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Rainfall Rate Field Space-Time Interpolation Technique for North West Europe

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Abstract—The ability to predict rain characteristics at small space-time scales is important, particularly in the planning, design, and deployment of wireless networks operating at frequencies above 10 GHz. For wide area networks, high space and time resolution rainfall data are often not available, and the cost of such measurements is prohibitive. This paper thus presents a new approach to address this problem using rain radar measurements to obtain rain estimates at finer resolutions than that available from the original measured data. This paper proposes three innovative methodologies: 1) the approach is not directly applied to measured rainfall rate data but focuses on the parameters of fitted lognormal distribution parameters and/or computed rain characteristics for each location; 2) to facilitate the application in wireless communication networks operating above 10 GHz, a set of databases and contour maps of rain parameters spanning North West Europe have been created. These conveniently and efficiently provide rain parameters for any location within the area under study; and 3) the proposed 3D space-time interpolation approach can extrapolate rain parameters at space-time resolutions that are shorter than those found in NIMROD radar databases. The results show that the approach presented in this paper can be used to provide {1 km, 5 mins} space-time rain rate resolution from {5 km, 15 mins} data for the whole North West Europe with error percentages of less than 4%. This is far superior to estimates provided by the International Telecommunication Union recommended model.

1. INTRODUCTION

The knowledge of the spatial and temporal variations of point rainfall rate is important for the planning and prediction of the performance of satellite and terrestrial networks operating at frequencies greater than 10 GHz especially as precipitation is one of the main causes of signal attenuation [1]. The advent of meteorological radar and satellite systems has significantly improved the space resolution of rain fields, but they are not suited to generating data over short time scales like rain gauges. Therefore, new techniques are needed in order to integrate the advantages of different apparatus and for predicting rainfall rate values at finer resolutions. In particular, shorter time scales are required than that currently available from wide area coverage measured rainfall rate databases. Traditional models, such as the stochastic model [2], Markov chain model [3], and numerical weather prediction (NWP) model [4], can no longer meet the ever-increasing demands for high resolution data. There are some limitations inherent in such models, and the two major ones are:

(1) the models are only applicable to areas where rainfall data with the required integration volumes have been measured, and the accuracy of the models in areas where no data is available is difficult to verify;

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(2) the application of the models is limited by integration volume. The modelling of rainfall fields is limited to space-time resolutions derived from rain radar and/or rain gauge measurements. Rainfall fields at finer space-time scales required for the next generation of wireless communication systems cannot be estimated using these models.

As a result, improvements are needed to compensate, enhance, and extend the performance of the traditional models. In particular, an increase in the use of high frequencies over short communication links has led to an increase in the need for rainfall rate estimates at finer resolutions. These are beyond the capabilities of traditional models. Thus, interpolation techniques have attracted considerable attention, and extensive studies have been carried out over the last few decades. For example, the Random Midpoint Displacement algorithm (RMD) developed by Voss [5] is one of the most widely used interpolation algorithms. The basic idea of this technique is to introduce new rain rate samples with the same underlying distribution as existing measurements at new locations or times. The one-dimensional (1D) time interpolation is also of interest as high frequency network planners and designers of physical layer fade mitigation techniques [6] require knowledge of rain variation over short time scales (of the order of seconds or less). Other interpolation models have been published in [7, 8]. One such model proposed by Paulson [9] is a stochastic numerical model that can interpolate the point of rainfall rate over short time durations down to 10 s.

The downscaling model, which is based on space-time averaging theory, is an alternative technique that has also attracted significant attention. According to [10], there are two fundamental requirements for the precipitation downscaling models, which are: 1) understanding of the statistical properties and scaling laws of rainfall fields, and 2) validation of the downscaling model that preserves the same statistical characteristics as observed in actual precipitation. In particular, the theory of fractals, which was first introduced by Mandelbrot in 1967 [11], has also attracted a significant amount of attention. This theory was not applied to the study of rainfall until the mid-1980s [12]. Rain has been shown to hold fractal properties over a range of scales. The intermittent and discontinuous nature of rain is reproduced by the fractal based models, which are strongly favoured for rainfall modelling. Many studies have been undertaken to interpolate the rain radar/gauge data to finer scales using the fractal theory, such as multifractal models [13], which can capture any single moment of the observed signal, especially higher order moments where there is greater interest [14, 15]. Some multifractal models use discrete cascade algorithms to produce data at finer scales from original sparse observations, for example in [16, 17].

The prediction at finer space-time resolution, however, has long been a challenging issue in rainfall field modelling. Results from 3-dimensional (3D) interpolation studies are quite poor because it is difficult to consider both spatial and temporal variabilities and irregularities of rain events in an appropriate way. The predominant idea in published literature is to try to find the underlying principle of how the space-time transformation can be achieved. A representative model was developed by Deidda [18] based on the assumption that Taylor's hypothesis [19] can be applied. The space-time rainfall field is assumed homogeneous and isotropic in 3-dimensions (2-dimensional, 2D space and 1D time). An advection velocity parameter is then introduced to connect the space scale and time scale. With the help of a velocity parameter, the statistical properties of rain at finer scales can be deduced from larger ones. Similar studies can be found in [20] in which rain has been studied in a range of space-time scales to define the transformation parameter.

