Progress In Electromagnetics Research M, Vol. 78, 125–133, 2019

Solution of Wideband Scattering Problems Using Hierarchical Ultra-Wideband Characteristic Basis Functions

Wen-Yan Nie[1] and Zhong-Gen Wang[2, *]

Abstract—In this paper, a hierarchical ultra-wideband characteristic basis function method (HUCBFM) is presented for high-precision analysis of wideband scattering problems. Unlike existing improved ultra-wideband characteristics basis function method (IUCBFM), HUCBFM reduces the number of characteristic basis functions (CBFs) necessary to express a current distribution. This reduction is achieved by combining primary CBFs (PCBFs) with the secondary level CBFs (SCBFs) to form a single hierarchical ultra-wideband characteristic basis function (HUCBF). As HUCBF incorporates the effects of PCBFs and SCBFs, the accuracy does not change significantly compared to that obtained by IUCBFM. Furthermore, the efficiencies of constructing the CBFs and filling the reduced matrix are improved. Numerical examples verify and demonstrate that the proposed method is credible both in terms of accuracy and efficiency.

1. INTRODUCTION

Many electromagnetic applications require the solution of the radiation from an antenna or scattering problems over a wide frequency band rather than at a single frequency. However, the solution using either the method of moments (MoM) [1] or other fast methods [2–4] requires the calculations to be executed at each frequency point. Several techniques have been proposed to alleviate this problem. In [5], model-based parameter estimation (MBPE) is used to obtain the wideband data from frequency and frequency-derivative data. In [6], impedance interpolation technology is used to analyze the wideband electromagnetic scattering problems. This method leads to a CPU time reduction at the expense of increased memory requirement. In [7], multilevel fast multipole method (MLFMM) is combined with the best uniform approximation to calculate the wideband radar cross section (RCS) of objects. In [8, 9], asymptotic waveform evaluation (AWE) technology is used to predict the RCS over a band of frequencies. However, this technique needs to store both the impedance matrix and its frequency derivatives, limiting its applicability to small-scale problems only. In [10], the AWE technique is combined with the pre-corrected fast Fourier transform (PFFT)/adaptive integral method (AIM) for a fast analysis of the wideband scattering problems. Most of the approaches mentioned above usually face the unpredictable problem of convergence rate as they use iteration method to solve the linear equations.

Another method used for the solution of electromagnetic scattering problems is characteristic basis function method (CBFM) [11–13]. This method uses the direct method for matrix equation solution. Several methods have been proposed to improve the accuracy, computational time, and memory problems of CBFM. In [14–16], an improved primary characteristic basis function method is proposed to improve the efficiency of constructing characteristic basis functions (CBFs). In [17, 18], adaptive cross approximation algorithm and AIM are used to accelerate reduced matrix construction. However, CBFs

Received 18 December 2018, Accepted 24 January 2019, Scheduled 31 January 2019
* Corresponding author: Zhong-Gen Wang (zgwang@ahu.edu.cn).
[1] College of Mechanical and Electrical Engineering, Huainan Normal University, Huainan, Anhui 232001, China. [2] College of Electrical and Information Engineering, Anhui University of Science and Technology, Huainan, Anhui 232001, China.

should be recalculated at each frequency point over the frequency band of interest. Hence, in [19], ultra-wideband CBFM (UCBFM) is proposed to analyze wideband electromagnetic scattering problems. The ultra-wideband CBFs (UCBFs) constructed at the highest frequency point can also be employed at lower frequencies without going through the time-consuming step of constructing the CBFs again. However, the errors in the calculation of the RCS using UCBFs are usually large at lower frequency points. In addition, the number of UCBFs constructed at the highest frequency point is unnecessarily high, and computational complexity increases when applying UCBFs to the lower frequency points. To mitigate these problems, an improved UCBFM (IUCBFM) is proposed in [20] to improve the accuracy at lower frequency points. This improvement is achieved by fully considering the mutual coupling effects among the sub-blocks to obtain the secondary level CBFs (SCBFs). In [21], an adaptive improved UCBFs (AIUCBFs) construction method is proposed, which can decrease the computational complexity at the lower frequency points. In [22], the unnecessary UCBFs are removed from the basis set as the frequency is decreased. In [23], a singular value decomposition (SVD) enhanced IUCBFM (SVD-IUCBFM) is proposed to reduce the number of matrix equation solutions. These methods have advantages of higher accuracy at lower frequency points and lower computational complexity in lower frequency bands. However, most of the approaches mentioned above need to calculate SCBFs that leads to an increase in the number of CBFs before the application of SVD.

