

# Systematic Detailed Design of Unequal-Split 3-Way Bagley Power Dividers Using Uniform Transmission Lines

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**Abstract**—In this paper, we systematically derive design equations for 3-way Bagley power dividers with arbitrary split ratios using interconnecting transmission lines with the same characteristic impedance. The exact value of the characteristic impedance for a specific dividing ratio is determined using these equations to achieve perfect input port matching. To validate the design procedure, two microstrip dividers with different split ratios, 1 : 3 : 1 and 1 : 10 : 1, are designed, simulated, fabricated, and measured. The desired split ratios are achieved at the design frequency, 1 GHz. Good agreement between simulated and measured results is obtained.

## 1. INTRODUCTION

Due to their significant importance in modern wireless systems, power dividers gained a noticeable attention in literature. Unequal-split power dividers are main components in RF/microwave subsystems, such as radar applications, amplifiers and antenna arrays beam-forming, where unequal energy is required at each antenna element. The Bagley power divider (BPD) is a multi-way equal-split divider that can be extended to any number of output ports [1]. The conventional BPD has interconnecting half-wavelength transmission lines (TLs) between its adjacent output ports, and quarter-wavelength TLs between its input port and adjacent output ports at the design frequency [1]. The large occupied area of this divider drove many researchers to build modified compact BPDs [2–5]. Dual-band BPDs were also investigated in many papers [6–9].

The *unequal-split* BPD appeared for the first time in [10], in which, 3 and 5-way unequal-split BPDs were proposed. The TLs between the adjacent ports in the proposed dividers in [10] have the length of quarter-wavelength at the design frequency. The characteristic impedances of these lines are determined using analytically derived equations to achieve a specific split ratio.

In [11], another approach to design unequal-split 3-way BPDs using uniform TLs was proposed, where the characteristic impedance is fixed for all TLs in the divider, and the lengths of these lines are set to obtain the desired dividing ratio. However, the main drawback of the proposed design equations in the latter method is their inability to determine the exact value of the fixed characteristic impedance to obtain dividers with perfect input port matching at the design frequency. In [12], using the same approach, the authors of [11] proposed 3- and 5-way unequal-split BPDs using uniform TLs, and with output ports terminated in different impedances.

In this paper, exact equations are systematically derived to design 3-way unequal-split BPDs with uniform TLs. The rest of this paper is arranged as follows: Section 2 presents the derived conditions for perfect input port matching and design equations for a specific dividing ratio; Section 3 presents simulation and measurements of the designed divider; and finally, conclusions and possible related future work are given in Section 4.

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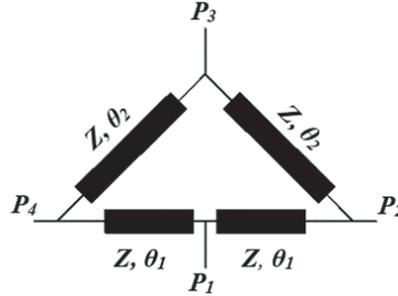
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## 2. THEORY AND DESIGN EQUATIONS

Figure 1 shows the schematic diagram of the 3-way unequal-split BPD with uniform TLs. The characteristic impedances of all TLs used in the divider are the same. The output powers at ports 2 and 4 are the same due to the design symmetry and all ports are terminated with  $Z_0$  ( $Z_0 = 50\Omega$ ). Firstly, expressions will be derived to find the possible values of  $Z$  that give zero reflection at port 1 (i.e.,  $S_{11} = 0$ ). Then, to split the input power between the output ports with a specific split ratio, expressions will be derived for the scattering parameters,  $S_{21}$  ( $S_{21} = S_{41}$ ) and  $S_{31}$ . The derived expressions will be then merged to obtain the final design equations for the 3-way unequal-split BPD with uniform TLs.