There are two primary gaps in published interpolation models. The first is the absence of high resolution rainfall data at desired space and time scales. Deidda et al. [21] pointed out that most of the existing rainfall studies at finer scales are purely focused on either space modelling [22] or time modelling [23]. However, both of these approaches have limitations. For example, the statistical behaviour of rain in time has implicit consideration of the spatial distribution and extension of the rain field itself, whereas the study in space is normally based on fixed time duration whilst the evolution in time of spatial patterns is ignored. However, accurate rainfall field estimation requires knowledge of rainfall rate variability in both space and time. There is not enough research in the area of space-time interpolation apart from works such as [24–26]. Thus, appropriate space-time interpolation techniques that can preserve the underlying statistical properties at finer scales are needed. Another important gap is the insufficient knowledge and description of rain characteristics at high space and time resolution. Most of the existing interpolation and/or downscaling models directly focus on rain precipitation itself,

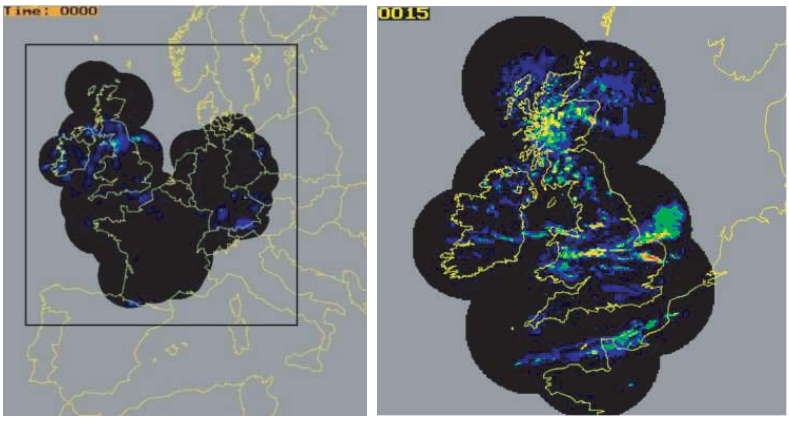
and no systematic study of rain characteristics at scales better than those provided by data from rain radars, satellite, or rain gauges has been reported. Currently published works only focus on a specific property of rain. For example, the authors' previous work [27] proposed an empirical equation that can extrapolate the probability of rain occurrence to a wide range of integration volumes in both space and time domains. In addition, Luini and Capsoni [28] investigated the variation of space correlation functions of rain using UK radar data and proposed a method to describe this property in changing space and time scales. Kundu and Bell [29] also developed a model that can reasonably yield the correlation function of rain in 3D space-time domain but in a very complicated form.

To further the development of rain-induced radio-waves attenuation models and to provide more accurate performance prediction of satellite and terrestrial high frequency network links over wide areas, there is an increasing need for a good understanding of the space-time characteristics of rainfall rate at finer scales. This is because rain events are complex processes that are highly variable in both space and time. So this paper presents a simple (which is different from the work in [30]) but accurate space-time interpolation approach for key properties of rain under study in space and time domains simultaneously. In this novel approach, a series of European maps are superimposed with each parameter at different spatial-temporal volumes. In particular, a simple but accurate approach for interpolating rain characteristics is proposed. It can predict the correlation coefficient values of the statistical model in a wide range of space-time resolutions with reasonable accuracy. This will deliver accurate rainfall field estimates yet involves very little computation time and, in doing so, provides considerable assistance for applications related to high frequency networks.

The remainder of this paper is organised as follows. Section 2 describes the data used in this study. Section 3 reviews the statistical model and describes the proposed technique on how to extrapolate the measurements into 3-dimensional space-time domains. The detailed results, including the 2D contour map of rain characteristics across North West Europe and the 3D space-time predictions at each location are presented in Section 4. Section 5 discusses the validation of the results achieved from the proposed interpolation approach. Section 6 presents the conclusions.

2. DATA DESCRIPTION

Five complete years of NIMROD rain radar data (from 2005 to 2009) have been analysed and used in this study. The NIMROD radar system produces a series of composite rain field maps every 15 mins. The measured rain rate samples are distributed over a 5 km squared Cartesian grid covering North West Europe. Each NIMROD map contains 700×620 data cells, but only the locations for which data are available have been analysed, see the outline in Fig. 1(a). The study area ranges approximately from latitude 43.1938° to 59.4306° and longitude between -11.7370° and 17.8364°. In addition, the



(a) (b)

Figure 1. Composite radar scan image: (a) radar image for North West Europe (the outline is the studied area), and (b) radar image for the British Isles.

NIMROD system also holds data for the British Isles which has better resolution of rain rate estimates (1 km in space and 5 mins in time) as shown in Fig. 1(b). The performance of any prediction model or approach needs to be validated through comparison with measured data (e.g., rain-gauge or rain radar data). The UK data, which has better resolution than the European NIMROD data, will be used to validate the technique developed using the European data in this paper.

3. METHODOLOGY

3.1. Review of the Statistical Model

The empirical equations that can yield estimates of the rain characteristics with reasonable accuracy throughout the whole integration range have been discussed in [30]. The proposed statistical model of the key rain characteristics is described briefly in this paper for completeness.

It is well accepted that rainfall rate, R in mm/h, at one location is modelled as a lognormal process with a mixed probability density function (pdf). For example, the joint pdf of two variables with both zero mean and unit variance and cross correlation given by r, takes the form

$$f(R_1, R_2) = \frac{1}{2\pi\sqrt{1-r^2}} \exp\left(-\frac{1}{2(1-r^2)}\left(R_1^2 - 2rR_1R_2 + R_2^2\right)\right) \tag{1}$$

According to [31], the general formula for a straight-line fit is given by:

$$Q_{inv} = \frac{\ln(R)}{\sigma} + \frac{\mu}{\sigma} \tag{2}$$

where $\{\mu, \sigma\}$ is the set of lognormal parameters that are used to study the statistics of rainfall rate at a location of interest. Recent developments have produced a single general empirical equation that fits both the space correlation and time correlation functions. The common function takes the form

$$\rho(x) = \frac{a}{a + x^n} \tag{3}$$

where a and n are coefficients that need to be determined from the data, and x can be either d (distance) in km or t (time lag) in mins.

