_L)^2 + (R_2 + R_L)(\omega_0 M_{12})^2}, \tag{5}$$

where $\omega_0 = 1/(L_1C_1)^{0.5} = 1/(L_2C_2)^{0.5}$.

According to Eq. (4), the transfer efficiency of the MRC-WPT system with the ferrite core η' can be calculated by

$$\eta' = \frac{R_L (\omega_0 M'_{12})^2}{R_1 (R_2 + R_{core} + R_L)^2 + (R_2 + R_{core} + R_L)(\omega_0 M'_{12})^2}, \tag{6}$$

where $\omega_0 = 1/(L_1C_1)^{0.5} = 1/(L_2 + L_{core})C'_2)^{0.5}$.

When the ferrite core is inserted within the resonant coil, the ferrite core will be magnetized, and the magnetic field through the coil is intensified. Thus, the mutual inductance between the coils with the ferrite core is larger than that between the air resonant coils, i.e., $M'_{12} > M_{12}$ in this paper [25]. Compared with Eq. (6) and Eq. (5), it can be seen that it is difficult to determine whether the performance of the MRC-WPT system with the ferrite core is better or not due to the increase in both the denominator and the numerator. In addition, the transfer efficiency of the system with the ferrite core is not only related to the parameters of the coils and the mutual inductance, but also dependent on the configuration of the ferrite core and the load. For a fixed MRC-WPT system, the transmission efficiency is merely in relation to the load.

To make the transfer efficiency of the system with the ferrite core higher than that of the system with the air resonant coils, Eq. (5) and Eq. (6) should satisfy the following relation:

$$\frac{R_L\left(\omega M'_{12}\right)^2}{R_1\left(R_2+R_{core}+R_L\right)^2+\left(R_2+R_{core}+R_L\right)\left(\omega M'_{12}\right)^2} > \frac{R_L\left(\omega M_{12}\right)^2}{R_1\left(R_2+R_L\right)^2+\left(R_2+R_L\right)\left(\omega M_{12}\right)^2}. \tag{7}$$

According to Eq. (7), the load can be expressed as:

$$R_L > R_L^*, \tag{8}$$

with

$$R_L^* = \frac{K}{\sqrt{K^2-1}}\sqrt{\frac{R_{core}^2}{K^2-1}+\frac{R_{core}\left(\omega_0 M_{12}\right)^2}{R_1}}+\frac{R_{core}}{K^2-1}-R_2, \tag{9}$$

where R_L^* is the critical load and $K = M'_{12}/M_{12}$.

From Eq. (9), it can be seen that the critical load is different for various transfer distances. For a fixed transfer distance, the transfer efficiency of the MRC-WPT system with the ferrite core would be higher than that with the air core when the load is larger than the critical load R_L^*. To make the efficiency of the MRC-WPT system with ferrite core keep higher than that in the counterpart with air resonant coils in the whole transfer region, the load of the system with the ferrite core must be bigger than the maximal critical load which is denoted by $R_L^{\max}$. It should be noted that K is larger than one in Eq. (9).

3. NUMERICAL SIMULATION

For validation, simulation has been carried out using Maxwell, which is a simulation software package based on the finite-element method. In the work, the resonant frequency is set to 0.8 MHz. The cylindrical ferrite core is applied, and its configuration is illustrated in Fig. 2. All coils are constructed with 1.4-mm-diameter copper wire, and the parameters of the simulation are shown in Table 1.

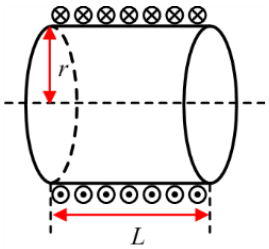

Figure 2. Configuration of the cylindrical ferrite core, where $r = 50\,\text{mm}$ and $L = 60\,\text{mm}$.

Table 1. Parameters of the simulation.

Parameters	System with air resonant coils	System with the ferrite core
Radius of Tx	$r_1 = 50\,\text{mm}$	$r_1 = 50\,\text{mm}$
Radius of Rx	$r_2 = 50\,\text{mm}$	$r_2 = 50\,\text{mm}$
Number of turns of Tx	$n_1 = 20$	$n_1 = 20$
Number of turns of Rx	$n_2 = 10$	$n_2 = 10$
Pitch between turns of Tx	$p_1 = 1.5\,\text{mm}$	$p_1 = 1.5\,\text{mm}$
Pitch between turns of Rx	$p_2 = 1.5\,\text{mm}$	$p_2 = 1.5\,\text{mm}$

Figure 3 shows the mutual inductance versus the transfer distance for the coils with air core and with the ferrite core. As demonstrated in Fig. 3, the mutual inductance decreases as the transfer