**Figure 1.** Schematic diagram of the 3-way unequal-split BPD with uniform TLs.

### 2.1. Input Port Matching Conditions

Figure 2 shows the equivalent circuit of the 3-way unequal-split BPD with uniform TLs.  $Z_{in}$  is the input impedance seen from port 1. In general,  $S_{11}$  is given as follows:

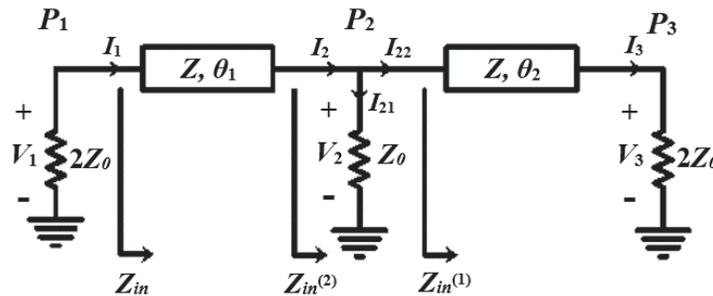
$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{\text{output ports are match-terminated}} \quad (1)$$

where  $V_1^+$  and  $V_1^-$  are the incident and the reflected voltages at port 1, respectively. From Figure 2,  $S_{11}$  can be written as follows:

$$S_{11} = \frac{Z_{in} - 2Z_0}{Z_{in} + 2Z_0} \quad (2)$$

$Z_{in}$  is given as follows:

$$Z_{in} = Z \frac{Z_{in}^{(2)} + jZ \tan(\theta_1)}{Z + jZ_{in}^{(2)} \tan(\theta_1)} \quad (3)$$



**Figure 2.** Equivalent circuit of the 3-way unequal-split BPD with uniform TLs.

where  $Z_{in}^{(2)} = Z_{in}^{(1)} // Z_o$ , and  $Z_{in}^{(1)}$  can be obtained by the following expression:

$$Z_{in}^{(1)} = Z \frac{2Z_o + jZ \tan(\theta_2)}{Z + j2Z_o \tan(\theta_2)} \quad (4)$$

After some mathematical manipulations, it can be shown that the following conditions must hold to get a perfect match at port 1:

$$\tan^2(\theta_1) = \frac{K^2 + 1}{K^2 - 3K^4} \quad (5a)$$

$$\tan(\theta_2) = \frac{-1}{K^2 \tan(\theta_1)} \quad (5b)$$

where  $K = Z/2Z_o$ . Equations (5) are considered as the input port matching conditions for the 3-way BPD with uniform TLs. Equation (5b) states that the TLs electrical lengths,  $\theta_1$  and  $\theta_2$ , should be in different quadrants for this divider (a fact that has not been stressed in [11]). These equations will be used to determine the exact  $K$  that should be chosen.

## 2.2. Transmission Parameters

Now, expressions will be derived for  $S_{21}$  and  $S_{31}$  to be used to obtain the desired split ratio. Referring to Figure 2, the relation between the voltages and currents at ports 1 and 2 can be written as follows:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_1) & jZ \sin(\theta_1) \\ jZ^{-1} \sin(\theta_1) & \cos(\theta_1) \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad (6)$$

The relation between  $V_1$  and  $I_1$  is:

$$I_1 = \frac{V_1}{2Z_o} \quad (7)$$

Substituting (7) in (6) yields to:

$$\frac{V_2}{V_1} = \cos(\theta_1) - jK \sin(\theta_1) \quad (8)$$

$S_{21}$  can be obtained as follows:

$$S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{\text{ports are match-terminated}} \quad (9)$$