In this paper, a hierarchical UCBFM (HUCBFM) is proposed to improve the efficiency of the IUCBFM. The HUCBFM is a method that reduces the number of CBFs necessary to express a current distribution. The method combines the SCBFs calculated for each block of the scatter with the primary CBFs (PCBFs) to form a single hierarchical ultra-wideband characteristic basis function (HUCBF). As the HUCBF incorporates the effect of PCBFs and SCBFs, a reduction in the number of CBFs can be achieved, but the accuracy does not change significantly compared to that of IUCBFM. Furthermore, dimensions of the reduced matrix and the reduced matrix filling time are significantly reduced.

This paper is composed of the following sections. The next section describes the improved ultra-wideband characteristic basis function method. In Section 3, the construction of hierarchical ultra-wideband characteristic basis functions (HUCBFs) is described. Section 4 presents the complexity of the two methods. In Section 5, some numerical results are given, while conclusions are drawn in Section 6.

2. IMPROVED ULTRA-WIDEBAND CHARACTERISTIC BASIS FUNCTION METHOD

To improve the calculation accuracy of UCBFM at lower frequency points, the construction of UCBFs is improved by considering the mutual coupling effects among sub-blocks. The IUCBFM [20] begins by dividing an object into M blocks. Then, it establishes a model at the highest frequency point and constructs the CBFs by using a series of plane waves (PWs). Suppose that N_θ and N_ϕ represent the numbers of PWs in the directions of θ and ϕ, respectively, giving a total of $2N_\theta N_\phi$ PWs, considering two polarization modes. For each plane wave excitation, PCBFs of each block can be solved by the following system:

$$\mathbf{Z}_{ii}\mathbf{J}_{ii}^{P} = \mathbf{V}_{ii}, \tag{1}$$

where $\mathbf{V}_{ii}$ represents the excitation vector of block i, for $i = 1, 2, 3, \ldots, M$; $\mathbf{Z}_{ii}$ is an $N_i \times N_i$ self-impedance matrix of block i; N represents the number of unknowns in the extended block i; and $\mathbf{J}_{ii}^{P}$ is the PCBFs matrix of dimensions $N_i \times 1$. The PCBFs of block i can be obtained by directly solving Eq. (1). After the PCBFs of each block are solved, the SCBFs are calculated subsequently using the following equations:

$$\mathbf{Z}_{ii}\mathbf{J}_{ii}^{S1} = -\sum_{j=1(j\neq i)}^{M} \mathbf{Z}_{ij}\mathbf{J}_{jj}^{P}, \tag{2}$$

$$\mathbf{Z}_{ii}\mathbf{J}_{ii}^{S2} = -\sum_{j=1(j\neq i)}^{M} \mathbf{Z}_{ij}\mathbf{J}_{jj}^{S1}, \tag{3}$$

where $\mathbf{Z}_{ij}$ represents the impedance matrix between i and j blocks, and $\mathbf{J}_{ii}^{S1}$ and $\mathbf{J}_{ii}^{S2}$ represent the first-level SCBFs and second-level SCBFs, respectively.

Following the procedure described above, a total of $2N_\theta N_\phi\,\mathbf{J}_{ii}^{P}$, $2N_\theta N_\phi\,\mathbf{J}_{ii}^{S1}$, $2N_\theta N_\phi\,\mathbf{J}_{ii}^{S2}$, can be obtained on each block. To reduce linear dependency among these CBFs, the SVD approach is used. Only those CBFs whose relative singular values are above a certain threshold, e.g., 0.001, are retained as improved UCBFs (IUCBFs). For simplicity, it is assumed that all of the blocks contain the same number of K IUCBFs after the application of SVD. The solution to the entire problem is expressed as a linear combination of the $M \times K$ IUCBFs as follows:

$$\mathbf{J} = \sum_{m=1}^{M}\sum_{k=1}^{K}\alpha_m^k(f)J_{mm}^{CBF_k}, \tag{4}$$

where $J_{mm}^{CBF_k}$ represents the kth IUCBFs of block m, and $\alpha_m^k(f)$ represents the unknown weight coefficients. The Galerkin method [11] is used to convert the traditional MoM equation $\mathbf{Z}\cdot\mathbf{J} = \mathbf{V}$ into a linear equation with coefficient matrix $\alpha(f)$. A $KM \times KM$ reduced matrix can be obtained as shown next:

