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Controlling Surface States of Planar Metamaterial Based on Moire Effect

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Abstract—The possibility to achieve a continuous tuning of the spectral properties in the case of two types of planar metamaterials based on the moire effect is demonstrated both experimentally and numerically. Tuning spectral characteristics are provided by changing geometric parameters of above-mentioned metamaterials. It is shown that for a one-dimensional moire metamaterial obtained by superposition of two microstrip photonic crystals with close periods, the position of the stopband in the spectrum can be controlled by changing these periods. We also consider the two-dimensional moire metamaterial formed by two identical periodic crossed structures with hexagonal symmetry. The ability to control the frequency of surface state mode by changing the crossing angle of these structures relative to each other has been shown experimentally and numerically. It is numerically demonstrated that, if the moire metamaterial is irradiated by the horn antenna, a surface wave propagating in the metamaterial plane appears in all directions beginning from its intersection point with the axis of the incident wave beam. In practice, moire metamaterials of this type can be considered as a promising prototype of microwave filters, whose spectral properties can be continuously and smoothly mechanically rearranged.

1. INTRODUCTION

Currently, the interest in research of the metamaterials based on moire effect continues to expand quite rapidly. The moire effect consists in the appearance of a pattern when two or more planar structures are superimposed with the same or similar parameters of translational symmetry. In the simplest case, we can talk about clear imposition of two identical periodic structures. Thus, the moire metamaterial as a special kind of quasi-periodic structure is formed. Obviously, its electromagnetic properties depend on the crossing angle of structures when they are superimposed (or/and displaced relative each other), on the difference between periods of these structures, etc. This can lead to the appearance of such metamaterial properties that are absent in individual structures which form above-mentioned metamaterial. A great illustration of this situation is obtained in the physics of graphene [1]. It is shown that, if the crossed graphene layers are superimposed, certain new energy states are formed on the boundary of the structure.

As well, there is a limited number of papers that study the electromagnetic properties of moire metamaterials in the optical [2, 3] and microwave [4] ranges. However, the electromagnetic properties of microwave moire metamaterials are only beginning to be explored. It occurs first and foremost due to the complexity of experiments and difficulties of theoretical description caused by their topology. In particular, the implementation of continuous/smooth frequency tuning of certain surface mode represented as a function of the crossing angle of two structures, forming moire metamaterial, has not yet been considered.

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The aim of this work is clarifying the possibility to control the surface states in some types of planar moire metamaterials by changing their geometric parameters in the microwave frequency range.

2. BAND GAP SPECTRUM OF ONE-DIMENSIONAL MOIRE METAMATERIAL

2.1. Problem Statement

As known, the moire pattern can be formed when two "combs" with close but different periods are superimposed. This effect is applied, for example, in the Vernier scale. In order to clarify the principle to control the band structure of the spectrum in one-dimensional (1D) moire metamaterials, we choose a structure, consisting of two combs in a microstrip design superimposed to each other with a different period. The combs are made from the alternating areas of a microstrip line with different widths that form a spatially limited periodic structure (Fig. 1(a)). As shown in [5, 6], each of them represents a so-called 1D photonic crystal (PC) in microstrip design. If electromagnetic wave is excited in such a 1D PC with the orientation of the field components indicated on Fig. 1(a), then a set of allowed (passband) and forbidden (stopband) bands are formed in PCs spectrum [5]. However, when these structures are superimposed, in the case of a small difference in their periods, such a moire metamaterial forms a new kind of quasi-periodic structure (Fig. 1(b)).