But,  $V_2^- = V_2$  since port 2 is matched terminated. Also,  $V_1^+ = V_1$  since  $S_{11} = 0 (V_1^- = 0)$ . Therefore:

$$S_{21} = \frac{V_2}{V_1} = \cos(\theta_1) - jK \sin(\theta_1) \quad (10)$$

Similarly, the equations between ports 2 and 3 are given as follows:

$$\begin{bmatrix} V_2 \\ I_{22} \end{bmatrix} = \begin{bmatrix} \cos(\theta_2) & jZ \sin(\theta_2) \\ jZ^{-1} \sin(\theta_2) & \cos(\theta_2) \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} \quad (11)$$

where:

$$I_3 = \frac{V_3}{2Z_o} \quad (12)$$

Substituting Eq. (12) in Eq. (11) yields to:

$$\frac{V_3}{V_2} = \frac{1}{\cos(\theta_2) + jK \sin(\theta_2)} \quad (13)$$

Therefore,  $S_{31}$  is obtained as follows:

$$S_{31} = \frac{V_3}{V_2} \times \frac{V_2}{V_1} = \frac{\cos(\theta_1) - jK \sin(\theta_1)}{\cos(\theta_2) + jK \sin(\theta_2)} \quad (14)$$

For a unit incident power at port 1, the powers at the output ports are given as follows:

$$P_2 = P_4 = |S_{21}|^2 = |S_{41}|^2 = \cos^2(\theta_1) + K^2 \sin^2(\theta_1) = M \quad (15a)$$

$$P_3 = |S_{31}|^2 = \frac{\cos^2(\theta_1) + K^2 \sin^2(\theta_1)}{\cos^2(\theta_2) + K^2 \sin^2(\theta_2)} = N \quad (15b)$$

Even though there was a small mistake in the proposed expressions for  $S_{21}$  and  $S_{31}$  in [11], their expressions for the output powers are similar to the derived ones above in Eqs. (15). However, these equations are not enough to determine the proper value for  $K$  to obtain perfect input port matching.

### 2.3. Design Equations

Assuming a lossless divider, at the design frequency, one can rewrite Equations (15) as follows:

$$|S_{21}|^2 + |S_{31}|^2 + |S_{41}|^2 = 1 \quad (16a)$$

$$N = 1 - 2M \quad (16b)$$

$$\frac{M}{\cos^2(\theta_2) + K^2 \sin^2(\theta_2)} = 1 - 2M \quad (16c)$$

The derived input port matching conditions in Eq. (5) are substituted in Eq. (16c) to find  $K$  given  $M$ . After some mathematical manipulations, the following expression can be used to find  $K$ :

$$K = \sqrt{\frac{M}{2 - 3M}} \quad (17)$$

It is worth mentioning that, in [11, 12], it seems that the value of  $K$  was chosen by trial-and-error, thus, a perfect match at the design frequency was not obtained. As a summary, the general design steps for a 3-way BPD with uniform TLs and a split ratio of  $P_2 : P_3 : P_4$  are as follows:

- 1) Compute  $M$  as follows:

$$M = \frac{P_2}{2P_2 + P_3}$$

- 2) Get the proper  $K$  using:

$$K = \sqrt{\frac{M}{(2 - 3M)}}$$

where  $K = Z/2Z_0$ .

- 3) Get  $\theta_1$  using:

$$\tan(\theta_1) = \pm \sqrt{\frac{K^2 + 1}{K^2 - 3K^4}}$$

- 4) Get  $\theta_2$  using:

$$\tan(\theta_2) = \frac{-1}{K^2 \tan(\theta_1)}$$

Step 1 shows that  $M < 1/2$  for any split ratio, consequently, from step 2,  $K < 1$ . Based on this constraint for BPDs with uniform TLs, Equation (16c) can be written as follows:

$$\frac{M}{1 - 2M} = \frac{M}{N} = \cos^2(\theta_2) + K^2 \sin^2(\theta_2) < 1 \quad (18)$$

Equation (18) states that BPDs with uniform TLs can be designed only when  $M < N$  (i.e.,  $P_2 = P_4 < P_3$ ). When the output power at port 2 or 4 is desired to be higher than the one at port 3, BPDs with uniform TLs cannot be designed.

Steps 3 and 4 show that if  $\theta_1$  is chosen to be in the first quadrant (Q1),  $\theta_2$  should be in the second one (Q2), and vice versa. Table 1 summarizes the parameters of uniform TLs BPDs with different split ratios. As can be noticed, choosing  $\theta_1$  to be in the second quadrant, and  $\theta_2$  in the first quadrant gives smaller overall divider size.