$$\mathbf{Z}^R(f)\cdot\alpha(f) = \mathbf{V}^R(f), \tag{5}$$

where $V_{ii}^R(f) = \mathbf{J}_{ii}^{\mathrm{T}}\cdot\mathbf{V}_{ii}(f)$, T represents transpose operation, and $\mathbf{Z}^R(f)$ represents the reduced impedance matrix of dimensions $KM \times KM$. Each element of $\mathbf{Z}^R(f)$ can be expressed as follows:

$$Z_{ij}^R(f) = \mathbf{J}_{ii}^{\mathrm{T}}\cdot\mathbf{Z}_{\mathbf{i}j}(f)\cdot\mathbf{J}_{jj}\quad i,j\le M. \tag{6}$$

where $\mathbf{Z}_{ij}(f)$ represents the impedance matrix between blocks i and j at frequency f. As the dimension of $\mathbf{Z}^R(f)$ is small, $\alpha(f)$ can be obtained by directly solving Eq. (5). Then, by substituting $\alpha(f)$ into Eq. (4), the surface current $\mathbf{J}$ at any frequency point can be obtained. Although IUCBFs can improve the calculation accuracy at lower frequency points, the calculation of SCBFs leads to an increase in both the number of matrix equation solutions and the number of CBFs.

3. HIERARCHICAL ULTRA-WIDEBAND CHARACTERISTIC BASIS FUNCTION METHOD

In the IUCBFM outlined in the previous section, a matrix of size $6N_\theta N_\phi \times M$ needs to be solved for constructing the CBFs, and subsequently, $6N_\theta N_\phi \times M$ CBFs are obtained. When dealing with large-scale targets, the enormous number of CBFs significantly increases the CPU time and memory requirements. To solve these problems, a HUCBFM is proposed in this section. The HUCBFM reduces the number of CBFs necessary to express a current distribution by combining PCBFs with the SCBFs to form a single HUCBF. Fig. 1 shows the schematic diagram to construct the HUCBFs. The HUCBFs of each block are constructed based on Eqs. (1), (2) and (3) as follows [14]:

$$\mathbf{Z}_{ii}\mathbf{J}_{ii}^{P(k)} + \mathbf{Z}_{ii}\mathbf{J}_{ii}^{S1(k)} + \mathbf{Z}_{ii}\mathbf{J}_{ii}^{S2(k)} = \mathbf{V}_{ii}^k - \sum_{j=1(j\ne i)}^{M}\mathbf{Z}_{ij}\mathbf{J}_{jj}^{P(k)} - \sum_{j=1(j\ne i)}^{M}\mathbf{Z}_{ij}\mathbf{J}_{jj}^{S1}, \tag{7}$$

$$\mathbf{Z}_{ii}\left(\mathbf{J}_{ii}^{P(k)} + \mathbf{J}_{ii}^{S1(k)} + \mathbf{J}_{ii}^{S2(k)}\right) = \mathbf{V}_{ii}^k - \sum_{j=1(j\ne i)}^{M}\mathbf{Z}_{ij}\mathbf{J}_{jj}^{P(k)} - \sum_{j=1(j\ne i)}^{M}\mathbf{Z}_{ij}\mathbf{J}_{jj}^{S1}, \tag{8}$$

$$\mathbf{Z}_{ii}\mathbf{J}_{ii}^{H(k)} = \mathbf{V}_{ii}^k - \sum_{j=1(j\ne i)}^{M}\mathbf{Z}_{ij}\mathbf{J}_{jj}^{P(k)} - \sum_{j=1(j\ne i)}^{M}\mathbf{Z}_{ij}\mathbf{J}_{jj}^{S1}. \tag{9}$$

where $\mathbf{V}_{ii}^k$ is the kth incident plane wave excitation, and $\mathbf{J}_{jj}^{P(k)}$ and $\mathbf{J}_{jj}^{S1(k)}$ represent PCBFs and the first-level SCBFs, respectively, which are obtained by using the kth incident plane wave irradiating block j. $2N_\theta N_\phi$ HUCBFs $\mathbf{J}_{ii}^{H(k)}$ can be obtained by directly solving Eq. (9). Subsequently, the SVD procedure can be utilized to remove the redundancy in the obtained HUCBFs before constructing the reduced matrix. Since the number of CBFs is much lower than that constructed by the IUCBFM, the time