Also in [30], an empirical equation has been proposed, which gives good estimates of the probability of rain occurrence (P_0) throughout the whole range of integration volumes. The mathematical equation is described by:

$$P_0(x) = 100 - b\exp(cx^e) \tag{4}$$

where b, c, and e are experimental coefficients that can be determined from study, and x denotes either spatial integration length L or temporal integration period T.

3.2. Data Integration

Following the authors' previous work [27], the rainfall rate data can be up-scaled from small integration volume to large ones using:

$$R_\lambda(x, y, t) = \frac{1}{\lambda^3 L^2 T} \iiint R(x', y', t')\, I\left(\frac{x - x'}{\lambda L}, \frac{y - y'}{\lambda L}, \frac{t - t'}{\lambda T}\right) dx'dy'dt' \tag{5}$$

where $R_\lambda(x, y, t)$ is the rain rate at position (x, y) derived from a spatial integration region of linear size λL and temporal integration period λT. $\lambda > 1$ is known as the scale parameter. More generally, the spatial and temporal regions could have different scale parameters, e.g.,

$$R_\lambda(x, y, t) = \frac{1}{\lambda^2 \varphi L^2 T} \iiint R(x', y', t')\, I\left(\frac{x - x'}{\lambda L}, \frac{y - y'}{\lambda L}, \frac{t - t'}{\varphi T}\right) dx'dy'dt' \tag{6}$$

The radar-derived rain rate data can be up-scaled to coarser resolutions based on the above equations. It is important to highlight the fact that each grid point is used only once for each integration procedure, and no overlapping regions and/or time periods are considered. The integrated data are tiled up without changing the size of the original rain map, but a new dataset with larger integration scale is created. The larger the integration volume is, the smaller the number of data samples will be. Parameters λ and φ must be integers. The volume of the integrated data is the integral times the original radar data, and it will be λL and φT, where $L = 5$ km and $T = 15$ mins.

3.3. Principle of 3D Interpolation

In [27–30], it is shown that the rain characteristics regularly change with increasing integration volume, in both space and time. This allows rain characteristics at other spatial or temporal integration volumes to be predicted. More importantly, 3D interpolation could be carried out if there are enough measured data with different space-time resolution combinations.

Figure 2 illustrates the grid of available rain data points from the NIMROD radar measurements. The dots represent the available space-time integration volume combination where the rain characteristics can be determined. The lines represent the range of integration volume where rain characteristics can be calculated from Equations (4) to (6). The proposed statistical model in [30] can only produce estimates of rain characteristics along the lines but not within the blank areas. Taking advantage of the regular distribution of the measurements, the key rain characteristics at other spatial-temporal integration volumes (blank areas) can be predicted using any existing interpolation technique with acceptable accuracy.

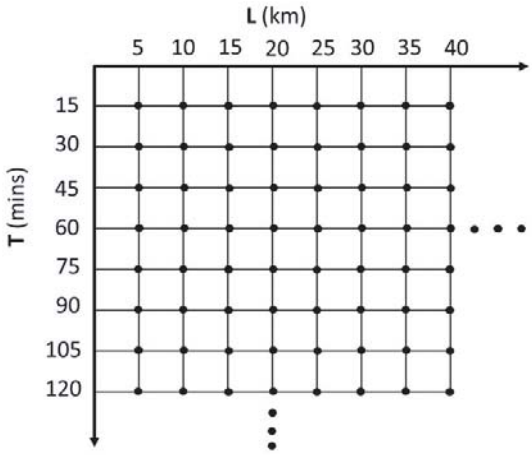


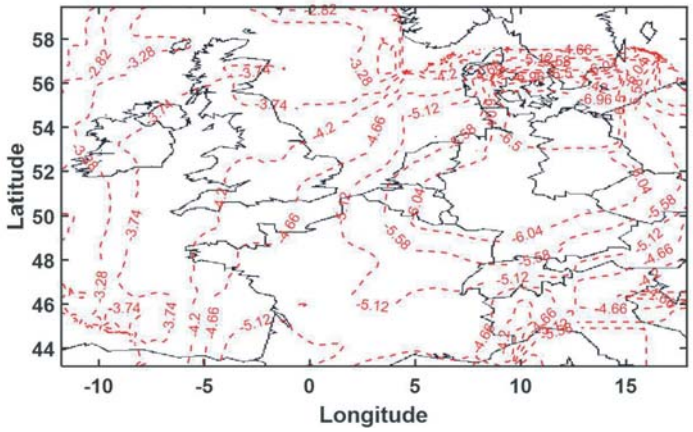
Figure 2. Grid of available rain data points from the NIMROD radar measurements.