Table 1. Parameters of BPDs with uniform TLs.

Split Ratio ( $P_2: P_3: P_4$ )	$K$	$\theta_1^{Q1}$ ( $^\circ$ )	$\theta_2^{Q2}$ ( $^\circ$ )	$\theta_1^{Q1} + \theta_2^{Q2}$ ( $^\circ$ )	or	$\theta_1^{Q2}$ ( $^\circ$ )	$\theta_2^{Q1}$ ( $^\circ$ )	$\theta_1^{Q2} + \theta_2^{Q1}$ ( $^\circ$ )
1 : 2 : 1	0.447	75.5	127.8	203.3		104.5	52.2	156.7
1 : 3 : 1	0.377	75.0	118.3	193.3		105.0	61.7	166.7
1 : 5 : 1	0.301	76.2	110.3	186.5		103.8	69.7	173.5
1 : 10 : 1	0.218	78.8	103.6	182.4		101.2	76.4	177.6
1 : 15 : 1	0.179	80.5	100.0	180.5		99.50	80.0	179.5

Figure 3 shows the theoretical input-output ports matching parameters,  $S_{11}$ ,  $S_{22}$  ( $S_{22} = S_{44}$ ) and  $S_{33}$ , obtained using the ADS circuit simulator for 1 GHz BPDs with uniform TLs and different split ratios based on the derived equations. Figure 4 shows the output ports isolation parameters,  $S_{23}$  ( $S_{23} = S_{34}$ ) and  $S_{24}$ .

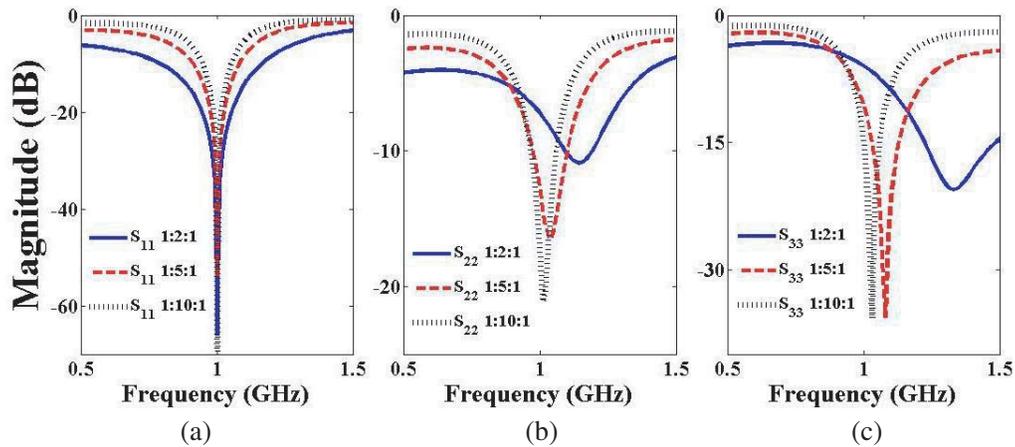


Figure 3. Circuit-simulated input-output ports matching parameters, (a)  $S_{11}$ , (b)  $S_{22}$  ( $S_{22} = S_{44}$ ) and (c)  $S_{33}$ , for 1 GHz BPDs with uniform TLs and different split ratios.

Perfect input port matching is obtained for all dividers as shown in Figure 3(a). As the dividing ratio becomes higher, matching at the output ports gets better as shown in Figures 3(b) and (c). This can be explained by noticing that  $\theta_1$  becomes closer to  $\theta_2$  as the split ratio increases. Thus, the input impedance seen from the output port becomes closer to the one seen from port 1 (the perfectly matched port), especially from port 3, which is on the opposite side of the input port. Figure 4 shows that ports 2 and 3 get more isolated for BPDs with higher split ratios, whereas poor isolation between ports 2 and 4 can be noticed for the BPD.

