

Computing with Large Time Steps for Electromagnetic Wave Propagation in Multilayered Homogeneous Media

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Abstract—We present an extension of Large Time Step (LTS) method to electromagnetic wave propagation involving multilayered homogeneous media. The LTS method proposed by LeVeque is an extension of Godunov’s method for the numerical solution of hyperbolic conservation laws. In this method, very large time steps are allowed by an increase in the numerical domain of dependence compared to conventional explicit methods constrained by the Courant-Friedrichs-Lewy stability criteria. This can lead to additional complexities when being applied to multilayered homogeneous media due to presence of material interfaces. Appropriate treatment of material interface boundaries is proposed in the present work in the context of finite volume time-domain method with LTS. Numerical examples are presented involving solution of time-domain Maxwell’s equations in a layered dielectric medium using LTS approach.

1. INTRODUCTION

The classical LTS method was proposed by LeVeque [1] to speed up the solution of nonlinear hyperbolic conservation laws like the Euler equations in gas dynamics [2–4] and shallow water equations in fluid mechanics [5–9] when being solved in a Godunov based finite volume framework. The LTS approach assumes waves arising out of solving Riemann problems at cell interfaces in a Godunov method to behave linearly. This linear behaviour allows superposition of waves to be used to update downwind cell values. This results in the Courant-Friedrichs-Lewy (CFL) criterion restriction on the explicit time step due to numerical stability considerations being bypassed. The large value of time step Δt , due to CFL number (ν) $\gg 1$ in LTS method, can considerably speed up numerical simulations involving hyperbolic waves even with the increased per-grid-point computational load due to inclusion of more Riemann waves for updating grid values. The time-domain Maxwell’s equations in finite difference time-domain (FDTD) or finite volume time-domain (FVTD) context can be computationally very expensive if the explicit time step Δt is small due to fine meshes resulting from point-per-wavelength (PPW) requirements.

The LTS method was extended to the solution of time-domain Maxwell’s equations, in free space in a FVTD framework in [10]. It was shown that Δt (and ν) can be unconditionally large for 1D problems given that the time-domain Maxwell’s equations constitute a system of linear hyperbolic conservation laws. Fewer time steps in the LTS method also result in enhanced accuracy because of reduced discretization errors. Extension to multidimensions through operator splitting is also unconditionally stable using LTS, but the accuracy may degrade for $\nu \gg 1$. This is attributed to splitting error arising from the fact that Jacobian matrices in the multidimensional form of the time-domain Maxwell’s equations do not commute [11].

In the present work, we extend the LTS method in an FVTD framework to multilayered homogeneous media. We consider perfect dielectrics in a 1D domain. Dielectric slabs result in material interfaces which require special treatment in the LTS framework. Material interfaces can separate

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material of constant or variable impedance. Propagating waves resulting from Riemann problem in the LTS method, encountering a constant impedance material interface, continue to update cells only in downwind direction of the interface. But the range of downwind cells is now dictated by the new wave speed downwind of the interface. Riemann waves encountering dielectric interface of variable impedance, on the other hand, produce reflected and transmitted waves at the interface. The amplitudes of the waves depend on reflection and transmission coefficients based on satisfying conservation at the material interface. Riemann waves encountering such an interface update both upwind and downwind cells to the interface. The range of cells, in both directions to be updated, depend on the local wave speed. Appropriate interface boundary conditions will form a part of the LTS framework involving layered media consisting of perfect dielectrics considered in this work. In a simple 1D domain Riemann waves, in-principle, can cross multiple dielectric interfaces in a single time step in an LTS method. This would be difficult to implement for practical problems in multidimension involving variable impedance media. The 1D problem solved here can also provide a condition for minimum thickness of dielectric slabs that can be efficiently solved using LTS for variable impedance media in more practical problems.

2. GOVERNING EQUATIONS

The time-domain Maxwell equations in differential form for a linear medium can be written as

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (1a)$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \quad (1b)$$

where \mathbf{E} is the electric field vector, \mathbf{H} the magnetic field vector, \mathbf{D} the electric flux density, \mathbf{B} the magnetic flux density, and \mathbf{J} the electric current density.

The constitutive relation for homogeneous and non-dispersive medium are given by

$$\mathbf{D} = \epsilon \mathbf{E} = \epsilon_0 \epsilon_r \mathbf{E}, \quad (2a)$$

$$\mathbf{B} = \mu \mathbf{H} = \mu_0 \mu_r \mathbf{H} \quad (2b)$$

where ϵ is the electric permittivity and μ the magnetic permeability of the medium. In Equation (2), $\epsilon_0 = 8.854 \times 10^{-12}$ (farad/meter) and $\mu_0 = 4\pi \times 10^{-7}$ (henrys/meter) are free space electric permittivity and magnetic permeability, while ϵ_r and μ_r are relative permittivity and permeability of the medium. $\epsilon_r = 1$ and $\mu_r = 1$ for free space. We initially present an overview of the LTS formulation for the solution of 1D Maxwell's equations in homogeneous medium. This formulation is then extended to multilayered homogeneous media and numerical examples solved.

3. LTS METHOD IN HOMOGENEOUS MEDIUM

The source free x -directed and y -polarized 1D Maxwell's equations are given by

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\epsilon} \frac{\partial H_z}{\partial x} \quad (3a)$$

$$\frac{\partial H_z}{\partial t} = -\frac{1}{\mu} \frac{\partial E_y}{\partial x} \quad (3b)$$

For the solution of Equation (3) using the LTS approach in an FVTD framework, the equations are written in conservative form as

$$\frac{\partial \mathbf{u}}{\partial t} + A \frac{\partial \mathbf{u}}{\partial x} = 0 \quad (4)$$

where conserved variable \mathbf{u} and Jacobian matrix $A = \partial \mathbf{f}(\mathbf{u})/\partial x$ are given as

$$\mathbf{u} = \begin{bmatrix} E_y \\ H_z \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 0 & 1/\epsilon \\ 1/\mu & 0 \end{bmatrix}. \quad (5)$$