4. EXPERIMENTAL RESULTS

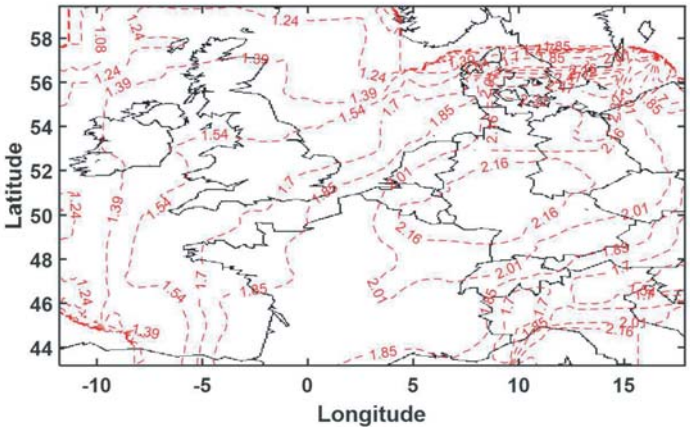
4.1. Contour Map of Rain Characteristics

The proposed statistical model can yield estimates of key rain characteristics (including the first order statistics, the spatial and temporal correlation coefficient values of rain rate, as well as the probability of rain/no rain) in either space domain (2D) or time domain (1D). Considerable computation is required to extract these statistics for a large area. In particular, based on the proposed model, the rain characteristics at any location within North West Europe where data are available can be obtained. The work in this paper has produced a set of multi-resolution databases of parameters and contour maps that cover the whole area under study. From these databases, a user can easily obtain the characteristic parameters of rain (or the distribution coefficients) at any location of interest and thus rainfall rate estimates using any existing rain field simulator, such as the ITU-R [32]. This is useful for a range of wireless communication applications, such as fade mitigation technique (FMT) [33] and site diversity [34].

Examples of contour maps of the lognormal rain rate distribution parameters $\{\mu, \sigma\}$ (see Equation (2)) are presented in Fig. 3. Fig. 3(a) is the map of μ values across North West Europe. Fig. 3(b) is the contour map of σ values, and the background is the map of the North West Europe coastline. It shows that the contour maps can provide the parameter value at any location within $-11.7370°$ to $17.8364°$ longitude and $43.1938°$ to $59.4306°$ latitude. A set of databases of $\{\mu, \sigma\}$ with different space-time scales has been created from which researchers or users can obtain parameter values



(a)



(b)

Figure 3. Contour maps of rain lognormal distribution parameters with spatial integration length of 5 km and temporal integration length of 15 mins: (a) μ values and, (b) σ values.

for any longitude and latitude location using the following equations

$$y_{(\text{longitude})} = 0.0658x - 19.8364 \tag{7}$$

$$y_{(\text{latitude})} = -0.0409x + 59.430 \tag{8}$$

where x denotes either row or column number of the NIMROD data grid, and y is the corresponding coordinate value in either latitude or longitude.

This offers convenience as almost no computation time is required. Similar databases of other rain characteristic parameters have also been created but not presented in this paper. Given these databases, the prediction of rain characteristics at other space-time resolutions, especially those not available from rain radar measurements, can be obtained.

4.2. Space-Time Predictions of Rain Characteristics

The existing NIMROD radar maps have been integrated to a range of integration volumes from {5 km, 15 mins} to {75 km, 120 mins}. The key characteristics of rain have been analysed to see how they vary with increasing space-time scale. Table 1 gives an example of the probability of rain (P_0) at Portsmouth (UK) for a range of integration volume combinations. It can be seen that P_0 changes systematically with increasing integration volume. Given this finding, the predictions at other resolutions, particularly those at finer resolutions than radar measurements, can be estimated through interpolation.

Table 1. Probability of rain occurrence for increasing spatial-temporal integration lengths ranging from 5 km to 75 km and 15 mins to 120 mins at Portsmouth.

	T (mins)							
L	15	30	45	60	75	90	105	120
5 km	15.0	22.9	25.9	28.5	30.5	32.6	34.3	36.1
10 km	23.4	28.61	32.3	35.3	37.7	40.1	41.9	43.8
15 km	28.6	33.7	37.5	40.3	42.9	44.9	46.7	48.6
20 km	32.6	37.9	41.6	44.7	47.1	49.4	51.4	53.1
25 km	35.2	40.6	44.1	47.2	49.5	51.7	53.7	55.2
35 km	42.5	48.3	52.2	55.4	57.9	60.1	61.2	63.1
40 km	45.9	49.4	53.2	56.1	59.6	62.6	64.4	67.9
45 km	48.8	55.6	59.5	62.2	64.7	66.6	68.2	69.6
50 km	49.9	56.2	60.2	62.9	65.6	67.2	69.5	71.8
55 km	56.5	62.2	65.8	68.5	70.7	72.2	73.5	75.3
65 km	60.9	65.5	68.5	70.6	72.3	74.2	75.1	76.5
75 km	63.9	69.1	72.4	74.4	76.3	77.8	78.7	79.9

In this study, the cubic spline interpolation algorithm, which is constructed by piecing together cubic polynomial at different intervals [35], has been chosen. It takes the form

$$S(x) = \begin{cases} s_1(x) & \text{if } x_1 \le x < x_2 \\ s_2(x) & \text{if } x_2 \le x < x_3 \\ \ldots & \\ s_{n-1}(x) & \text{if } x_{n-1} \le x < x_n \end{cases} \tag{9}$$

where s_i is a third degree polynomial defined by:

$$s_i(x) = a_i(x - x_i)^3 + b_i(x - x_i)^2 + c_i(x - x_i) + d_i \tag{10}$$

Cubic spline is often used for 1D interpolation. The data in each row and column of the database (see the example in Table 1) can be treated as samples in one dimension. It enables the use of cubic spline interpolation to estimate parameter values at other scales, based on the measured values. This is similar to the basic principle of the statistical model proposed in [30]. The first step is to extract the multi-scale parameters from the database for the location of interest. The chosen algorithm is then used to interpolate the data to a different spatial or temporal integration volume.