### 3. DESIGN AND MEASUREMENTS

Two BPDs with uniform TLs are designed, simulated, fabricated, and measured. The operating frequency of the two dividers is 1 GHz and the FR-4 substrate is considered ( $\epsilon_r = 4.4$  and thickness = 1.5 mm). Figure 5 shows the layout of the designed 1 : 3 : 1 BPD with uniform TLs and a photograph of the fabricated prototype.  $\theta_1$  and  $\theta_2$  are set in the second and the first quadrants, respectively, to attain compactness.

Figure 6 shows the theoretical  $S$ -parameters of the designed divider along with full-wave simulated ones (obtained using HFSS) and measured results using an E5071C ENA Keysight vector network analyzer. Full-wave simulation results show an input port matching of  $-19$  dB at the design frequency.

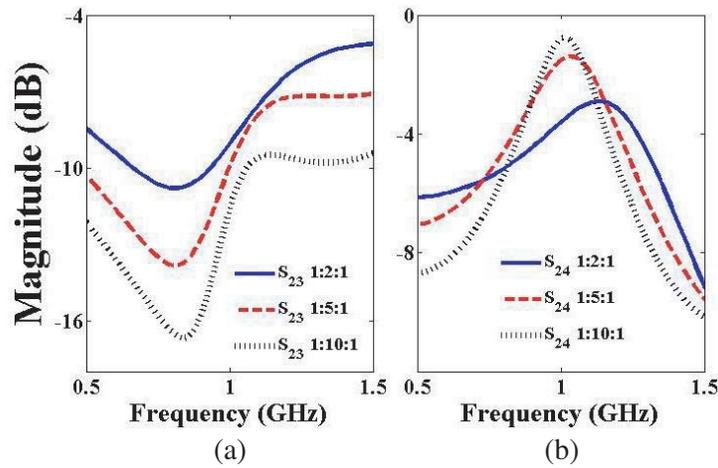


Figure 4. Circuit-simulated output ports isolation parameters, (a)  $S_{23}$  ( $S_{23} = S_{34}$ ) and (b)  $S_{24}$ , for 1 GHz BPDs with uniform TLs and different split ratios.

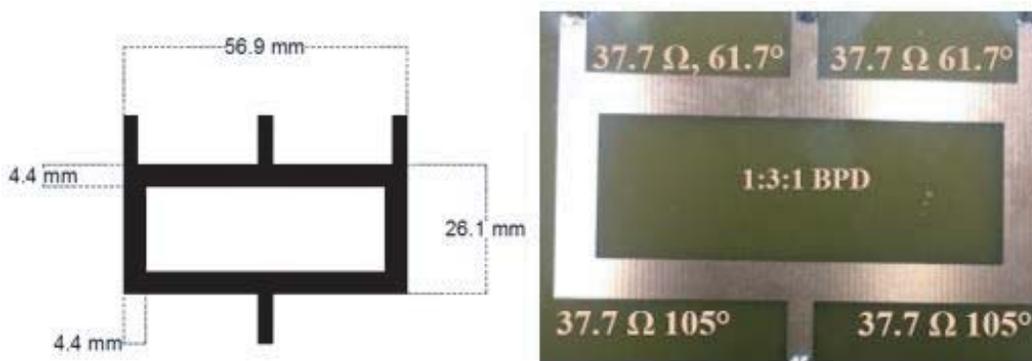


Figure 5. Layout and a photograph of 1 : 3 : 1 BPD using uniform TLs.