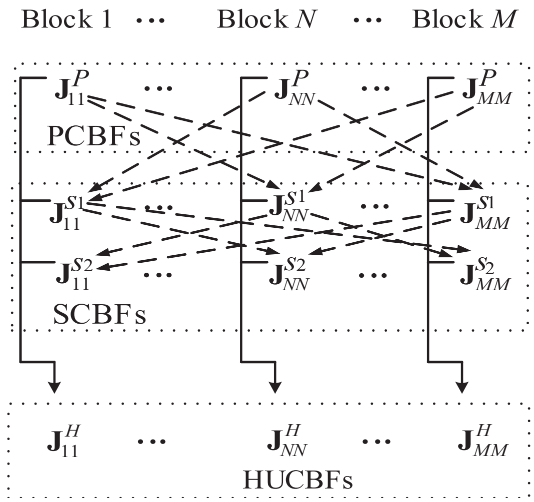

Figure 1. Schematic diagram of HUCBFs construction.

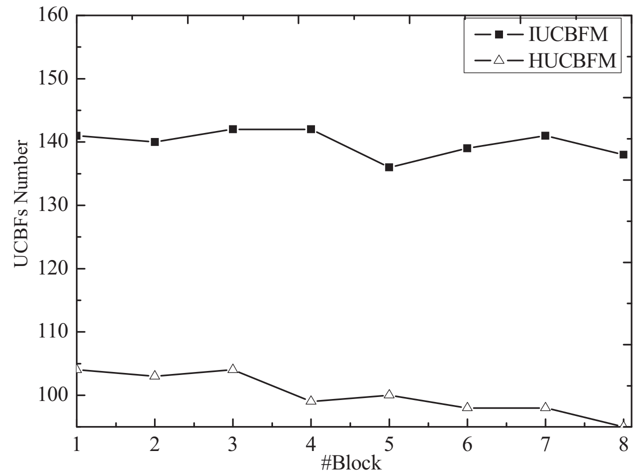

Figure 2. UCBFs number of each block.

taken by the SVD procedure is also reduced in the HUCBFM. In addition, as the mutual interaction of surrounding blocks is properly considered in the HUCBFM, a high accuracy can be obtained when the same number of PWs is set to irradiate each block.

4. COMPLEXITY ANALYSIS

The computations involved in the application of the IUCBFM or HUCBFM can be divided into three parts: 1) The UCBFs construction. 2) Constructing the reduced matrix. 3) Solving the reduced matrix. The most computationally intensive part of the IUCBFM is associated with the construction of the UCBFs and the reduced matrix construction procedure. In the following, the computational complexity of each of these parts is analyzed for both IUCBFM and HUCBFM:

(1) Ultra-wideband CBFs construction: To solve Eq. (1), the LU factorization is usually used. In IUCBFM, the construction of the UCBFs mainly includes two parts: first is the construction of the PCBFs and SCBFs, and the second is the SVD procedure. The complexity of the UCBFs construction is $O(6MN_\theta N_\phi(N_i)^3) + 4M(M-1)N_\theta N_\phi N_i N_j + 6MN_\theta N_\phi(N_i)^2$. N_i and N_j represent the numbers of the RWG basis functions in blocks i and j, respectively. In HUCBFM, the computational complexity of UCBFs construction is $O(6MN_\theta N_\phi(N_i)^3) + 4M(M-1)N_\theta N_\phi N_i N_j + 2MN_\theta N_\phi(N_i)^2$.

(2) Reduced matrix construction: In IUCBFM, the complexity of the reduced matrix construction is $O((KM)^2 N_i N_j)$. In HUCBFM, the complexity of the reduced matrix construction is $O((K_{new}M)^2 N_i N_j)$. The number of HUCBFs retained on each block after SVD is K_{new}, where K_{new} is always smaller than K.

(3) Reduced matrix solution: In IUCBFM, the complexity of the reduced matrix solution is $O((KM)^3)$. In HIUCBFM, the complexity is $O((K_{new}M)^3)$.

Compared with the IUCBFM, the computational complexity of HUCBFM is greatly decreased in UCBFs construction, reduced matrix construction and reduced matrix solution.