The technique proposed in this paper takes advantage of the databases created, which contain the calculated rain parameters for a range of integration volumes between {5 km, 15 mins} and {75 km, 120 mins}. Typically, the software that implements the proposed method extracts rain characteristics for all available integration volumes at the location of interest for further processing. Taking the extracted data at the location of interest, the interpolation algorithm then processes the data and yields estimates for other space-time resolutions. This applies to locations for which radar measurements are available (the black areas in Fig. 1).

Figure 4 presents an example of the predicted P_0 at other spatial-temporal integration volumes along with values calculated from measured datain Table 1 for Portsmouth. It is clear that the outcome of the 3D interpolation is a surface constructed from numerous curves both in space and time domains. The dots are the measured values at a range of spatial-temporal integration volumes that are multiples of the radar data resolution, whilst the surface is produced by the interpolation algorithm. The multi-scale data are regularly spaced, which reduces complexity and improves the accuracy of the extrapolation procedure. Interestingly, the results show that P_0 value increases systematically with increasing spatial-temporal integration volume. In addition, the technique allows values at resolutions smaller than {5 km, 15 mins} to be obtained. These are constrained by the assumption that $P_0 \to 0$ as either $\lambda \to 0$

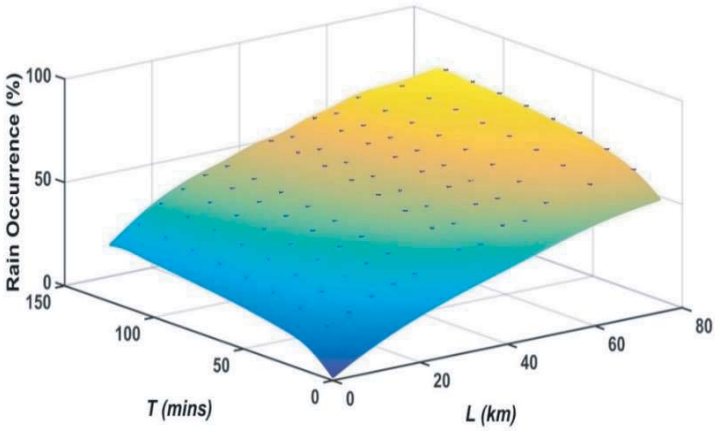


Figure 4. An example of 3D space-time extrapolation of P_0 at Portsmouth.

or $\varphi \to 0$. This enables the predictions to be plotted smoothly to form a 3D surface. The resolution of the key characteristics of rain in this paper offers significant improvements over previous methods (e.g., [27, 30]). This is important for rainfall field studies and particularly at finer scales that are essential for the design and planning of the next generation of short link high frequency network systems. The salient point of the technique proposed in this paper is that estimates can be obtained for space and time resolutions up to 50 m and 6 s, respectively. Predictions at finer scales than this threshold are not acceptable due to the occasional emergence of negative values. Obviously, this is impossible as P_0 should be equal to or greater than 0. The validity of interpolated parameters at scales smaller than {1 km, 5 mins} could not be tested due to lack of data.

5. VALIDATION

The validity of interpolated parameters has been tested. The technique developed using {5 km, 15 mins} Europe NIMROD data has been used to predict the correlation functions of {1 km, 5 mins} data over the British Isles. These are compared to the values calculated from the measured UK NIMROD data with the same resolution. In this paper, Portsmouth has been selected as an example of a location to show the performance of the proposed method.

Figure 5 to Figure 7 compare the studied rainfall rate characteristics estimated by interpolating the

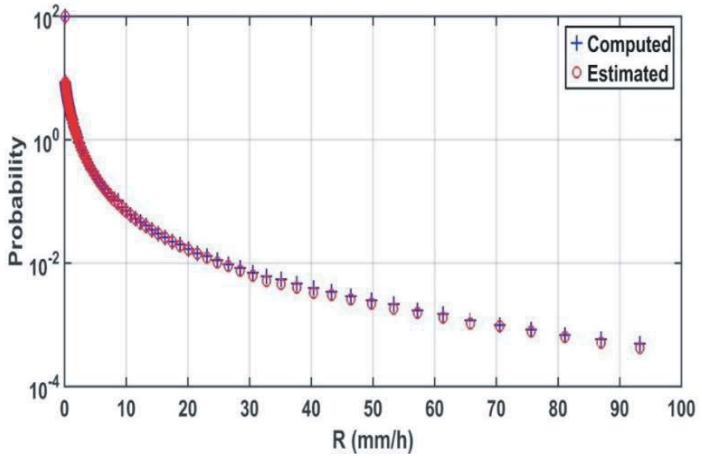


Figure 5. A comparison of probability of rain (P_0) estimated from data interpolated from 5 km to 1 km and calculated directly for 1 km measured data.

5 km Europe NIMROD data to 1 km data. The parameters' values have also been calculated directly from 1 km UK NIMROD data. The predicted $\{\mu, \sigma\}$ values are $\{-3.75, 2.12\}$, and the calculated values from the high resolution data are $\{-3.85, 2.04\}$, whilst the predicted and measured P_0 values are 12.5% and 12.1%, respectively. Although the predicted and measured $\{\mu, \sigma, P_0\}$ values are marginally different (with 2.6%, 3.92%, 3.31% differences, respectively), the associated 0.1%, 0.01%, and 0.001% exceeded rain rates values are similar, see the distribution presented in Fig. 5. In particular, the proposed approach gives excellent approximation of the first-order rainfall rate statistics, especially for rain rates lower than 40 mm/h for which the accuracy is greater than 90%. The root mean square error between the calculated and estimated values in Figs. 5, 6, and 7 are 4.29%, 0.0037, and 0.0043, respectively. The probability of heavy rain events is extremely low such that there is insufficient data to assess.