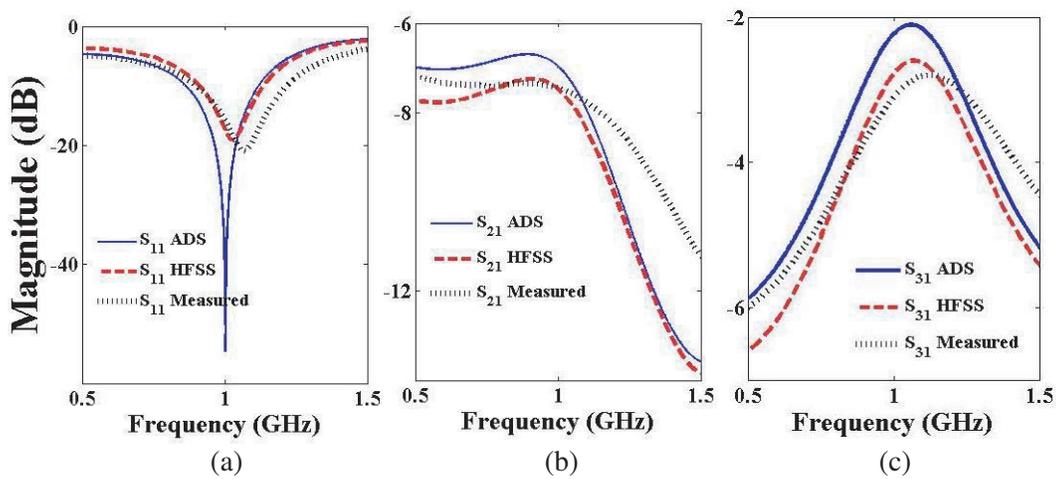


Figure 6. Theoretical (ADS), simulated (HFSS) and measured  $S$ -parameters, (a)  $S_{11}$ , (b)  $S_{21}$  ( $S_{21} = S_{41}$ ) and (c)  $S_{31}$ , for 1 GHz BPD with uniform TLs and 1 : 3 : 1 split ratio.

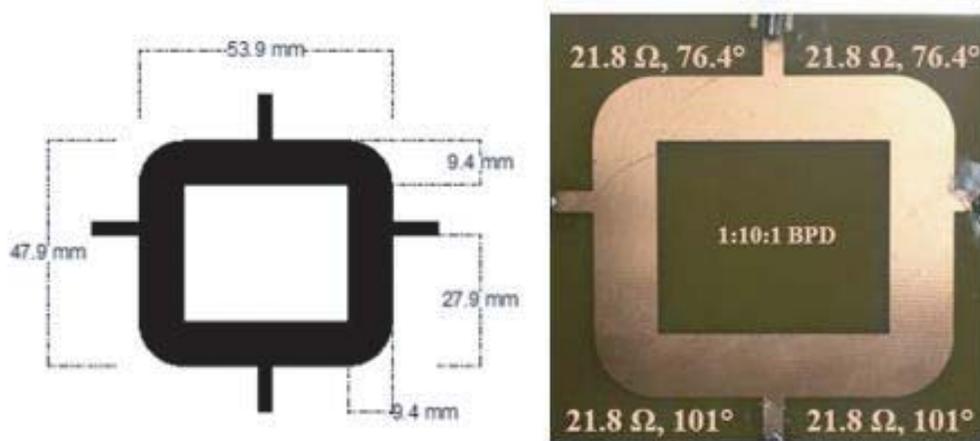


Figure 7. Layout and a photograph of 1 : 10 : 1 BPD using uniform TLs.