5. NUMERICAL RESULTS

In this section, a comparison is carried out between the performances of the existing IUCBFM and the proposed HUCBFM. All simulations are completed on a personal computer with an Intel(R) Core(TM) i7-3970 CPU with 3.5 GHz (only one core is used) and 16 GB RAM. The compiler used is Visual studio

2013. The threshold of the SVD is set to 10^{-3}. In order to decrease the computational complexity at the lower frequency points, the adaptive UCBFs construction method is used [21]. The equivalent dipole-moment method (EDM) [24] is applied to accelerate the impedance matrix element filling procedure. We use the relative error of the RCS to estimate the accuracy of the proposed method, and the relative error is defined as follows:

$$\mathrm{Err}(\%)=100 \times \sqrt{\sum_n \left|\mathrm{RCS}^x - \mathrm{RCS}^{ref}\right|^2} \Big/ \sqrt{\sum_n \left|\mathrm{RCS}^{ref}\right|^2} \tag{10}$$

where RCS^x is the RCS calculated by the IUCBFM or HUCBFM, and RCS^{ref} is the RCS calculated by the FEKO.

First, a PEC sphere with radius of 0.1 m over a frequency range of 0.3 GHz to 3 GHz is considered. The geometry is divided into 4914 triangular patches with an average length of $\lambda/10$ at 3 GHz, thus resulting in 10900 unknowns. The sphere is divided into 8 blocks, with each block extended by $\Delta = 0.15\lambda$. Referring to [20], each block is illuminated by multi-angle PWs from $0^\circ \leq \theta < 180^\circ$ and $0^\circ \leq \phi < 360^\circ$ with $N_\theta = 8$ and $N_\phi = 8$, resulting in a total of 128 PWs. The numbers of UCBFs constructed by using IUCBFM and HUCBFM retained on each block at the highest frequency point are shown in Fig. 2. It can be observed easily that the number of UCBFs obtained using the HUCBFM is significantly smaller than that obtained using the IUCBFM. The $\theta\theta$ polarization bistatic RCSs calculated at 900 MHz using IUCBFs and HUCBFs are shown in Fig. 3. The results calculated

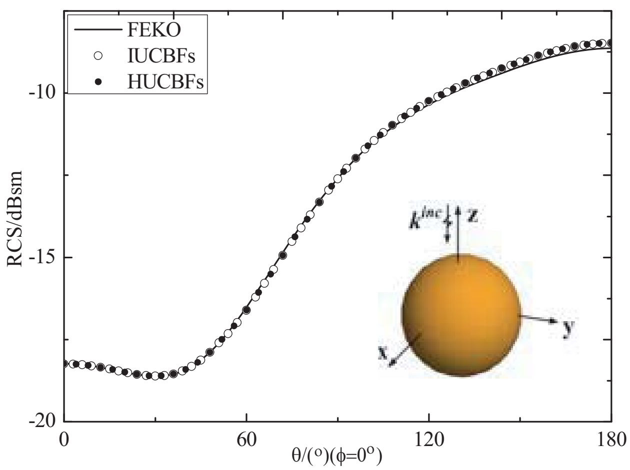

Figure 3. Bi-static RCS of the sphere at 900 MHz.

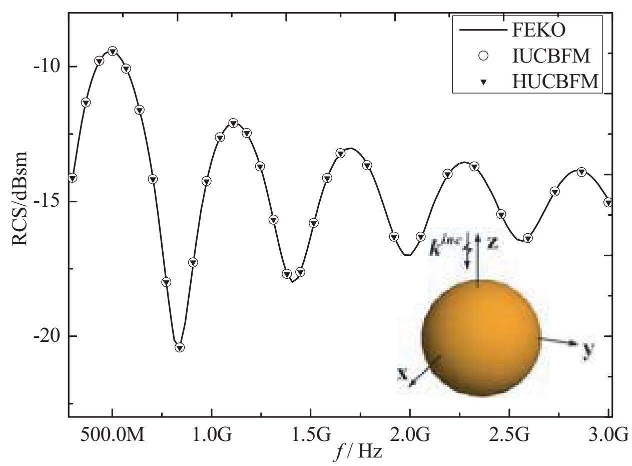

Figure 4. Wideband RCS of the PEC sphere.

Table 1. Numbers of UCBFs and calculation time of different procedures for the two methods.