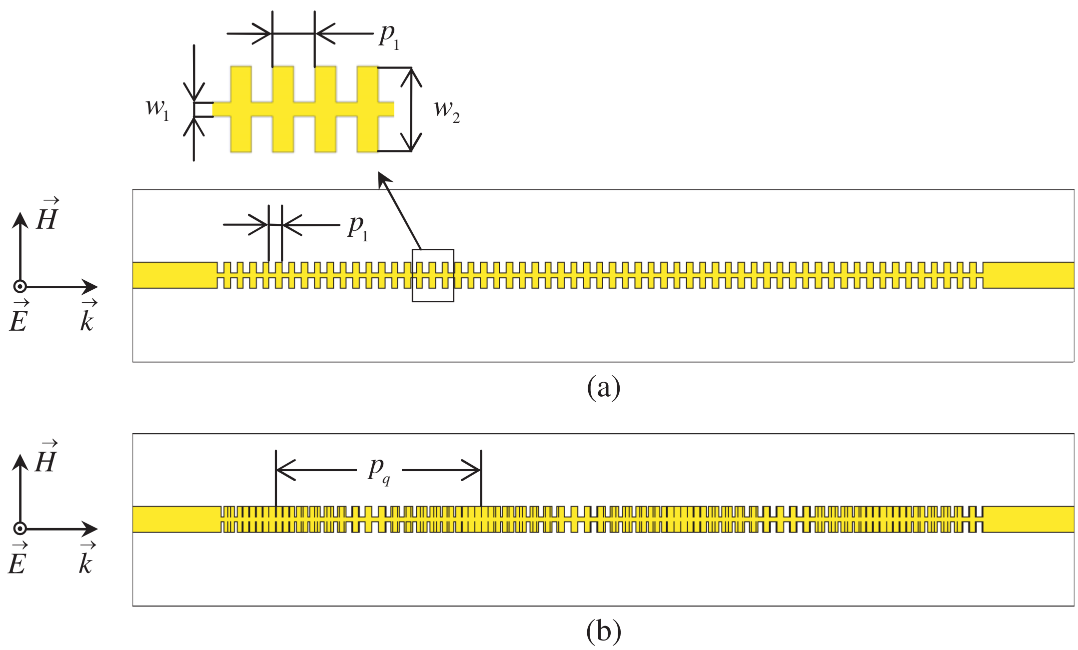


Figure 1. (a) A schematic representation of the planar structures with 90 mm long in a microstrip design: photonic crystal with a period of $p_1 = 1.5$ mm; (b) moire metamaterial formed by superposition of two photonic crystals with periods $p_1 = 1.5$ mm and $p_2 = 1.6$ mm.

It is expected to have slightly different spectrum parameters, which obviously has the band nature. Such a moire metamaterial has periodical alternating regions, for which the averaged width of the microstrip line differs significantly. Obviously, with a maximum degree of mismatch between wide and narrow sections of the superimposed structures, the average width of the combined structure (the moire metamaterial) is maximal. In the opposite case, i.e., with a minimal degree of mismatch between wide and narrow sections of the superimposed structures, the average width of the moire metamaterial is minimal. It is easy to show that the quasi-period of such a moire metamaterial p_q depends on the periods p_1 and p_2 of the PCs, as follows:

$$p_q = \frac{p_1 p_2}{|p_2 - p_1|}. \tag{1}$$

It can be seen from Eq. (1) that if the period difference of photonic crystals becomes smaller, then the quasi-period becomes larger. Thus, it is possible to choose the periods of subwavelength PCs in such a way that the quasi-period of the obtained moire metamaterial approximately corresponds to one or several half-wavelengths at the selected frequency. In this case, in the spectrum of the transmission coefficient of waves passed through the metamaterial, some new stopbands should be formed. They should lie at significantly lower frequencies than the stopbands of each formative PC.

2.2. Calculation Results and Data Analysis

In order to demonstrate the possibility of controling the position of the stopband in the spectrum of one-dimensional moire metamaterial, the following parameters are chosen for the numerical calculation: the total length of the structure is 90 mm; the period of the first PC $p_1 = 1.5$ mm; the width of the narrow section of the microstrip line $w_1 = 0.5$ mm; the width of the wide section of the microstrip line $w_2 = 3.0$ mm. The lengths of the wide and narrow sections of the microstrip line on each period for both PCs are chosen the same. The metamaterial is placed on the upper side of laminate substrate with a thickness of 1.5 mm. The laminate possesses permittivity $\varepsilon' = 3.66$ and tg$\delta = 0.004$, and it is metallized from the lower side. Metal elements are made of copper with a thickness of 0.035 mm. The calculation is carried out by the finite-difference time-domain (FDTD) method. Fig. 2 shows the calculated transmission spectrum of such a moire metamaterial for several period values of the second formative PC p_2 in the frequency range 0–15 GHz.