The eigenvalues and eigenvectors matrix of A are

$$\Lambda = \begin{bmatrix} -c & 0 \\ 0 & c \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} -Z & Z \\ 1 & 1 \end{bmatrix} \quad (6)$$

where $c = 1/\sqrt{\mu\epsilon}$ is the wave speed, and $Z = \sqrt{\mu/\epsilon}$ is the impedance of the medium. In an FVTD framework [11], the spatial domain is subdivided into a number of finite volumes (cells), and over each cell the conservative form of Maxwell's equations (4) is solved in an integral form. For a finite volume $v_i = [x_{i-1/2}, x_{i+1/2}]$, the integral form of Eq. (4) over a time interval t_n to t_{n+1} can be written in a cell centered discrete form as

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \frac{\Delta t}{\Delta x} \left(A^+ \Delta \mathbf{U}_{i-1/2}^n + A^- \Delta \mathbf{U}_{i+1/2}^n \right) \quad (7)$$

where \mathbf{U}_i^n is the approximated cell average value of \mathbf{u} over the cell interval $[x_{i-1/2}, x_{i+1/2}]$ at time t_n , and $A^\pm \Delta \mathbf{U}_{i\mp 1/2}^n$ is the numerical flux function at cell interface $x_{i\mp 1/2}$. A^\pm can be defined as $A^\pm = R\Lambda^\pm R^{-1}$ where, $\Lambda^\pm = \text{diag}(\lambda^\pm)$, and $\Delta \mathbf{U}_{i-1/2} = \mathbf{U}_i - \mathbf{U}_{i-1}$. $\lambda^+ = \max(\lambda, 0)$, $\lambda^- = \min(\lambda, 0)$ where λ are the eigenvalues of the Jacobian matrix A as in Equation (6) and $\Lambda = \Lambda^+ + \Lambda^-$. For the solution of Equation (7) using a wave propagation form of LeVeque's LTS method, the variable \mathbf{U}_i is updated at each time step by incorporating the solution of Riemann problem at each cell interface of the upwind grid cells [3, 12]. The waves arising out of a Riemann problem at an interface $x_{i-1/2}$ is written as [11]

$$\mathbf{U}_i^n - \mathbf{U}_{i-1}^n = \sum_{p=1}^m \alpha_{i-1/2}^p \mathbf{r}^p = \sum_{p=1}^m \Delta \mathbf{u}_{i-1/2}^p \quad (8)$$

The 1D problem in Equation (3) has $m = 2$ with wave speeds $(-c, c)$. $\alpha_{i-1/2}^p$ are the scalar coefficient or amplitudes of eigenvectors \mathbf{r}^p , which represents the linear waves in the eigenvector expansion in Equation (8), and are expressed as

$$\alpha_{i-1/2}^1 = \frac{-(E_y|_i - E_y|_{i-1}) + Z(H_z|_i - H_z|_{i-1})}{2Z} \quad (9a)$$

$$\alpha_{i-1/2}^2 = \frac{(E_y|_i - E_y|_{i-1}) + Z(H_z|_i - H_z|_{i-1})}{2Z} \quad (9b)$$

where $E_y|_i$ is the field E_y defined at the i th grid point. In homogeneous medium, the p th wave propagates with wave speed λ^p and propagates a distance $|\lambda^p|\Delta t$ in each time step Δt . Thus, if the p th wave propagates an entire cell interval $[x_{i-1/2}, x_{i+1/2}]$ the cell average gets incremented by a quantity $\alpha_{i-1/2}^p \mathbf{r}^p$. If the wave traverses a part of the cell, a fraction of $\alpha_{i-1/2}^p \mathbf{r}^p$ contributes to increment the cell average [3, 11, 12]. As shown in Figure 1, at each time step, the p th wave contributes to $\lceil \nu \rceil$ downwind grid cells by updating the cell averages, where $\nu = \lambda_{\max}|\Delta t/\Delta x$ is the CFL number, and $\lceil \nu \rceil$ maps ν to the least integer $\geq \nu$. λ_{\max} is the maximum wave speed [12]. The classical LTS method due to LeVeque is also shown in the form of a flowchart in Figure 2. In the flowchart $\lceil \nu \rceil$ is the integer counterpart of ν .

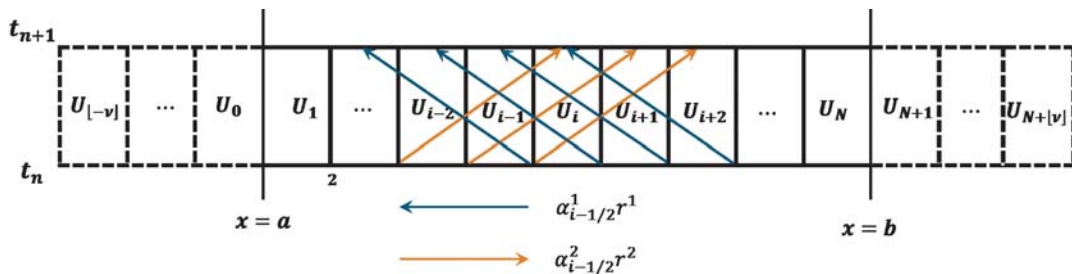


Figure 1. Wave propagation in computational domain using LTS (schematic).