	IUCBFM			HUCBFM		
Frequency sub-band (GHz)	0.3–0.975	0.975–1.65	1.65–3.0	0.3–0.975	0.975–1.65	1.65–3.0
Number of PWs	128	128	128	128	128	128
SVD time (s)	19.29	19.63	21.09	9.37	9.45	9.51
Number of UCBFs	462	711	1129	329	521	801
UCBFs construction time (s)	527.99	536.18	541.64	439.41	441.83	443.79
Reduced matrix filling (s)	19.39	30.59	59.04	13.22	21.46	34.69
Total time (s)	1647.31	1785.65	2072.62	1445.51	1530.51	1668.89
Relative error (%)	2.88	2.61	1.99	2.95	2.78	2.01

using the HUCBFs agree well with the results obtained by the IUCBFs. The given frequency band is adaptively divided into 3 sub-bands using the adaptive UCBFs construction method. The broadband RCSs with 10 frequency sampling points in each sub-band, computed by the conventional IUCBFM and the HUCBFM are compared in Fig. 4, which are in good agreement. The numbers of UCBFs and the computational times for different procedures are summarized in the Table 1. It can be seen that the HUCBFM needs a smaller number of UCBFs than the conventional IUCBFM. Furthermore, the UCBFs constructing time and reduced matrix filling time are remarkably reduced, and the gains are about 19% and 37%, respectively.

Table 2. Numbers of UCBFs and calculation time of different procedures for the two methods.

	IUCBFM				HUCBFM			
Frequency sub-band (GHz)	0.1–0.7125	0.7125–1.325	1.325–2.55	2.55–5.0	0.1–0.7125	0.7125–1.325	1.325–2.55	2.55–5.0
Number of PWs	128	128	128	128	128	128	128	128
SVD time (s)	34.33	34.81	34.89	34.64	17.11	16.53	16.62	16.73
Number of UCBFs	362	476	729	1340	210	321	524	923
UCBFs construction time (s)	986.56	997.75	989.5	1010.51	839.12	830.11	829.12	839.15
Reduced matrix filling (s)	23.51	35.13	69.78	115.88	19.86	24.75	49.13	75.87
Total time (s)	2478.76	2643.97	2919.17	3520.07	2291.71	2403.81	2622.57	2931.86
Relative error (%)	4.08	3.64	3.11	2.96	4.29	3.84	3.36	3.27

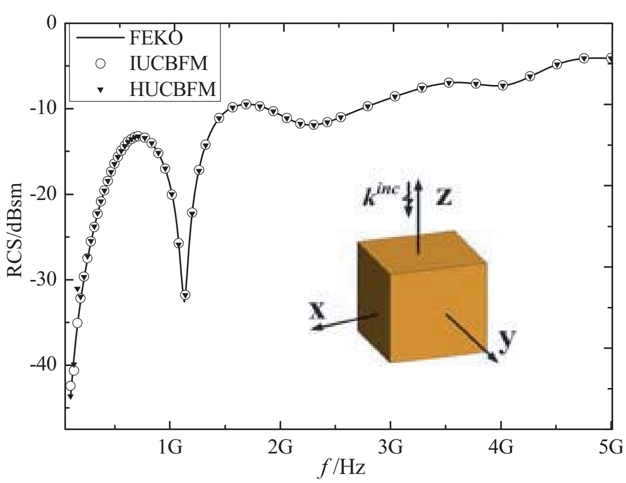

Figure 5. Wideband RCS of the cube.

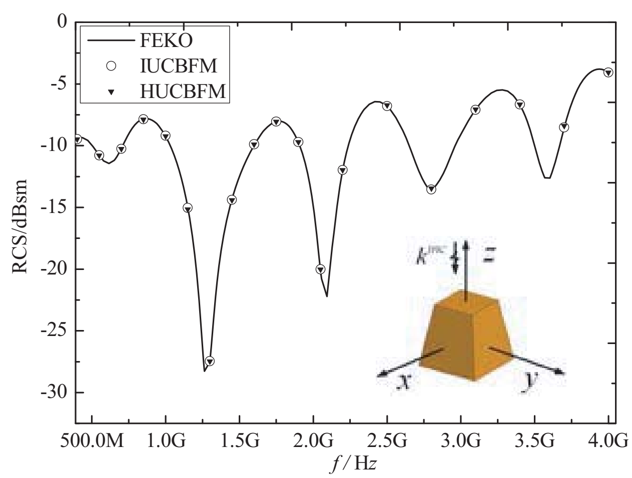

Figure 6. Wideband RCS of the hexahedron.

Second, a PEC cube with a side length of 0.1 m over a frequency range of 0.1 to 5 GHz is considered. The discretization in triangular patches is conducted at 5 GHz, which leads to a total number of 13607 unknowns. The geometry is divided into 8 blocks. The broadband RCSs computed by the IUCBFM and HUCBFM are shown in Fig. 5. The numbers of UCBFs and the computational times of the two methods are shown in Table 2. As shown in Fig. 5 and Table 2, the HUCBFM leads to a relatively small number of UCBFs in each sub-band resulting in substantial time-saving without compromising the accuracy.