One can see that the dip in the spectrum of the moire metamaterial (the region of the lowest value on the transmission coefficient curve) corresponds, most likely, to the stopband. It is seen that with increase of the difference between PCs periods, this dip shifts to higher frequencies. To show that this dip really corresponds to the first stopband, the frequency dependence f_{sb} of the middle dip versus period of the second PC p_2 is presented (Fig. 3, squares). For photonic crystals, the period size approximately corresponds to half of wavelength at the frequency of the first stopband. It is easy to estimate that frequency f_r, which corresponds to this condition for several values of the quasi-period p_q, is equal to:

$$f_r = \frac{c}{2p_q\sqrt{\varepsilon_{eff}}}, \tag{2}$$

where c is the speed of light in vacuum, and ε_{eff} is the effective permittivity of the microstrip line,

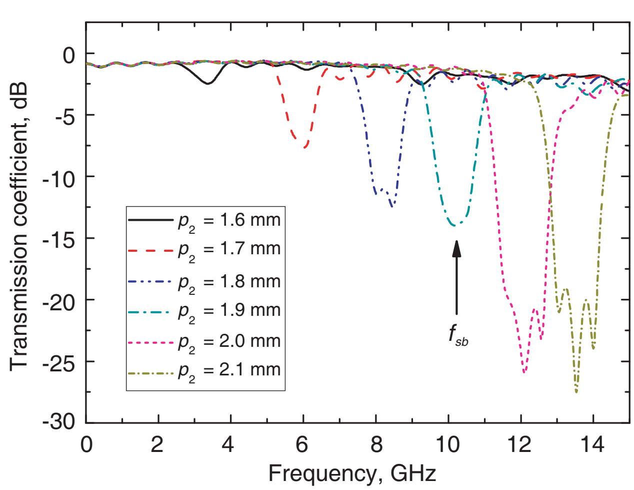


Figure 2. Transmission coefficient spectrum of the moire metamaterial in microstrip design, formed by the imposition of two photonic crystals (PC) for several values of the second PC period p_2 and with the first PC period $p_1 = 1.5$ mm.

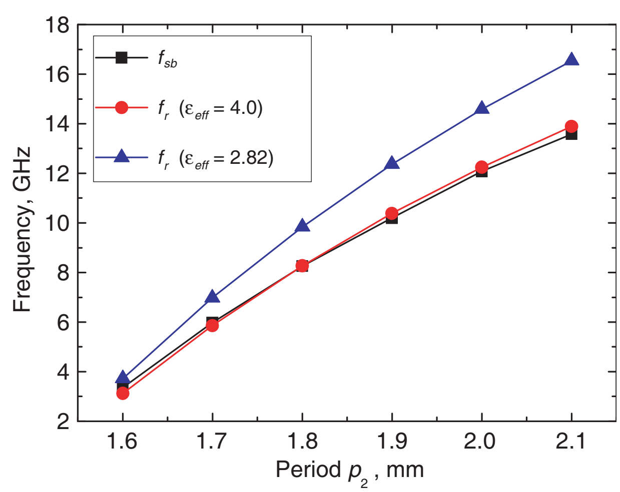


Figure 3. The frequency that corresponds to the middle of the stop band f_{sb} in the transmission spectrum of the moire metamaterial formed by imposition of two photonic crystals (PC) on the period value p_2 of the second PC.

which for a homogeneous line can be approximately calculated using the formula given in [7]. Applying this formula (for a homogeneous microstrip line) with a strip width $W = 3\,\mathrm{mm}$, we obtain $\varepsilon_{eff} \approx 2.82$. Let us construct the frequency dependence $f_r(p_2)$ versus the period of second PC p_2 for two values of permittivity: $\varepsilon_{eff} = 2.82$ (Fig. 3, triangles) and for $\varepsilon_{eff} = 4.0$ (Fig. 3, circles). It is easy to see that for the case of $\varepsilon_{eff} = 4.0$, the calculated frequency f_r well approximates the revealed middle frequencies f_{sb} of the band gap. Consequently, the wavelength into the structure at the frequency that corresponds to the middle of the stopband is smaller than in a homogeneous microstrip line.

Note that when the period of the second PC increases (see Fig. 2), the depth of the stopband also increases. This can be easily explained: with the period of second PC increasing the quasi-period according to formula (1) decreases, and at a fixed length of the resulting moire metamaterial the number of such quasi-periods also increases. As is well known, the depth of the stopband for a spatially bounded photonic crystal is in the first approximation proportional to this number.

Thus, the frequency position of the stopband in the spectrum of 1D moire metamaterial in microstrip design can be effectively controlled by varying the periods of 1D photonic crystals, which form it.