Figure 6 shows a comparison between the predicted spatial correlation coefficient values and the calculated values. For temporal correlation coefficient values of rain rate, there is a small difference between the predicted and calculated values for short time lags up to approximately 150 mins (see Fig. 7). However, the result is still acceptable as the trends are very similar, especially for large time lags. This shows that the proposed technique has potential and requires considerably less computational efforts than direct estimates from rain radar data. However, the rain characteristics at scales finer than

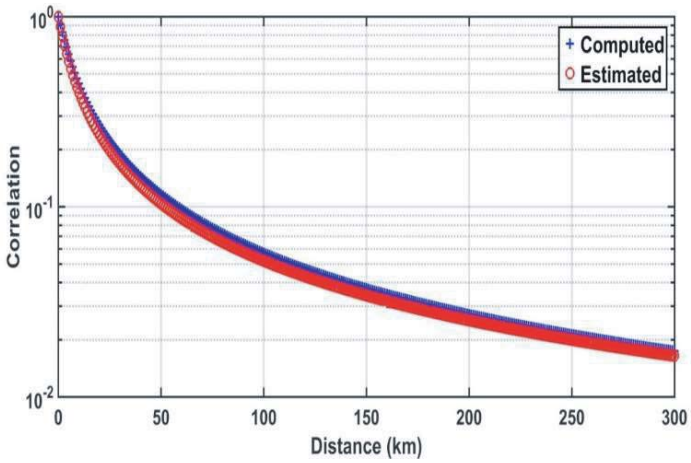


Figure 6. A comparison of spatial correlation coefficient values of rainfall rate estimated from data interpolated from 5 km data to 1 km and calculated directly from 1 km measured data.

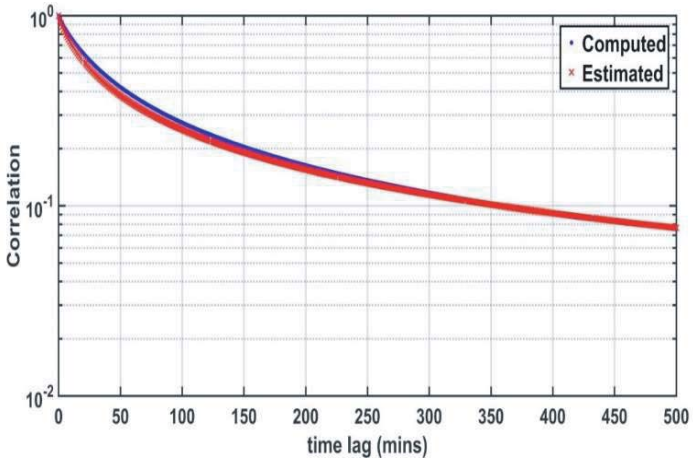


Figure 7. A comparison of temporal correlation coefficient values of rainfall rate estimated from data interpolated from 15 mins data to 5 min and computed directly from 5 min measured NIMROD data.

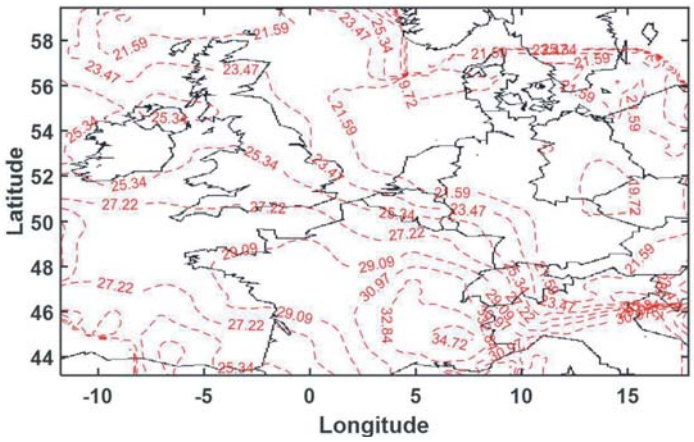
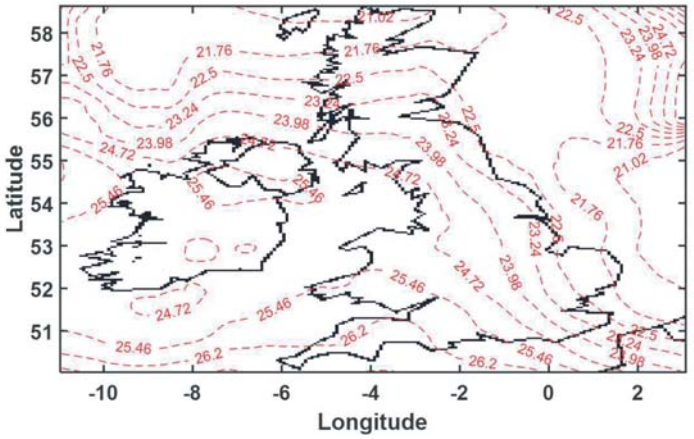
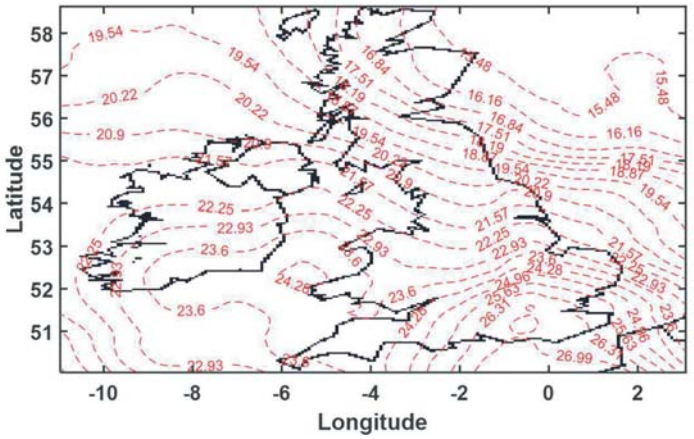


Figure 8. Contour map of 0.01% exceeded rain rates over 5 mins predicted by interpolating from 5 km Europe NIMROD to 1 km.