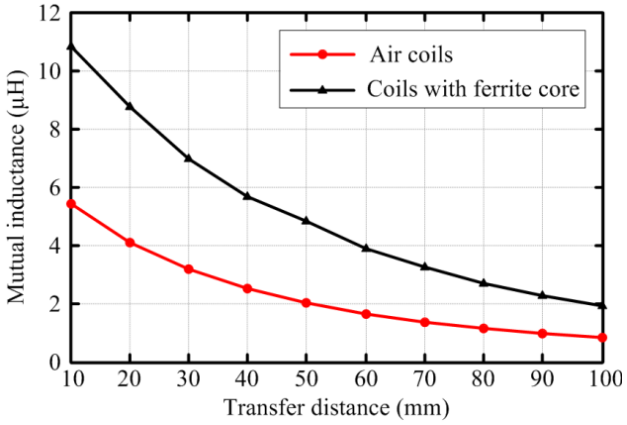

Figure 3. Simulated mutual inductance versus the transfer distance for the air resonant coils and the coils with the ferrite core.

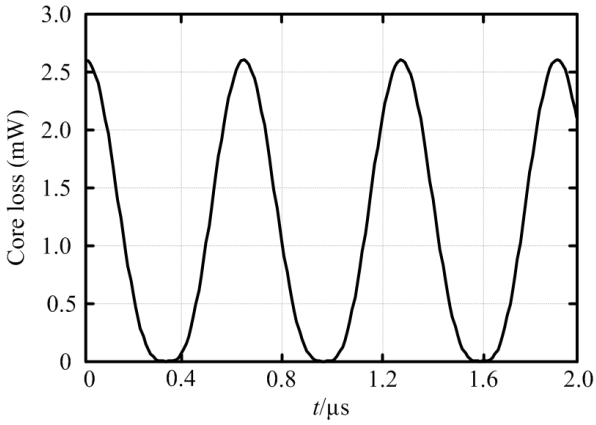

Figure 4. Core loss versus time for the ferrite core.

distance increases. However, the mutual inductance of the coils with the ferrite core is generally larger than that of the air resonant coils. This is because the magnetic field through the coil is enhanced due to the magnetization of the ferrite core. When the transfer distances are 10 mm and 100 mm, the mutual inductances for the coils without ferrite core are 5.43 μH and 0.82 μH, whereas the values are 10.83 μH and 1.92 μH in the coils with ferrite core. The mutual inductance is improved by 99% and 134% respectively compared with the air resonant coils. In the whole transfer region, the average mutual inductance for the coils with ferrite core is higher than that for the air resonant coils by 128%.

To obtain the equivalent resistance of the ferrite core R_{core}, the ferrite core loss is analyzed in Maxwell, a simulation software package based on the finite-element method, and the result is shown in Fig. 4. In the work, the flowing in the receiving loop is 10 mA, and the driving power is 52 mW. From Fig. 4, it can be seen that the average core loss P_{av} is 1.31 mW. Based on the analysis in [26], the equivalent resistance of the ferrite core is calculated to be $R_{core} = 13.1\,\Omega$.

According to Eq. (9), the mutual inductance shown in Fig. 3 and the equivalent resistance of the ferrite core, the critical load of the MRC-WPT system with the ferrite core for different transfer distances can be calculated which is illustrated in Table 2. It can be seen from Table 2 that the critical load decreases with the increase in the transfer distance. To improve the efficiency, the load of the system with the ferrite core must be larger than 115.40 Ω, i.e., $R_L^{\max} = 115.40\,\Omega$ in this paper.

Figure 5 compares the efficiency of the system with air core and with ferrite core for different loads.

Table 2. Critical loads for different transfer distances.

Distance (mm)	M_{12} (μH)	M'_{12} (μH)	$K = M'_{12}/M_{12}$	R_L^* (Ω)
10	5.43	10.83	2.00	115.40
20	4.08	8.74	2.14	84.97
30	3.16	6.98	2.21	65.42
40	2.50	5.68	2.27	51.58
50	2.01	4.84	2.41	40.87
60	1.65	3.89	2.36	34.08
70	1.36	3.24	2.38	28.31
80	1.14	2.68	2.35	24.21
90	0.96	2.25	2.34	20.83
100	0.82	1.92	2.34	18.20