3. SURFACE STATES IN A TWO-DIMENSIONAL MOIRE METAMATERIAL

3.1. Problem Statement

Another way to design a moire pattern is the superposition of two identical periodic planar structures crossed at some angle. In order to control the frequencies of surface states in two-dimensional metamaterials based on the moire effect, two structures with hexagonal symmetry superimposed at a certain angle have been chosen. Each structure presents itself a two-periodic array of thin metal equilateral hexagons with side a, located in the nodes of a hexagonal lattice with a period p (Fig. 4(a)). When two such structures are superimposed with some crossing angle α, a quasi-periodic structure is formed in the moire metamaterial (Fig. 4(b)). It is easy to see that the unit cell of structure also has a hexagonal symmetry (as in the initial structures), but it has a larger size.

Different distances between the centers of the metal hexagons of two formative structures will be observed in different points of such a metamaterial. The area of mutual overlap is minimum at the greatest distances between the centers of metal hexagons of different structures. At the smallest

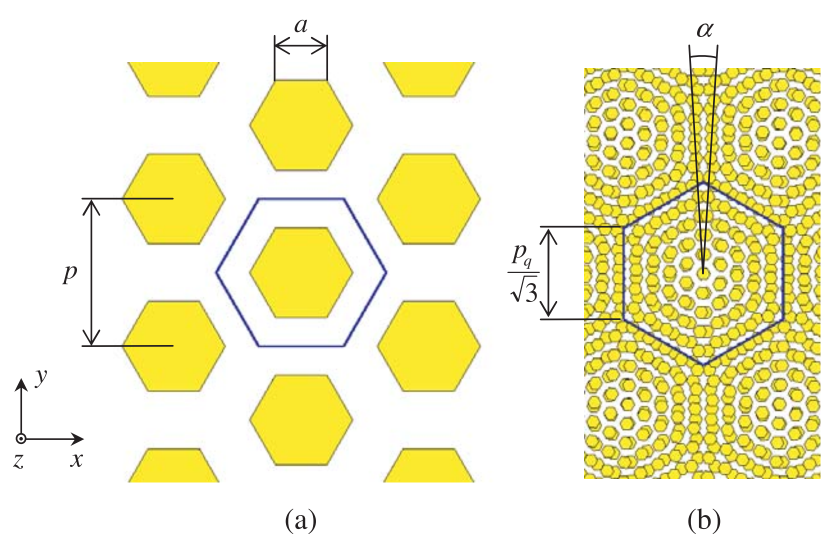


Figure 4. A schematic representation of elements which form the moire metamaterial. The moire metamaterial represented with superimposing of two identical planar structures with crossing angle α: (a) the unit cell of each planar structure; (b) the unit cell of the quasi-periodic structure at $\alpha = 7°$. The hexagonal unit cells are marked by thick lines (blue — online).

distances between the centers of metallic hexagons, they almost overlap with each other (Fig. 4(b), the central region). It can be shown that the quasi-period p_q of such a combined structure (the hexagonal moire metamaterial) depends on the period of the formative structures p, and for a small crossing angle α it is equal [1] to:

$$p_q = \frac{p/2}{\sin(\alpha/2)}. \tag{3}$$

From Formula (3) it can be seen that with decrease of the crossing angle of formative structures, the value of the quasi-period increases. Our numerical calculation has shown that in a certain range of ratios between the side of hexagon a and the period of the forming structure p in the hexagonal moire metamaterial, almost continuous metallic stripes of irregular shape ("current paths") are formed along the unit cells borders of the moire metamaterial (Fig. 4(b)). As a result, the surface modes appear in the metamaterial, and this process has the percolative characteristic.

In particular, if a/p values are too large, the metal stripes are very wide, up to the case that most of the metallic elements overlap with each other. In such a structure, there are practically no conditions for the formation of the flow of large closed (ring-like) conductive currents on the surface of the moire metamaterial. Therefore, no surface modes arise here.

Further, as a/p decreases, at a certain threshold value (the "upper percolation threshold"), the above-mentioned metal strips of irregular shape ("current paths") are quite narrow and almost continuous, so closed conductive currents of appreciable magnitude can flow on moire metamaterial surface. Then, surface modes of noticeable magnitude arise in the moire metamaterial.

Subsequently, there is a tendency to the formation of gaps in metal strips at certain crossing angles of the structures, with a decrease of a/p, at certain "lower percolation threshold". As a result, the intensity of the surface modes decreases sharply.