Finally, a PEC hexahedron over a frequency range of 0.4 GHz to 4 GHz is considered. The lengths of bottom and top sides are 0.2 m and 0.1 m, respectively, and 0.2 m high. The discretization in triangular patches is conducted at 4 GHz, which leads to 21479 unknowns. The geometry is divided into 8 blocks. The broadband RCS with 6 frequency sampling points in each sub-band, computed by the IUCBFM and HUCBFM are shown in Fig. 6. It can be seen from Fig. 6 that the RCSs calculated by the two methods agree well with calculation results of the FEKO. The numbers of UCBFs and the computational times of the two methods are shown in Table 3. It is evident that the HUCBFM outperforms the conventional IUCBFM, both in UCBFs construction and RCS computational time. Especially, the numbers of UCBFs and the reduced matrix filling time are remarkably reduced.

Table 3. Numbers of UCBFs and calculation time of different procedures for the two methods.

	IUCBFM			HUCBFM		
Frequency sub-band (GHz)	0.4–1.3	1.3–2.2	2.2–4.0	0.4–1.3	1.3–2.2	2.2–4.0
Number of PWs	128	128	128	128	128	128
SVD time (s)	83.27	83.38	83.06	38.39	39.02	39.26
Number of UCBFs	606	923	1463	453	680	858
UCBFs construction time (s)	2575.76	2596.09	2585.40	2227.63	2225.28	2217.09
Reduced matrix filling (s)	143.39	217.29	348.03	106.27	159.36	199.59
Total time (s)	5144.78	5683.41	6457.80	4363.78	4788.81	5141.39
Relative error (%)	3.98	3.68	3.35	4.21	3.83	3.47

6. CONCLUSION

This paper puts forward an effective numerical method for analyzing the wideband scattering of PEC objects. In this method, hierarchical ultra-wideband characteristic basis functions (HUCBFs) are obtained by combing primary CBFs (PCBFs) with the secondary level CBFs (SCBFs). This combination incorporates the effects of both PCBFs and SCBFs such that the number of UCBFs and the computation time for UCBFs are reduced significantly. Furthermore, the reduced matrix dimensions and reduced matrix filling time are decreased. The numerical results demonstrate that the proposed method can calculate the wideband radar cross section more efficiently than the conventional improved ultra-wideband characteristics basis function method (IUCBFM) without compromising the accuracy.

ACKNOWLEDGMENT

This work was supported by the Natural Science Foundation of Anhui Province under Grant No. 1808085MF166, the Anhui Provincial Key Science Foundation for Outstanding Young Talent under No. gxyq2017060, the Postdoctoral Science Foundation of Anhui Province under Grant No. 2017B214, the Overseas Visiting and Training Program for Outstanding Young Talents of Anhui Province under No. gxgwfx2018025, and the Natural Science Foundation of Anhui Provincial Education Department under Grant No. KJ2016A669.