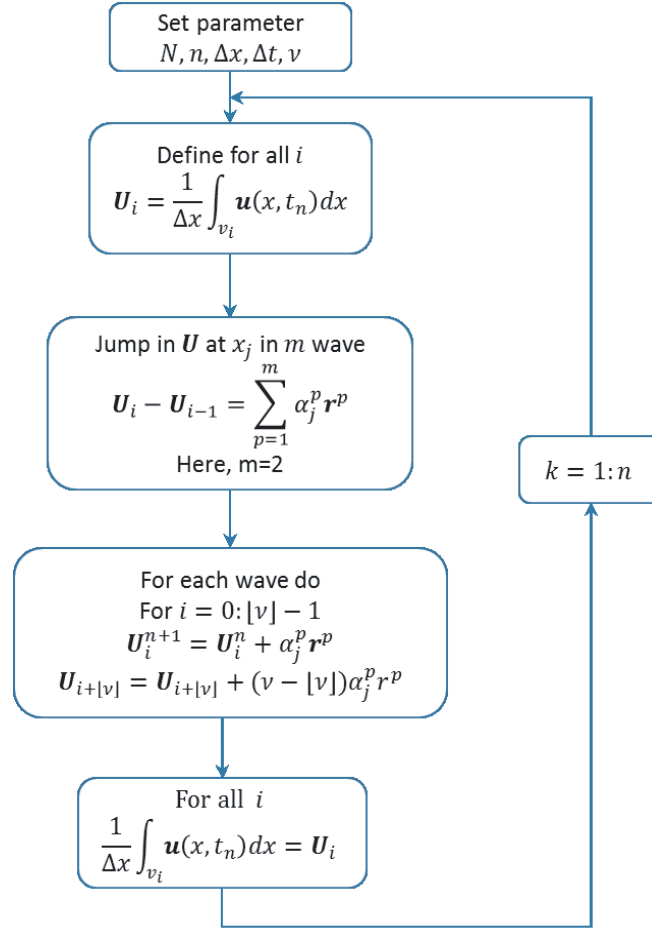


Figure 2. LTS algorithm, homogeneous medium.

4. EXTENSION OF THE LTS METHOD TO MULTILAYERED HOMOGENEOUS MEDIA

For multilayered homogeneous media, the permittivity ϵ and permeability μ are considered constant in a finite volume cell but can vary across cell faces. Equation (4) is now written as

$$\frac{\partial \mathbf{u}}{\partial t} + A(x) \frac{\partial \mathbf{u}}{\partial x} = 0 \quad (10)$$

where,

$$A(x) = \begin{bmatrix} 0 & 1/\epsilon(x) \\ 1/\mu(x) & 0 \end{bmatrix}.$$

The eigenvalues and eigenvectors of Jacobian matrix $A(x)$ are given by

$$\Lambda(x) = \begin{bmatrix} -c(x) & 0 \\ 0 & c(x) \end{bmatrix} \quad \text{and} \quad R(x) = \begin{bmatrix} -Z(x) & Z(x) \\ 1 & 1 \end{bmatrix} \quad (11)$$

where wave speed $c(x)$ and impedance $Z(x)$ are defined as

$$c(x) = \frac{1}{\sqrt{\mu(x)\epsilon(x)}} \quad \text{and} \quad Z(x) = \sqrt{\frac{\mu(x)}{\epsilon(x)}}. \quad (12)$$

For the solution of Eq. (10) in an FVTD framework, \mathbf{u} is approximated over each grid cell and similarly updated by solving the Riemann problems at each interface of the grid cell. The wave speed

and impedance in each grid cell i are $c_i = 1/\sqrt{\mu_i\epsilon_i}$ and $Z_i = \sqrt{\mu_i/\epsilon_i}$ with ϵ_i and μ_i the cell-wise values. For the conventional Godunov’s method ($\nu \leq 1$), a Riemann problem can straddle a material interface and influence evolution of the solution only in constituent cells containing that interface. The solution to the Riemann problem would then consist of single wave moving into either material for the 1D problem [11]. In the LTS framework multiple left and right moving waves can encounter a material interface. Interaction with the interface can create transmitted and reflected waves which will appropriately affect the domain traversed by the waves for $\nu > 1$.

4.1. Constant Impedance Media

A special case of constant impedance $Z(x) = Z$ arises when the relation between $\mu(x)$ and $\epsilon(x)$ in the entire region of interest is $\mu(x) = Z^2\epsilon(x)$. In such cases, the eigenvectors $\mathbf{r}^p(x) = \mathbf{r}^p$ are constant in space. Hence, waves in Riemann problems are defined identically as in Eq. (8) with the strength of the waves $\alpha_{i-1/2}^p$ as in Eq. (9). The matrix of eigenvalues $\Lambda(x)$ which represent wave speeds varies spatially with x for the multilayered homogeneous media. The wave speed in the Riemann problem at an arbitrary cell interface $x_{i-1/2}$ can be defined in an upwind sense as $\lambda_{i-1/2}^1 = -c_{i-1}$ and $\lambda_{i-1/2}^2 = +c_i$. As shown in Figure 3, if the p th wave propagates entirely either in medium 1 or 2, with a wave speed $\lambda_{i-1/2}^p$ in time Δt , then the wave propagates $\lambda_{i-1/2}^p\Delta t$ distance and contributes to $[\nu_i]$ downwind grid cells to update cell averages, with ν_i defined as $|\lambda_{i-1/2}^p|\Delta t/\Delta x$. On the other hand, if the p th wave with wave speed $\lambda_{i-1/2}^p$ encounters a material interface after propagating $k < [\nu_i]$ downwind grid cells, the wave transmits completely from one layer to another but with a change in wave speed $\lambda_{i-1/2}^{p'} = \eta\lambda_{i-1/2}^p$. For a right running wave ($p = 2$), $\eta = \sqrt{\mu_1\epsilon_1/\mu_2\epsilon_2}$ is the refractive index between two layers. Due to change in speed of wave to $\lambda_{i-1/2}^{p'}$, the wave propagates $(k + \eta(\nu_i - k))\Delta x$ distance and updates cell averages in a total of $[k + \eta(\nu_i - k)]$ downwind cells instead of $[\nu_i]$ cells in time Δt ($\nu > 1$). In this case the entire wave is transmitted from one layer to another with compression/expansion of the original profile.

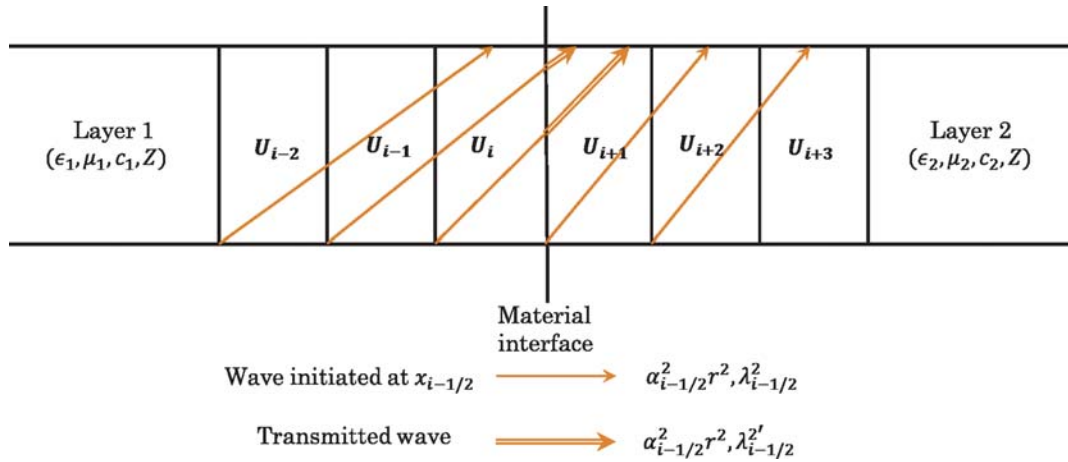


Figure 3. Right moving wave propagation in constant Z computational domain using LTS (schematic).