Our numerical calculations have shown that for such a type of moire metamaterial, there is only a narrow range of a/p values (approximately found as 0.33–0.38) at which surface states (modes) appear. Within this range, gaps of the transmission spectrum, whose frequencies depend on the crossing angle of the formative structures, should occur. We will discuss this phenomenon in more detail in the next subsection.

3.2. Numerical Evaluation of Frequency-Angular Characteristics of Surface States in Moire Metamaterial

In order to control the frequencies of surface states in two-dimensional moire metamaterials based on superimposed structures, the following parameters are chosen: the period of the hexagonal lattice of formative structures is $p = 2$ mm; the size of the side of copper hexagons is $a = 0.7$ mm; and their thickness is 0.035 mm. Structures are put close to each other with a crossing angle α. The calculation is carried out by the FDTD method for the frequency range of 1–15 GHz and crossing angles range $\alpha = 3$–12° (Fig. 5) with a normal incidence of electromagnetic waves.

One can see from Fig. 5 that in a certain range of frequencies and crossing angles of structures (marked as rectangular area in the figure), there is a resonant minimum (see blue regions in inset of Fig. 5) for transmission of waves, which shifts to higher frequencies with increasing crossing angle. This shift of the resonance should be explained by the fact that the length of the above-mentioned formed metal strips ("currents paths") depends on the crossing angle between the structures. This resonance in the spectrum corresponds to one of the surface state modes in this structure. Note that in the range of angles of 8.5–11.5° a monotonous increasing the frequency of this surface state mode is observed with increase of crossing angle. Note that other more high-frequency resonance gaps are also observed in the spectrum. They are caused by the flowing of current through other shorter paths in the metamaterial. However, they have a significantly lower intensity and a narrower range of tuning angles.

Note that while irradiating such hexagonal moire metamaterials with a spherical wave front (for example, from a horn antenna), a kind of very interesting surface waves at certain resonant frequencies may be formed. They propagate from the point on the metamaterial, which is closest to the irradiation source.

At the same time there is a minimum of the transmission coefficient in the spectrum is observed, and a significant part of the incident energy passes to the surface wave. Fig. 6 shows an example

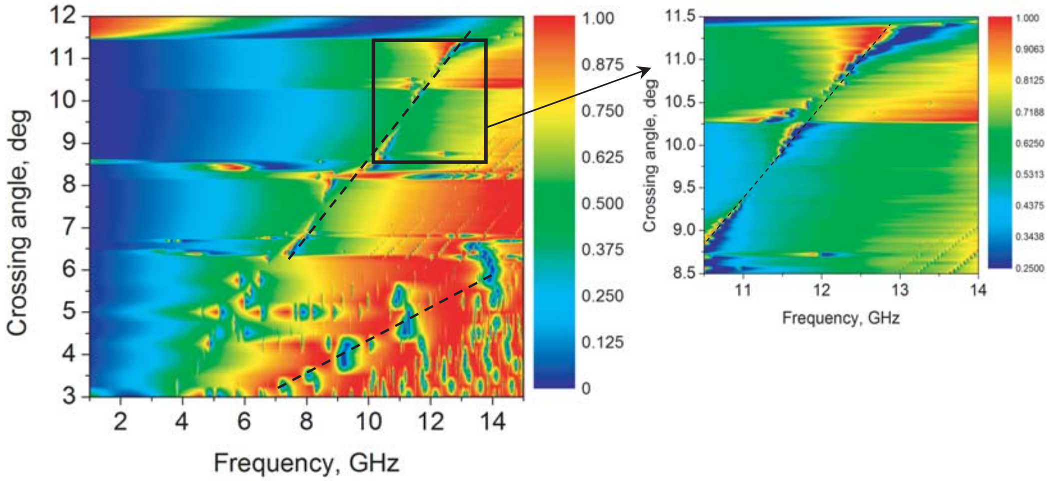


Figure 5. The transmission coefficient versus the frequency and crossing angle of structures that form the moire metamaterial. Dashed lines mark the most intensive modes of surface states.