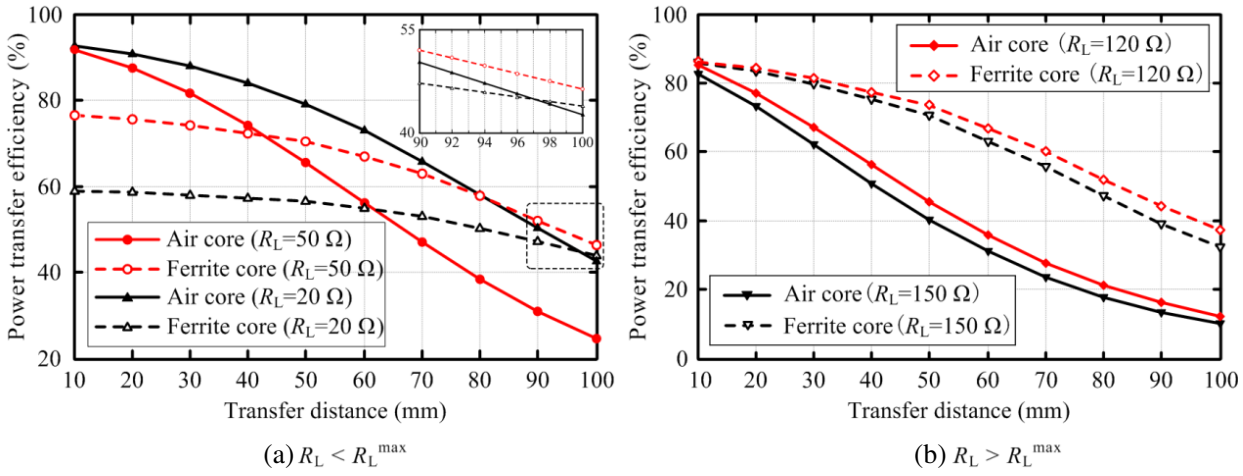

Figure 5. Power transfer efficiency versus the transfer distance for the system with air core and with ferrite core. (a) Load is smaller than the maximal critical load. (b) Load is larger than the maximal critical load.

As shown in Fig. 5, the efficiency decreases with the increase in the transfer distance for the two different systems. The efficiency of the system with the ferrite core is lower than that of the system with air core when the transfer distance is within 43 mm in the case of $R_L = 50\,\Omega$. As the load decreases to $20\,\Omega$, the efficiency for the system with the ferrite core is higher only at the distance of 100 mm than the system with air resonant coils, as shown in Fig. 5(a). Moreover, it can be observed from Fig. 5(b) that the transmission efficiency is significantly improved in the whole transfer region when the load is larger than the maximal critical load $R_L^{\max}$, which is well consistent with Table 2.

4. EXPERIMENTAL VALIDATION

An experimental prototype of MRC-WPT system is established to validate the performance of the system with different loads, as given in Fig. 6. The transmitting coil is connected in series to the power amplitude whose input signal is supplied by the function generator. The receiving coil is wound on a cylindrical ferrite core and is connected with the load. Compensated capacitances are used for the magnetic resonance. The experimental parameters of the coils and the ferrite core are the same as that in the simulation. Fig. 7 shows the variations in the mutual inductance versus the transfer distance for the coils with air core and with ferrite core.

As demonstrated in Fig. 7, the mutual inductance decreases as the transfer distance increases for different types of coils configurations. Comparing Fig. 7 with Fig. 3, an excellent agreement can be

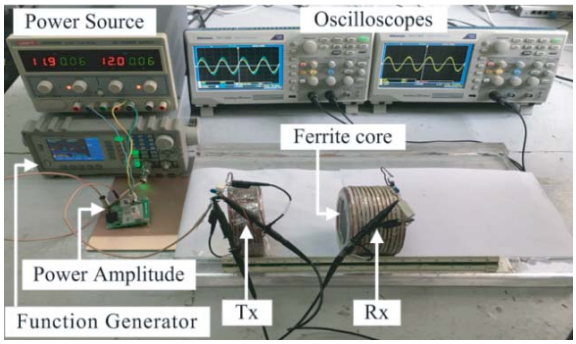

Figure 6. Photograph of the experimental setup.

observed between the measured and simulated results. The average mutual inductance is improved by 121% when the ferrite core is introduced.

Figure 8 shows a comparison of the efficiencies for the system with air core and with ferrite core. As illustrated in Fig. 8(a), when the load is set to 50 Ω the efficiency of the system using the ferrite core is lower within the transfer distance of 40 mm. In the case of $R_L = 20\,\Omega$, the transmission efficiency of the system with the ferrite core is slightly higher than that of the system with air core at the distance

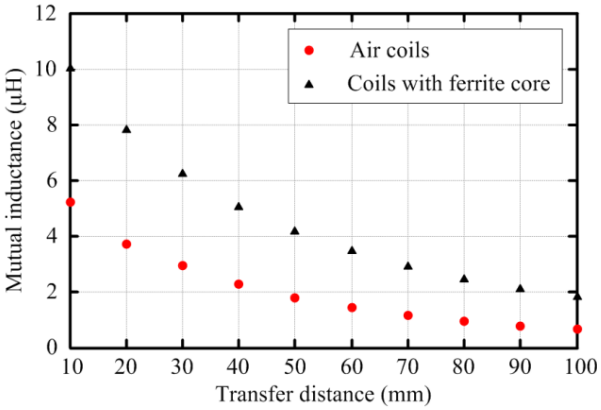

Figure 7. Measured mutual inductance versus the transfer distance for the air resonant coils and the coils with the ferrite core.