(a)



(b)

Figure 9. Contour maps of 0.01% exceeded rain rates (over 5 mins) from 1 km (a) data from interpolated approach and, (b) measured UK NIMROD data.

{1 km, 5 mins} cannot be validated due to lack of data. Therefore, this paper presents an opportunity to other researchers who have high resolution data.

Figure 8 shows the map of 1 km spatial resolution of 0.01% exceeded rain rates over 5 mins across North West Europe predicted using data interpolated from the 5 km Europe NIMROD data. The results are plausible for most areas except for the Grand Massif alpine area of France. Fig. 9 presents the map of 0.01% exceeded rain rates across the British Isles. Fig. 9(a) shows the estimates from the data derived by interpolating the rain characteristics parameters from the Europe data, and Fig. 9(b) shows the map of values obtained from the 1 km UK NIROMD. Note that the rain rates with 0.01% exceedance in both figures tend to decrease towards the edge of the radar scan region. This is thought to be due to non-availability of data at areas along the radar scan margins. The similarity between Fig. 9(a) and Fig. 9(b) shows that the estimates from the proposed technique can be used to produce reasonable rain rates with 0.01% exceedance.

Figure 10 presents the contour map of 0.01% exceeded rain rate for an average year as given by the ITU-R P 837-7 model [32]. The rain rates are higher than the figures from the Europe NIMROD data interpolated from 5 km to 1 km and calculated from 1 km UK NIMROD data (see Fig. 8). This suggests that the ITU-Rmodel tends to overestimate rain rates. This also leads to an overestimation of rain volumes over oceans as obtained from the ERA-40 data produced by ECMWF (i.e., the maps upon which the ITU-R rain rate models relies on). This is why the ITU recommends users to use their

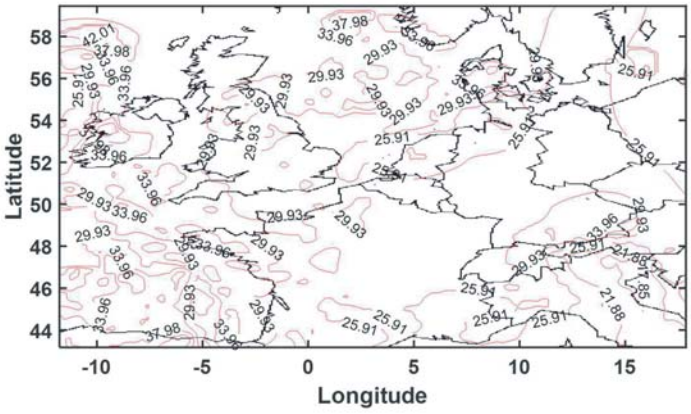


Figure 10. Contour map of 0.01% exceeded rain rates for an average year given by ITU-R P837-7.

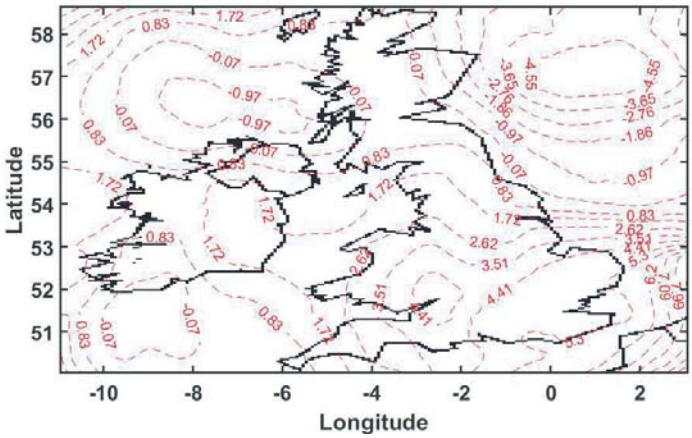


Figure 11. Contour map of 0.01% exceeded rain rates (over 5 mins) difference between values computed from interpolated data using the proposed approach and the values from measured 1 km UK NIMROD data.