REFERENCES

1. Harrington, R. F., *Field Computation by Method of Moments*, IEEE Press, New York, 1992.
2. Coifman, R., V. Rokhlin, and S. Wandzura, "The fast multipole method for the wave equation: A pedestrian prescription," *IEEE Ant. Propag. Mag.*, Vol. 53, No. 3, 7–12, 1993.
3. Song, J. M., C. C. Lu, and W. C. Chew, "Multilevel fast multipole algorithm for electromagnetic scattering by large complex objects," *IEEE Trans. Antennas Propag.*, Vol. 45, No. 10, 1488–1493, 1997.
4. Bleszynski, E., M. Bleszynski, and T. Jaroszewicz, "Adaptive integral method for solving large-scale electromagnetic scattering and radiation problems," *Radio Sci.*, Vol. 31, No. 5, 1225–1251, 1996.
5. Burke, G. J., "Using model based parameter estimation to increase the efficiency of computing electromagnetic transfer functions," *IEEE Trans. Mag.*, Vol. 25, No. 4, 2807–2809, 1988.
6. Newman, E. H., "Generation of wide band from the method of moments by interpolating the impedance matrix," *IEEE Trans. Antennas Propag.*, Vol. 36, No. 12, 1820–1824, 1988.
7. Chao, T., Y. J. Xie, and Y. Y. Wang, "Fast solutions of wide-band RCS pattern of objects using MLFMM with the best uniform approximation," *Journal of Electronics & Information Technology*, Vol. 31, No. 11, 2772–2775, 2009.
8. Reddy, C. J., M. D. Deshpande, and C. R. Cockrell, "Fast RCS computation over a frequency band using method of moments in conjunction with asymptotic evaluation technique," *IEEE Trans. Antennas Propag.*, Vol. 46, No. 8, 1229–1233, 1998.
9. Wang, X., S. X. Gong, and J. L. Guo, "Fast and accurate wide-band analysis of antennas mounted on conducting platform using AIM and asymptotic waveform evaluation technique," *IEEE Trans. Antennas Propag.*, Vol. 59, No. 12, 4624–4633, 2011.
10. Nie, X. C., N. Yuan, L. W. Li, and Y. B. Gan, "Fast analysis of RCS over a frequency band using pre-corrected FFT/AIM and asymptotic waveform evaluation technique," *IEEE Trans. Antennas Propag.*, Vol. 56, No. 11, 3526–3533, 2008.
11. Prakash, V. V. S. and R. Mittra, "Characteristic basis function method: A new technique for efficient solution of method of moments matrix equations," *Microw. Opt. Technol. Lett.*, Vol. 36, No. 2, 95–100, 2003.
12. Degiorgi, M., G. Tiberi, and A. Monorchio, "An SVD-based method for analyzing electromagnetic scattering from plates and faceted bodies using physical optics bases," *IEEE Antennas and Propagation Society International Symposium*, 147–150, Jul. 2005.
13. Tanaka, T., Y, Inasawa, Y, Nishioka, and H. Miyashita, "Improved primary characteristic basic function method for monostatic radar cross section analysis of specific coordinate plane," *IEICE Transactions on Electronics*, Vol. E99-C, No. 1, 28–35, 2016.
14. Tanaka, T., Y. Inasawa, Y. Nishioka, and H. Miyashita, "Improved primary-characteristic basis function method considering higher-order multiple scattering," *IEICE Transactions on Electronics*, Vol. E100-C, No. 1, 45–51, 2017.
15. Tanaka, T., Y. Inasawa, Y. Nishioka, and H. Miyashita, "Accuracy improvement of characteristic basis function method by using multilevel approach," *IEICE Transactions on Electronics*, Vol. E101-C, No. 2, 96–103, 2018.
16. Li, C. L., Y. F. Sun, and G. H. Wang, "Merged characteristic basis function method for analysis of electromagnetic scattering characteristics from conducting targets," *Progress In Electromagnetics Research Letters*, Vol. 69, 15–21, 2017.
17. Maaskant, R., R. Mittra, and A. G. Tijhuis, "Fast analysis of large antenna arrays using the characteristic basis function method and the adaptive cross approximation algorithm," *IEEE Trans. Antennas Propag.*, Vol. 56, No. 11, 3440–3451, 2008.
18. Wang, X., D. H. Werner, and J. P. Turpin, "Investigation of scattering properties of large-scale aperiodic tilings using a combination of the characteristic basis function and adaptive integral methods," *IEEE Trans. Antennas Propag.*, Vol. 61, No. 6, 3149–3160, 2013.

19. Degiorgi, M., G. Tiberi, and A. Monorchio, "Solution of wide band scattering problems using the characteristic basis function method," *IET Microwaves Antennas and Propagation*, Vol. 6, No. 1, 60–66, 2012.
20. Nie, W. Y. and Z. G. Wang, "Solution for wide band scattering problems by using the improved ultra-wide band characteristic basis function method," *Progress In Electromagnetics Research Letters*, Vol. 58, 37–43, 2016.
21. Nie, W. Y. and Z. G. Wang, "Analysis of wide band scattering from objects using the adaptive improved ultra-wide band characteristic basis functions," *Progress In Electromagnetics Research Letters*, Vol. 60, 45–51, 2016.
22. Koc, S. N. and A. Köksal, "Wideband analysis of planar scalable antennas and PEC bodies using CBFM," *Turkish Journal of Electrical Engineering & Computer Sciences*, Vol. 24, 1652–1662, 2016.
23. Nie, W. Y. and Z. G. Wang, "Efficient computation of wideband RCS using singular value decomposition enhanced improved ultrawideband characteristic basis function method," *International Journal of Antennas and Propagation*, Vol. 2016, Article ID 6367205, 1–6, 2016.
24. Yeo, J., S. Köksoy, V. V. S. Prakash, and R. Mittra, "Efficient generation of method of moments matrices using the characteristic function method," *IEEE Trans. Antennas Propag.*, Vol. 52, No. 12, 3405–3410, 2004.