4.2. Variable Impedance Media

In the case of a change in impedance, along with the speed $c(x)$, the eigenvector matrix $R(x)$ suffers from a change across a material interface. The eigenvectors at an arbitrary cell interface $x_{i-1/2}$ can again be defined as

$$r_{i-1/2}^1 = \begin{bmatrix} -Z_{i-1} \\ 1 \end{bmatrix} \quad \text{and} \quad r_{i-1/2}^2 = \begin{bmatrix} Z_i \\ 1 \end{bmatrix}. \quad (13)$$

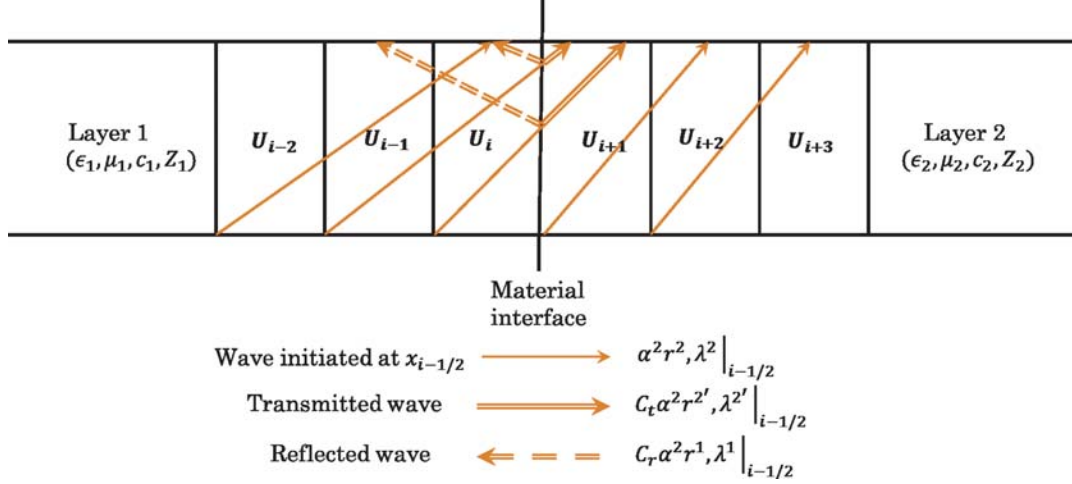


Figure 4. Right moving wave propagation in variable Z computational domain using LTS (schematic).

With these spatially varying eigenvectors $\mathbf{r}_{i-1/2}^p$, local waves in each Riemann problem are defined as previously in Equation (8), and the strength of waves $\alpha_{i-1/2}^p$ can be written as [11]

$$\alpha_{i-1/2}^1 = \frac{-(E_y|_i - E_y|_{i-1}) + Z_i(H_z|_i - H_z|_{i-1})}{Z_{i-1} + Z_i} \quad (14a)$$

$$\alpha_{i-1/2}^2 = \frac{(E_y|_i - E_y|_{i-1}) + Z_{i-1}(H_z|_i - H_z|_{i-1})}{Z_{i-1} + Z_i}. \quad (14b)$$

As shown in Figure 4, if the p th wave originating from the Riemann problem at $x_{i-1/2}$ propagates either entirely in medium 1 or 2, in time Δt with wave speed $\lambda_{i-1/2}^p$, then the p th wave updates $[\nu_i]$ downwind grid cells with $\alpha_{i-1/2}^p \mathbf{r}_{i-1/2}^p$ as in homogeneous medium or constant impedance case. However, if the p th wave interacts with a material interface after propagating $k < [\nu_i]$ downwind grid cells during time Δt only a part of the incident wave is transmitted while the rest is reflected back from the material interface. This is in order to maintain conservation when a jump in impedance is encountered. The strength (amplitude) of transmitted and reflected wave are given by $C_t \alpha_{i-1/2}^p$ and $C_r \alpha_{i-1/2}^p$, respectively. The transmission coefficient C_t and reflection coefficient C_r for a right moving wave ($p = 2$) originating at an arbitrary interface $x_{i-1/2}$ and encountering a material interface of refractive index $\eta = \sqrt{\mu_1 \epsilon_1 / \mu_2 \epsilon_2}$ are

$$C_t = \frac{Z_{i-1} + Z_i}{Z_{i-1} + \eta Z_i} \quad \text{and} \quad C_r = \frac{\eta Z_i - Z_i}{Z_{i-1} + \eta Z_i}. \quad (15)$$

For right moving waves ($p = 2$), the transmitted wave propagates $\eta(\nu_i - k)\Delta x$ distance and updates $[\eta(\nu_i - k)]$ downwind grid cells of the material interface. Cell averages are augmented by $C_t \alpha_{i-1/2}^2 \mathbf{r}_{i-1/2}^{2'}$ where $\mathbf{r}_{i-1/2}^{2'}$ is defined as in Eq. (13) with $Z_i = \eta Z_i$. Similarly, reflected wave propagates $(\nu_i - k)\Delta x$ distance and updates $[\nu_i - k]$ cells upwind of the material interface with $C_r \alpha_{i-1/2}^2 \mathbf{r}_{i-1/2}^1$. The range of updates are calculated taking into account local wave speed ($\lambda_{i-1/2}^1$). The effect of left moving waves ($p = 1$) encountering a material interface with refractive index $\eta = \sqrt{\mu_2 \epsilon_2 / \mu_1 \epsilon_1}$ can similarly be taken into account in the LTS method.