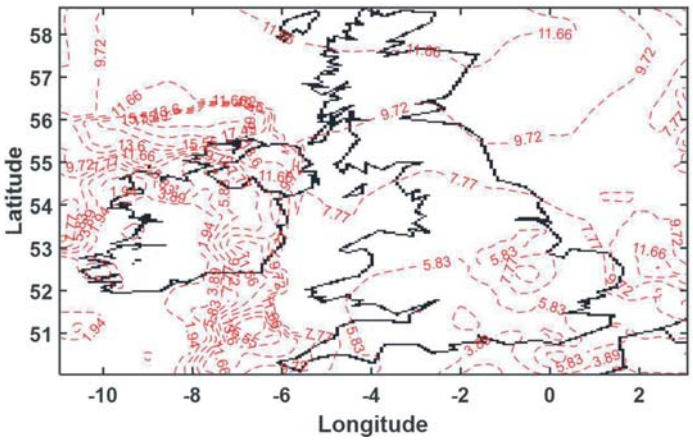
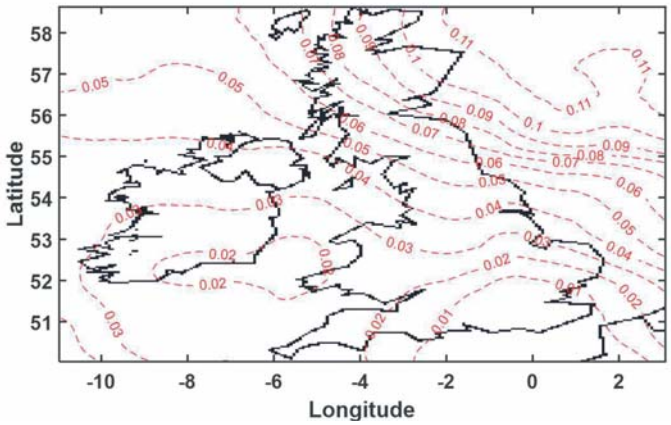
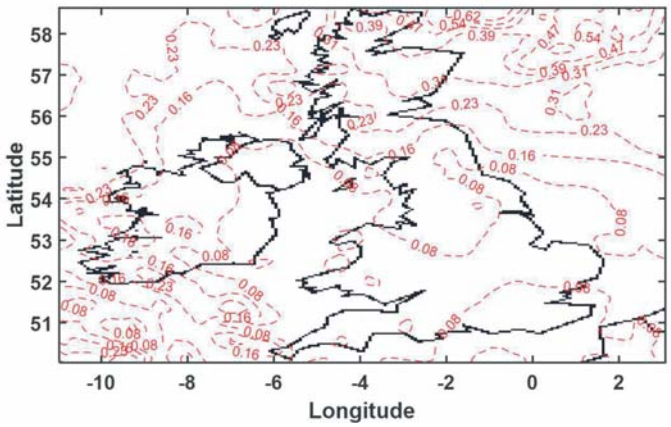


Figure 12. Contour map of 0.01% exceeded rain rates (over 5 mins) difference between values calculated from measured data and ITU-R P 837-7 model estimates.



(a)



(b)

Figure 13. Contour maps of error of 0.01% exceeded rain rate between the measured 1 km UK NIMROD data and estimated values: (a) proposed technique, and (b) ITU-R P 837-7.

own data in order to produce better results.

The differences among the Europe contour map, UK contour map, and ITU-R contour map have been studied in detail to investigate the accuracy of the proposed approach. The contour map in Fig. 11 illustrates the difference in the 0.01% exceedance rain rates between the interpolated Europe data and UK measured data. It shows that the proposed approach tends to overestimate the rain rates over land, but underestimates over some of the ocean/sea areas, like the North Sea region. However, the difference is acceptable as in the range of 0.5–5 mm/h for most areas. In a few areas, the difference can be up to 10 mm/h for example in the junctional region between UK and France, but this is rare.

Figure 12 presents the difference between the values calculated from measured data and values predicted using the ITU-R P 837-7 model. The contour map shows that the ITU-R P 837-7 model tends to overestimate rain rate compared to the proposed approach for most areas. The difference can be up to 20 mm/h for some regions. This indicates that the proposed approach gives more plausible estimates than the ITU-R model, although it is restricted to the areas under study.

The error between values calculated from the interpolated data or predicted by the ITU-R model and values calculated from measured data can be computed using the following function [36]:

$$\text{Error} = \left| \ln \left(\frac{R_{\text{measured}}}{R_{\text{predicted}}} \right) \right| \tag{11}$$

where R_{measured} and $R_{\text{predicted}}$ are the measured and predicted rainfall rates with 0.01% exceedance, respectively.

Figure 13 presents the error contour maps of 0.01% exceeded rain rate over the UK for both the proposed approach and the ITU-R model. Fig. 13(a) shows that the error of the proposed approach is between 2×10^{-5} and 0.12. It indicates that this approach can yield good estimates across the area under study. However, the error from the ITU-R model can be up to 1.32, see Fig. 13(b).

The mean error $\overline{\text{Error}}$ is calculated by:

$$\overline{\text{Error}} = \frac{1}{n} \sum_{i=1}^{n} \text{Error}_i \tag{12}$$

where Error_i is the error for individual locations, and n is the number of locations. The calculated $\overline{\text{Error}}$ for ITU-R model is 0.19, which is roughly 4 times that of the proposed approach for which $\overline{\text{Error}}$ is 0.048.

6. CONCLUSIONS

A simple but efficient interpolation approach has been presented in this study. Instead of the radar-/rain gauge derived estimates, the analysed rain characteristics and rain correlation coefficients are utilised as inputs to predict the rainfall fields at a wide range of space-time scales. Multi-resolution databases with estimated parameter values and maps of North West Europe have been created to allow users to access the key rain characteristics at any location within the area under study. This could assist potential users, such as satellite planners and designers, as the key characteristics of rain can be easily obtained without long computation time. In particular, an approach to extrapolate the fitted coefficients and/or rain characteristics in 3D space-time domain with arbitrary integration volume has been proposed. Although these quantities can be estimated at any combination of spatial and temporal integration volumes by interpolation or/and extrapolation, the results have only been tested down to 1 km spatial resolution due to the absence of higher resolution data, like rain-gauge measurements. The comparison between the predicted values and calculated values from UK NIMROD data shows a close and consistent match indicating an acceptable level of accuracy of the proposed approach.

In addition, the contour maps of 0.01% exceeded rain rates across the North West Europe and the British Isles have been generated and compared using data interpolated from 5 km to 1 km and estimated directly from 1 km UK NIMROD radar data. The comparison with ITU-R P 837-7 estimates shows that rain rate estimates using this proposed approach yield higher accuracy than the ITU-R model. The ITU-R model tends to overestimate precipitation values.

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