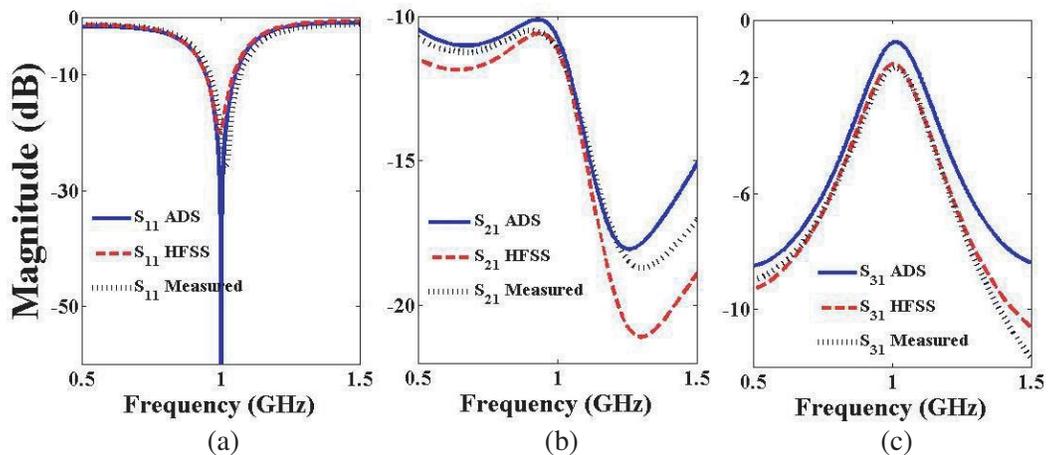


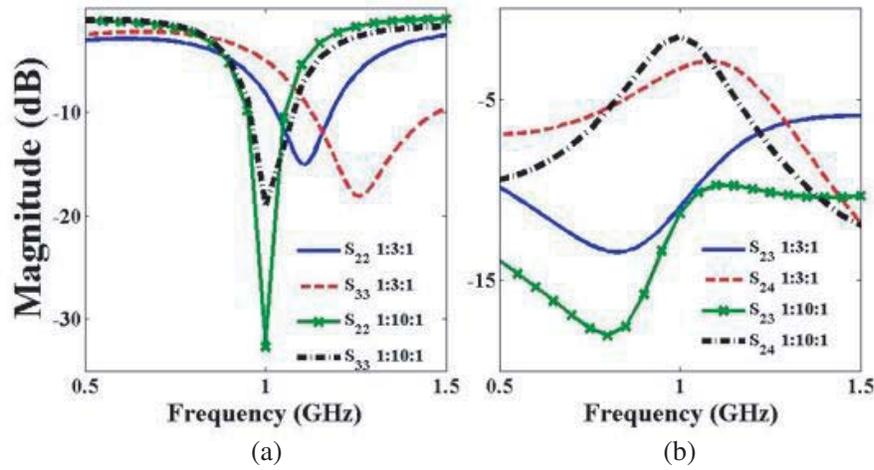
Figure 8. Theoretical, simulated and measured  $S$ -parameters, (a)  $S_{11}$ , (b)  $S_{21}$  ( $S_{21} = S_{41}$ ) and (c)  $S_{31}$ , for 1 GHz BPD with uniform TLs and 1 : 10 : 1 split ratio.

The transmission parameters,  $S_{21}$  ( $S_{21} = S_{41}$ ) and  $S_{31}$ , have the magnitudes of  $-7.5$  dB and  $-2.7$  dB at 1 GHz, respectively. Such values are considered close to the theoretical ones ( $S_{21} = S_{41} = -7$  dB,  $S_{31} = -2.22$  dB). Measured results match the simulated ones with small difference due to conductor losses and the soldering of connectors to the lines.

Figure 7 shows the layout and a photograph of the designed 1 : 10 : 1 BPD with uniform TLs. Simulation and measurement results are shown in Figure 8. Simulated  $S$ -parameters,  $S_{11}$ ,  $S_{21}$ , and  $S_{31}$ , have the values of  $-20$  dB,  $-11.2$  dB, and  $-1.5$  dB at the design frequency, respectively. The theoretical values of the 1 : 10 : 1 BPD transmission parameters are  $S_{21} = -10.8$  dB and  $S_{31} = -0.8$  dB. A very good agreement is observed between the measured and the simulated results. Figure 9 shows output ports matching and isolation parameters of the two dividers.

At 1 GHz,  $S_{22}$  and  $S_{33}$  have the magnitudes of  $-8$  dB and  $-5$  dB, respectively, for the 1 : 3 : 1 divider, whilst, for the 1 : 10 : 1 BPD, they have the magnitudes of  $-32$  dB and  $-19$ , respectively. Output ports isolation parameters,  $S_{23}$  and  $S_{24}$ , are around  $-10$  dB and  $-3$  dB, respectively, for the designed dividers.