A limitation on ν (and Δt) can result from the condition that a single wave is allowed to cross a single dielectric interface per time step in variable impedance media. This would be relevant for practical problems in multidimensions. This will set a limit on the minimum dielectric slab thickness that can be treated efficiently by LTS. For $\nu = K$ the minimum dielectric slab size is $L = K \Delta x / \sqrt{\mu_r \epsilon_r}$. K is based on free space considerations, and dielectric slab has property μ_r, ϵ_r .

5. NUMERICAL RESULTS

We present results for EM wave propagation in layered media involving constant and variable impedance. The incident field is a Gaussian pulse expressed as

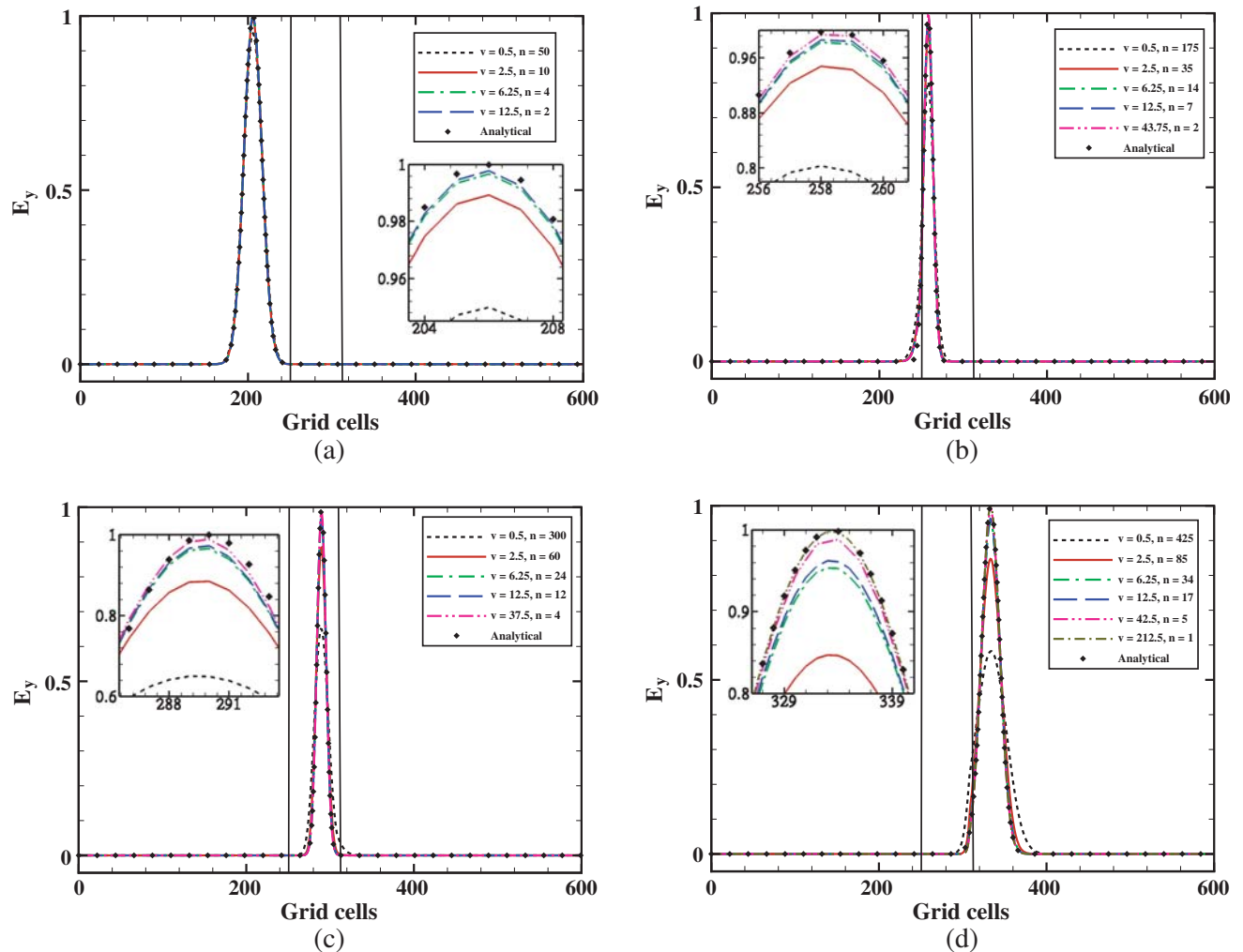
$$E_y(x, t) = E_0 \exp[\beta(x - x_p - ct)^2] \tag{16}$$

with

$$\beta = \frac{\ln(0.001)}{(w \cdot \Delta x)^2}. \tag{17}$$

E_0 is the peak amplitude of the pulse, x_p the original location of the peak, and $2w\Delta x$ defines the incident pulse width. The incident pulse width is set to 400 ps, and original location of the peak is $x_p/c = 4.5w\Delta x$ ps. The computational domain has 600 grid cells with $\Delta x = 1.5$ mm [13].

Initially, we consider the case of constant impedance, for which relative permittivity $\epsilon_r = 2$ and relative permeability $\mu_r = 2$ are set between cells 250 to 309 (Figure 5). Except this, the medium is considered free space with $\epsilon_r = 1$ and $\mu_r = 1$. Computational results are presented after time 0.1251, 0.4378, 0.7505, 1.0632, 1.1258, and 1.5636 ns. Figure 5 shows the electric field propagation. The required number of time steps for varying ν is denoted by n in Figure 5. As shown in Figure 5(b), when the pulse encounters interface 1, it propagates from free space to dielectric medium and is compressed due to change in wave speed. In contrast, when the pulse hits interface 2 (Figure 5(d)), it transits into free space, and the pulse width increases back to the original state (Figure 5(f)). Numerical results are compared with analytical solution for increasing $\nu \gg 1$. The LTS method improves the