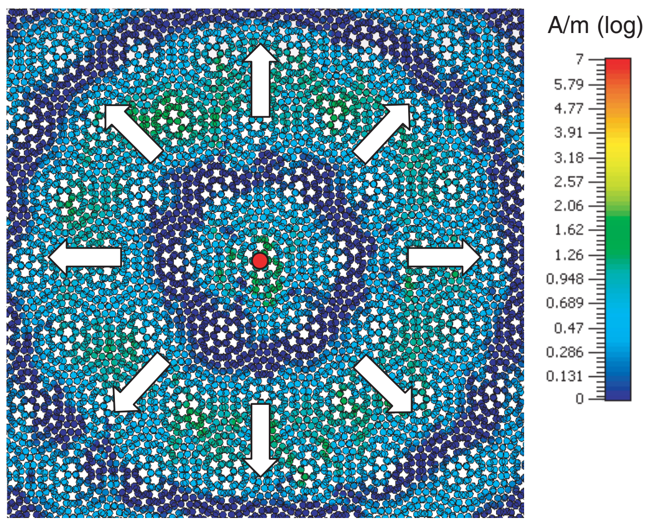


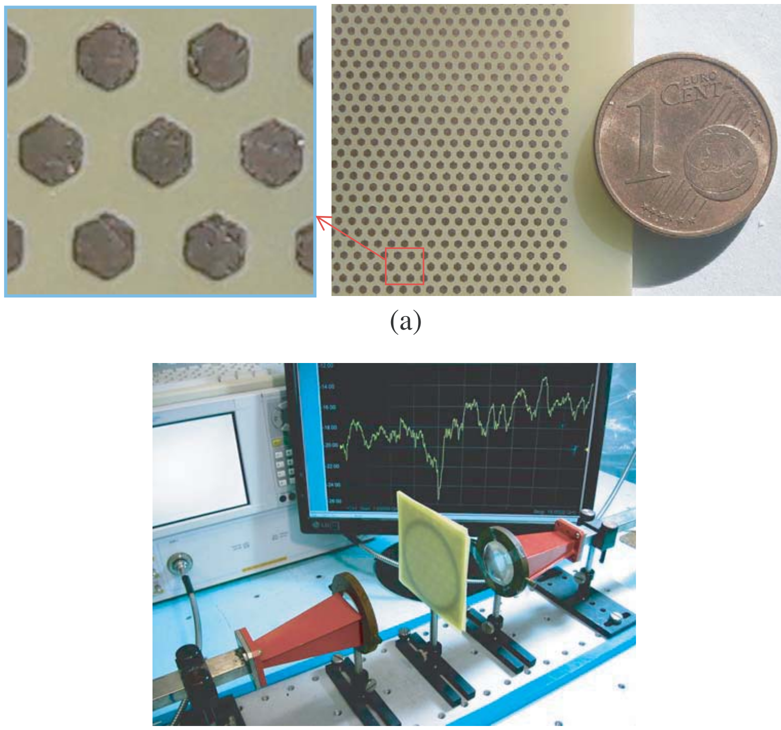
Figure 6. The distribution of the surface current amplitude for the moire metamaterial, formed by the superposition of two identical planar hexagonal structures crossed at an angle $\alpha = 10°$, with its normal irradiation by the horn antenna at the resonant frequency $f_{res} = 12.088$ GHz.

of the distribution of the surface current magnitude for moire metamaterial formed by two hexagonal structures with a period $p = 2$ mm crossed at angle $\alpha = 10°$ at the resonant frequency $f_{res} = 12.088$ GHz with normal incidence of waves from the rectangular horn antenna with an aperture of $33\,\text{mm} \times 47\,\text{mm}$ from a distance of 100 mm to the metamaterial plane.

Thus, it can be seen that at the resonant frequency with a certain crossing angle of the formative structures in the moire metamaterial, a surface wave is formed. It spreads in concentric circles from the center of the structure (red dot in Fig. 6), through which the axis of the horn antenna passes. The arrows indicate the direction of propagation of the surface wave.

3.3. Test Experiments and Data Analysis

With the purpose of experimental verification of the revealed resonant properties of two-dimensional metamaterials using the moire effect, the hexagonal planar structure has been fabricated (Fig. 7(a)).



(a)

(b)

Figure 7. (a) The view of a two-periodic array of equilateral thin metal hexagons on a dielectric (FR-4 laminate) substrate, that used as one of the crossed (superimposed) structures for the moire metamaterial; (b) The view of the experimental setup to study the moire metamaterial.