BPDs with uniform TLs have shorter total TLs electrical length than the one obtained using the quarter-wavelength TLs method proposed in [10]. Moreover, microwave components with uniform TLs are easier to fabricate and may have less discontinuities than the components with thick and thin lines.



**Figure 9.** (a) Output ports matching parameters,  $S_{22}$  and  $S_{33}$ , and (b) isolation parameters,  $S_{23}$  and  $S_{24}$ , of the designed dividers.

#### 4. CONCLUSIONS

In this paper, exact expressions were systematically derived to design 3-way unequal-split BPDs with uniform TLs and perfect input port matching. 1 : 3 : 1 and 1 : 10 : 1 BPDs were designed based on the derived equations to operate at 1 GHz. Simulated  $S$ -parameters,  $S_{11}$ ,  $S_{21}$ , and  $S_{31}$ , had the magnitudes of  $-19$  dB,  $-7.5$  dB, and  $-2.7$  dB at the design frequency for the 1 : 3 : 1 BPD, and the values of  $-20$  dB,  $-11.2$  dB and  $-1.5$  dB for the 1 : 10 : 1 divider. Moreover, the designed dividers were fabricated and tested. Simulated and measured results were in good agreement and close to the theoretical ones.

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#### REFERENCES

1. Pozar, D., *Microwave Engineering*, 3rd Edition, John Wiley, New York, 2005.
2. Sakagami, I., T. Wuren, M. Fujii, and M. Tahara, "Compact multi-way power dividers similar to the Bagley polygon," *Proceedings of the IEEE MTT-S International Microwave Symposium, (IMS '07)*, Hawaii, 2007.
3. Sakagami, I., T. Wuren, M. Fujii, and Y. Tomoda, "A new type of multi-way microwave power divider based on Bagley polygon power divider," *Asia-Pacific Microwave Conference (APMC) Proc.*, Yokohama, 2006.
4. Wuren, T., K. Taniya, I. Sakagami, and M. Tahara, "Miniaturization of 3- and 5-way Bagley polygon power dividers," *Proceedings of Asia-Pacific Microwave Conference (APMC)*, Dec. 2005.
5. Oraizi, H. and S. Ayati, "Optimum design of a modified 3-way Bagley rectangular power divider," *Mediterranean Microwave Symposium (MMS)*, Morocco, 2010.
6. Elles, D. and Y. Yoon, "Compact dual band three-way Bagley polygon power divider using composite right/left handed (CRLH) transmission lines," *Proceedings of the IEEE MTT-S International Microwave Symposium (IMS '09)*, Vancouver, Canada, 2009.
7. Sakagami, I. and T. Wuren, "Compact multi-way power dividers for dual-band, wide-band and easy fabrication," *Proceedings of the IEEE MTT-S International Microwave Symposium (IMS'09)*, Boston, 2009.

8. Al-Shamaileh, K., A. Qaroot, and N. Dib, "Non-uniform transmission line transformers and their applications in the design of compact multi-band Bagley power dividers with harmonics suppression," *Progress In Electromagnetics Research*, Vol. 113, 269–284, 2011.
9. Qaroot, A., K. Shamaileh, and N. Dib, "Design and analysis of dual-frequency modified 3-way Bagley power dividers," *Progress In Electromagnetics Research C*, Vol. 20, 67–81, 2011.
10. Abu Al-Nadi, O., N. Dib, and K. Al-Shamaileh, "Design and analysis of unequal split Bagley power dividers," *International Journal of Electronics*, Vol. 102, No. 3, 500–513, Apr. 2014.
11. Yoon, Y.-C. and Y. Kim, "Unequal Bagley power divider using uniform transmission lines," *2017 Progress In Electromagnetics Research Symposium — Spring (PIERS)*, 1184–1187, St. Petersburg, Russia, May 22–25, 2017.
12. Yoon, Y.-C. and Y. Kim, "Bagley power divider with uniform transmission lines for arbitrary power ratio and terminated in different impedances," *Progress In Electromagnetics Research C*, Vol. 77, 195–203, 2017.