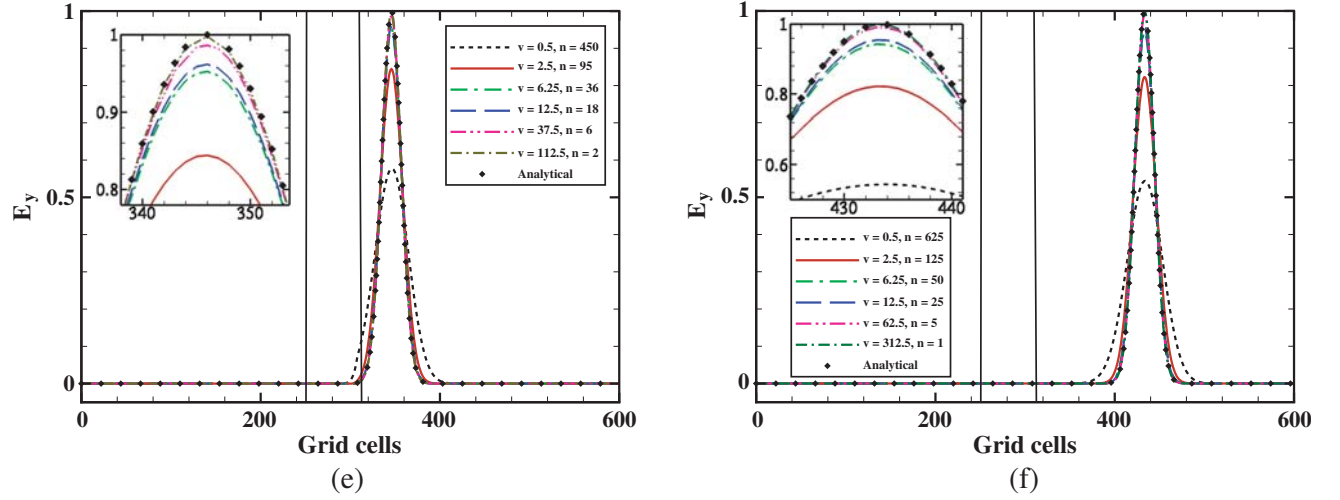


Figure 5. LTS method, Gaussian pulse, $Z(x) = Z_0$. (a) $t = 0.1251$ ns, (b) $t = 0.4378$ ns, (c) $t = 0.7505$ ns, (d) $t = 1.0632$ ns, (e) $t = 1.1258$ ns, (f) $t = 1.5636$ ns.

Table 1. Performance of LTS algorithm with varying ν , $Z(x) = Z_0$.

ν	Peak (V/m)	$\ \text{error}\ _{L_2}$	CPU time (s)
0.5	0.5437	7.8448×10^{-2}	2.8829×10^{-2}
2.5	0.8223	2.8698×10^{-2}	1.2073×10^{-2}
6.25	0.9427	8.9766×10^{-3}	7.6969×10^{-3}
12.5	0.9547	7.0125×10^{-3}	7.0941×10^{-3}
62.5	0.9905	1.5680×10^{-3}	6.7842×10^{-3}
312.5	0.9983	5.0818×10^{-4}	4.8998×10^{-3}

resolution and preserves better amplitude of the Gaussian pulse with increasing Courant number ν due to lower discretization errors. The lower discretization errors are a consequence of the fewer explicit time steps required in LTS than that in a standard $\nu \leq 1$ propagation. The quantitative performances of LTS method in terms of peak amplitude of transmitted wave and overall $\|\text{error}\|_{L_2}$ for varying ν at $t = 1.5636$ ns are also tabulated in Table 1. The required CPU time (processor Intel®Core™ i5-2500 CPU @3.30 GHz), measured in s , is reduced with increasing ν as shown in the table. The LTS method requires the change in wave speed because material change is factored in the range of downwind cells to be updated by waves arising out of upwind Riemann problems.

In the next example, we consider a case of varying impedance $Z(x)$. The relative permittivity is set to $\epsilon_r = 4$ between cells 250 and 309, while in other grid cells it is set to free space ($\epsilon_r = 1$) [13]. In this case, when the Gaussian pulse propagates from one layer to another, a part of the wave is transmitted while the rest reflects back from the interface. Figure 6 shows the propagated electric field at time 0.1251, 0.4378, 0.7505, 1.0632, 1.1258, and 1.5636 ns. In the case of variable impedance, Riemann waves interacting with an interface will affect both downwind (transmitted wave) and upwind (reflected wave) cells. The LTS method will require the range of both upwind and downwind cells to be updated by individual Riemann waves. The magnitudes of the update will depend on reflection and transmission coefficients based on direction of travel and impedances involved. Table 2 gives measured amplitude of transmitted and reflected waves from respective interfaces along with $\|\text{error}\|_{L_2}$ of the signal and the required CPU time for varying ν at 1.5636 ns. In this table, the amplitude of transmitted wave is E_t while E_{r_1} and E_{r_2} are the amplitudes of reflected waves from interfaces 1 and 2, respectively. The results again show the amplitudes of transmitted and reflected waves at interfaces 1 and 2 to be better