The period of the hexagonal lattice of formative structures is chosen as $p = 1.125\,\text{mm}$, and the size of the side of metal hexagons is $a = 0.375\,\text{mm}$. The hexagons are made of copper with a thickness of 0.035 mm, placed on a layer of FR-4 laminate ($\varepsilon' = 4.35$, $\text{tg}\delta = 0.022$) with a thickness of 1.5 mm. The metamaterial consists of two structures that are put close to each other by the sides with copper hexagons at some crossing angles (which can be varied smoothly). It is important to ensure the absence of air gaps between the structures, since they strongly influence the magnitude of the resonance minimum in the spectrum.

The experimental setup (Fig. 7(b)) consists of the metamaterial under study with a size of $100\,\text{mm} \times 100\,\text{mm}$ placed between two irradiating rectangular horn antennas [8]. Aperture of horns is $33\,\text{mm} \times 47\,\text{mm}$, and their length equals 100 mm. Antennas are located on the same axis that passes perpendicularly to the metamaterial plane through its center. The measurement circuit is fitted to the Vector Network Analyzer *Agilent* $N5230A$ by coaxial-waveguide adaptors and coaxial cables. Using the Network Analyzer, parameter S_{21}, which in fact is the transmission coefficient for waves passed normally through the metamaterial in the frequency range 7–15 GHz, is registered.

In order to compare the experimental data with the results of numerical simulation, the frequency-angular dependence of the transmission coefficient of wave with normal incidence passing through the metamaterial is registered (Fig. 8). Red circles mark those theoretical values of frequencies of the minimum transmission coefficient at several selected crossing angles of the formative structures, which are then compared with the experimentally obtained data.

During measuring the resonant frequencies, which are caused by the appearance of the surface states, certain control crossing angles of the formative structures are selected. It can be seen that there is good qualitative agreement between the experimentally obtained frequencies of the surface state (Fig. 8, black squares) and the results of numerical simulation (Fig. 8, red circles).

Note that, as from the results of numerical simulation, the depth of the resonant minimum can be increased by using a substrate material with lower losses for this frequency range.

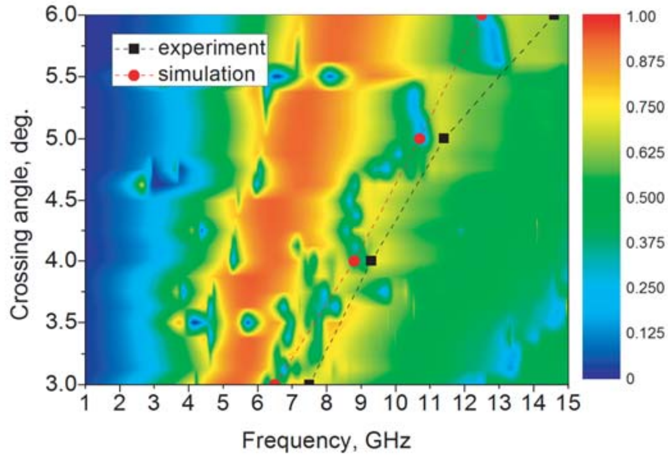


Figure 8. Experimental and calculated frequency of the surface state mode in the hexagonal moire metamaterial, depending on the crossing angle of formative structures.

4. CONCLUSIONS

Thus, the possibility to achieve a continuous tuning of the spectral properties in the case of two types of planar metamaterials based on the moire effect by changing their geometric parameters is experimentally and numerically demonstrated. It has been shown that:

1. The quasi-periodic moire metamaterial, obtained by superposition of two one-dimensional periodic photonic crystals in microstrip design with close periods, demonstrates unexpectedly deep stopbands in the transmission spectrum. In this case, the position of the stopband can be controlled by varying periods of photonic crystals, which form the moire metamaterial.

2. The quasi-periodic moire metamaterial formed by two identical superimposed crossing structures with hexagonal symmetry is studied. The resonant minima, corresponding to surface states, are observed in the transmission spectrum. The possibility to control the frequency of such a surface state by varying the crossing angle between the formative structures has been shown both experimentally and numerically. The range of ratio between the size and period of elements in the formative structures, at which the frequency of tunable surface state in moire metamaterial can be excited, has been numerically determined.

3. It is numerically demonstrated that at certain parameters of studied moire metamaterial, irradiated with horn antenna, the surface wave spreading in the plane of the metamaterial in all directions from the intersection point with the axis of incident wave beam takes place.

In conclusion, we note that from an application point of view, metamaterials of such a type can become the prototype of microwave filters, whose spectral characteristics can be continuously and smoothly mechanically tuned. Note that the implementation of magnetoactive elements in such structures is of special interest.

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