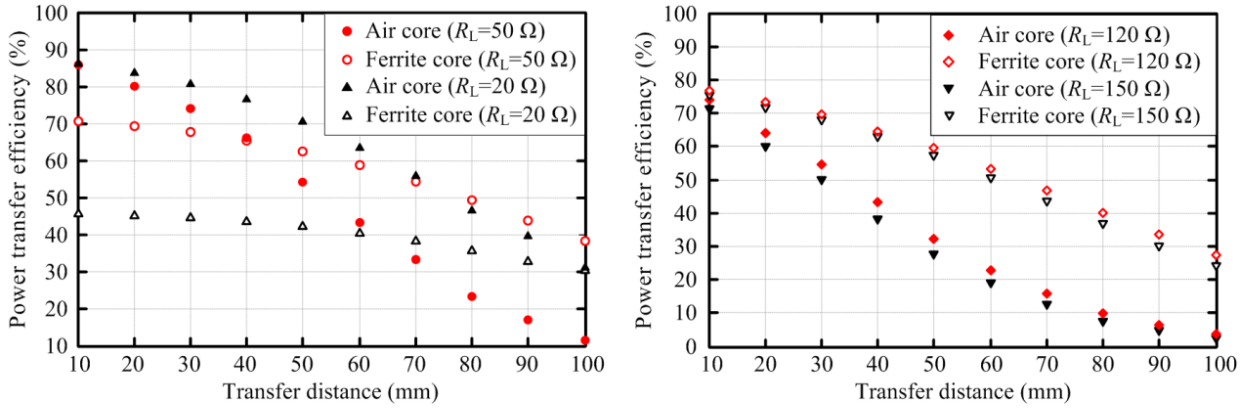

Figure 8. Measured efficiency for the system with air core and with ferrite core. (a) Load is smaller than the maximal critical load. (b) Load is larger than the maximal critical load.

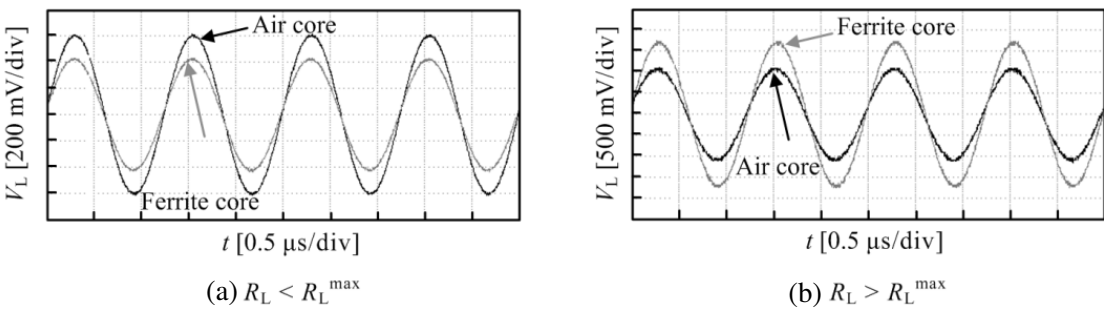

Figure 9. Measured voltage across the load for various loads. (a) Load is 50 Ω. (b) Load is 150 Ω.

of 100 mm whereas the efficiency is lower in the remaining transfer distances for the system with ferrite core. In addition, when the load satisfies the requirement for the load ($R_L > R_L^{\max}$), the efficiency for the system with the ferrite core is larger than that for the system with air core in the whole transfer region, as shown in Fig. 8(b).

Also, the voltage across the load is measured for various loads as shown in Fig. 9. The transfer distance is 30 mm. According to Fig. 9(a), the voltage obtained from the system using the ferrite core is lower than the system using air core. When the load increases to 150 Ω, the measured voltage is improved by adding the ferrite core to the system, as shown in Fig. 9(b).

5. CONCLUSION

According to the equivalent circuit model of the MRC-WPT system, the transmission efficiencies of the system with air resonant coils and with ferrite core are calculated respectively in this work. The requirement of load for improving transfer efficiency is derived when the ferrite core is introduced to the system. The different systems with various loads are simulated and tested. The results indicate that the performance of the system with the ferrite core is superior to the system with an air core when the load is larger than the critical load. However, when the load is very small, the MRC-WPT system with air core is a better alternative. This research provides a guideline for choice of different MRC-WPT systems in various load conditions.

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