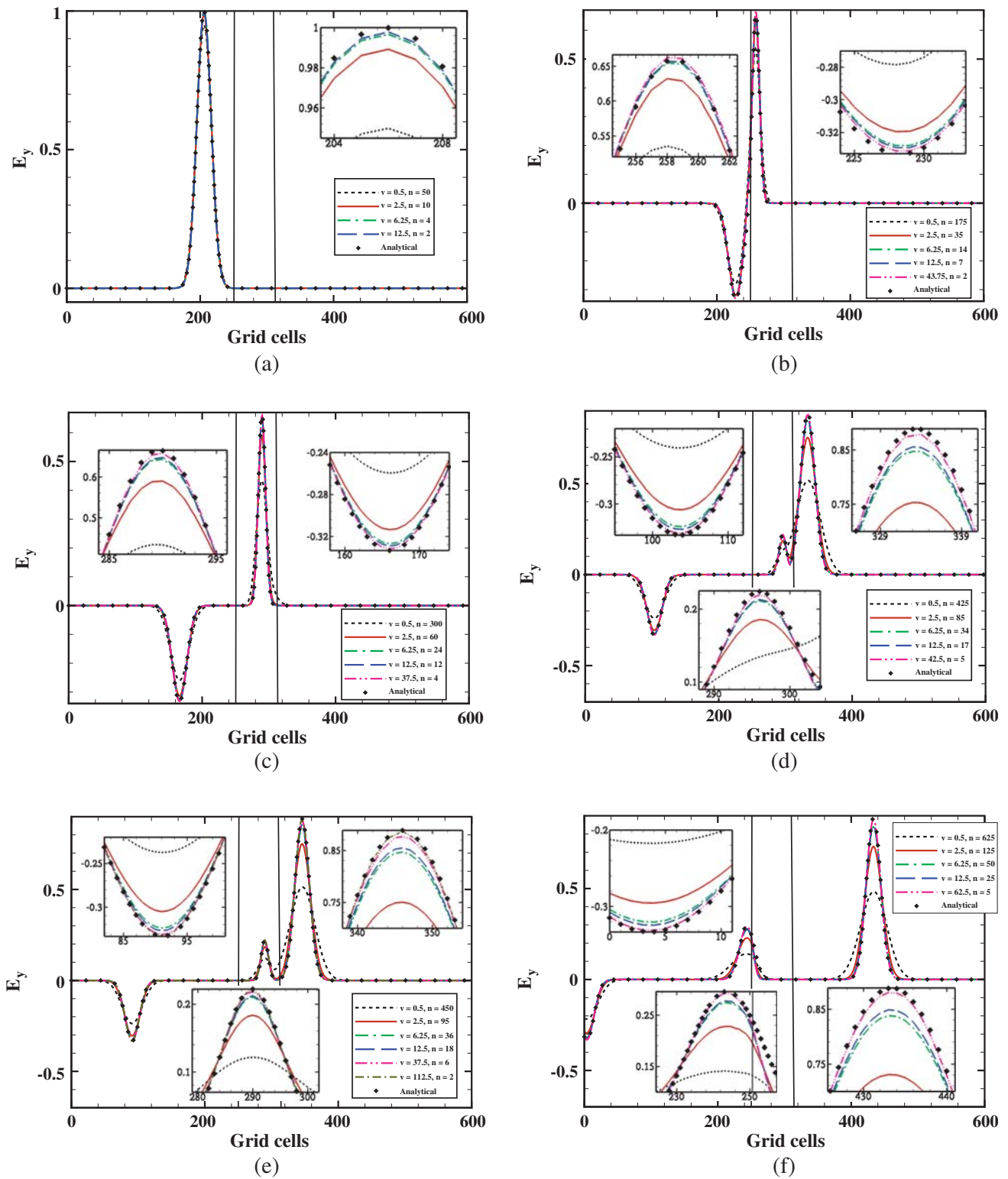


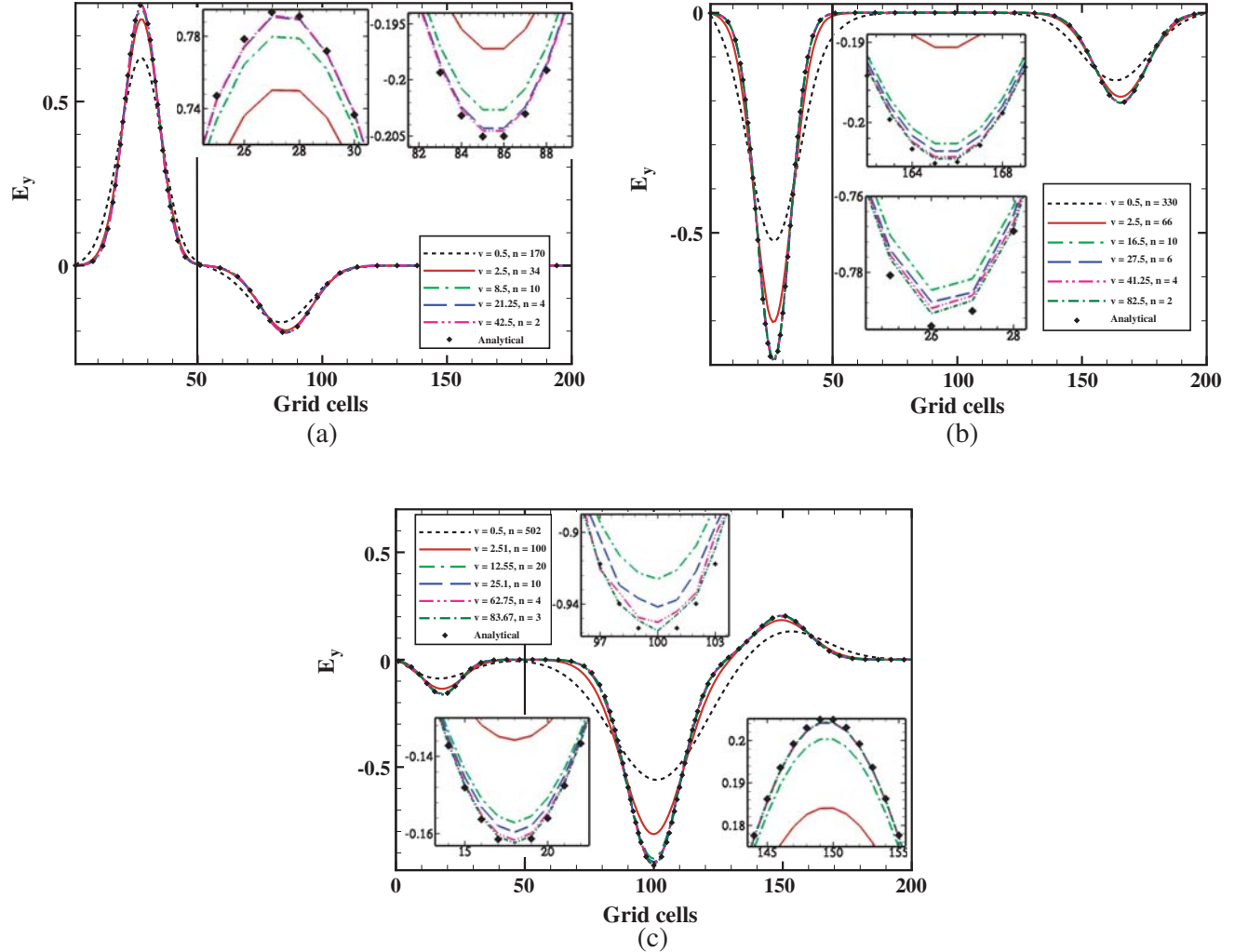
Figure 6. LTS method, Gaussian pulse, $Z(x) \neq Z_0$. (a) $t = 0.1251$ ns, (b) $t = 0.4378$ ns, (c) $t = 0.7505$ ns, (d) $t = 1.0632$ ns, (e) $t = 1.1258$ ns, (f) $t = 1.5636$ ns.

Table 2. Performance of LTS algorithm with varying ν , $Z(x) \neq Z_0$.

ν	E_t (V/m)	E_{r_1} (V/m)	E_{r_2} (V/m)	$\ \text{error}\ _{L_2}$	CPU time (s)
0.5	0.4834	-0.2173	0.1410	7.6155×10^{-2}	2.6202×10^{-2}
2.5	0.7309	-0.2956	0.2282	2.8872×10^{-2}	1.1435×10^{-2}
6.25	0.8380	-0.3204	0.2746	1.2244×10^{-2}	8.6917×10^{-3}
12.5	0.8486	-0.3245	0.2779	1.1051×10^{-2}	7.2581×10^{-3}
62.5	0.8804	-0.3313	0.2930	9.1832×10^{-3}	7.1695×10^{-3}
Analytical	0.8889	-0.3333	0.2963	-	-

preserved for $\nu \gg 1$. Overall CPU time again decreases with increasing ν . In this case, the Courant number ν is limited to 120 based on the self imposed limitation restricting interaction of waves with multiple interfaces to one per time step. The number of time steps decreases with increasing ν again resulting in lower discretization error as $\nu \gg 1$ due to LTS.

The previous examples involved unbounded computational domain. In the next example, the

**Figure 7.** LTS method, Gaussian pulse, $Z(x) \neq Z_0$, PEC boundaries. (a) $t = 0.1418$ ns, (b) $t = 0.2752$ ns, (c) $t = 0.4186$ ns.

Gaussian pulse propagates in a perfect conducting cavity [14]. The cavity has 200 grid cells with $\Delta x = 0.0005$ m, and the first fifty cells have dielectric material with relative permittivity $\epsilon_r = 2.3$. Other grid cells in the cavity are considered to be free space ($\epsilon_r = 1$). The Gaussian pulse is initialized at the centre of the domain and is expressed as $e^{-w^2 t^2}$, where $w = 4.14 \times 10^{10}$ 1/s. As time advances the pulse propagates left [14]. The representation of the electric fields after time $t = 0.1418, 0.2752,$ and 0.4186 ns are shown in Figure 7. As in the previous case of variable impedance media, when waves originating in a Riemann problem hit a material interface, part of the wave is transmitted, and part reflects back from the interface. These transmitted and reflected waves update appropriate range of downwind and upwind grid cell relative to the interface depending on locally defined wave speed. The computational domain is also bounded with the perfect electric conducting (PEC) boundaries. As shown in Figures 7(b) and 7(c), the outgoing wave from the boundaries completely reflect back into the computational domain so as to have $n \times \mathbf{E} = E_y = 0$ at PEC boundaries. This PEC boundary condition implementation was earlier described in detail by the authors in [10] for homogeneous medium. From the results the profile of electric field again agrees very well with the analytical solution for $\nu \gg 1$. The measured amplitude of transmitted and reflected waves, $\|\text{error}\|_{L_2}$, and CPU time for varying ν are again tabulated in Table 3 at $t = 0.4186$ ns. In this table E_t is the amplitude of transmitted wave. The incident pulse initially hits the interface, and the amplitude of reflected wave is E_{r_1} in the table. As shown in Figure 7(b), the left moving transmitted wave after reflection from PEC boundary again hits the dielectric interface. A part of the wave is again reflected from the interface, and amplitude of this reflected wave is represented by E_{r_2} in the table. As in the homogeneous case [10], the discretization error decreases with increase in ν due to decrease in the number of operations with increasing Δt .

Table 3. Performance of LTS algorithm with varying ν , $Z(x) \neq Z_0$, PEC boundaries.

ν	E_t (V/m)	E_{r_1} (V/m)	E_{r_2} (V/m)	$\ \text{error}\ _{L_2}$	CPU time (s)
0.5	-0.5594	0.1316	-0.0877	1.1744×10^{-1}	6.7667×10^{-3}
2.51	-0.8122	0.1841	-0.1358	3.9937×10^{-2}	3.1229×10^{-3}
12.55	-0.9260	0.2003	-0.1571	9.0897×10^{-3}	2.0558×10^{-3}
25.1	-0.9415	0.2041	-0.1596	5.9795×10^{-3}	1.9650×10^{-3}
62.75	-0.9500	0.2043	-0.1617	4.8474×10^{-3}	1.5927×10^{-3}
83.67	-0.9548	0.2044	-0.1624	4.6716×10^{-3}	1.4546×10^{-3}
Analytical	-0.9379	0.2053	-1.1631	-	-

6. CONCLUSIONS

The classical LTS approach originally proposed for nonlinear hyperbolic conservation law was previously adopted in an FVTD framework to accelerate numerical solution of time-domain Maxwell’s equations in free space [10]. This allowed the use of very large time steps much larger than that dictated by conventional stability criterion. This method is now extended to EM wave propagation with dielectric interfaces. The LTS method is shown to be unconditionally stable for constant impedance media for 1D domain with increasing ν similar to that for homogeneous problems. But the LTS method requires special handling of interface boundaries. In the case of constant impedance, Riemann waves only update downwind grid points, but the range of cells will be dictated by incorporating the change in wave speed across dielectric interfaces. On the other hand, in the case of variable impedance, Riemann waves on encountering dielectric interface result in both transmitted and reflected waves. This results in updating both upwind and downwind cells of dielectric interface taking into account coefficients of reflection and transmission across the interface. A possible limitation of ν (and Δt) can result from the condition that waves are allowed to cross a single dielectric interface in a time step for more practical problems in multidimensions. This sets a limit to the thickness of dielectric slab that can be treated efficiently by LTS in variable impedance media. Extension to multidimensions can in-principle be brought through an operator splitting approach. This extension is analysed in detail by the authors for LTS involving homogeneous medium in [10]. The same analysis holds true for multilayered homogeneous media.

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