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An Improved Taguchi's Method for Electromagnetic Applications

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Abstract—An improved Taguchi's method (ITM) is proposed in this paper. The dynamic reduced rate function linked with the contributions of each parameter is used to increase the convergence speed. An extra procedure is added in the ITM to determine whether the experiment results in orthogonal array meet the termination criterion which is neglected in the traditional Taguchi's method (TM). Three experiments, including the syntheses of linear arrays and the designs of an E-shaped antenna and an ultra-wide band monopole, are conducted to investigate the performance of the proposed method, and the results are compared with those of traditional TM and other meta-heuristic methods. The results show that the same or even better results are obtained by the improved TM with fast convergence speed.

1. INTRODUCTION

Optimization methods have drawn a lot of attention because of their wide applications, and most of them can be divided into two categories: global and local techniques. A global optimization method, Taguchi's method (TM), was developed based on an orthogonal array (OA) and a reduced function (RF) for the syntheses of linear antenna arrays [1]. The main role of RF is reducing the optimization ranges to coverage to the best design parameters after several iterations. Two RFs, the exponential reduced function (ERF) and Gaussian reduced function (GRF), were proposed in [2] simultaneously.

TM has the advantages of fast convergence speed and good search ability to find the best solution. Moreover, the method has been applied in electromagnetic (EM) optimization problems, such as the synthesis of linear arrays, ultra-wideband (UWB) antenna and planar filter designs [2–5]. The experiment results have shown that the best results are obtained with fast convergence speed. However, the convergence rate of TM is highly related to the value of reduced rate (RR) which is manually set between 0.5 and 0.95 to ERF or a positive integer greater than 1 to GRF. An improved TM was proposed in [6] that the value of RR is linked to a parameter's contribution toward performance variance for design of a line-start permanent magnet synchronous motor. However, no studies have investigated the algorithm performance for electromagnetic applications and the performance comparisons with the other classical algorithms.

Compared with traditional optimization techniques, TM guarantees to find the optimum solution with fast convergence speed [2]. However, the best solution of current iteration is confirmed by the analysis of mean's (ANOM) response table, and the confirmation experiment result is regarded as the final result of the current iteration. This procedure may neglect the situation that the result of the trial conducted in OA has met the termination requirement, so extra experiment should be conducted.

In this paper, an improved Taguchi's method (ITM) that combines the dynamic reduced function in [6] and the analysis of the trial results in OA is proposed. An extra procedure is added in the proposed algorithm to determine whether the trial result in OA meets the termination requirement. The proposed algorithm is applied to the problem of the linear antenna array synthesis and antenna

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designs. For the first problem, the amplitudes of twenty equally spaced isotropic elements are optimized for null control in specified directions, and the locations of ten elements are optimized for sidelobe level (SLL) suppression. For the second problem, ITM is combined with full wave simulation software to design an E-shaped patch antenna and an ultra-wide band monopole antenna, respectively. The optimized results are compared with that of TM in [1] and other traditional meta-heuristic methods, such as particle swarm optimization (PSO) and differential evolution (DE). For the linear antenna array synthesis, the results show the efficiency of ITM that at least 80% and 50% reductions in the number of fitness function evaluations have been achieved compared with that of meta-heuristic methods and TM, respectively. For antenna designs, the performances of ITM are better than the results of TM with less number of objective function evaluations.

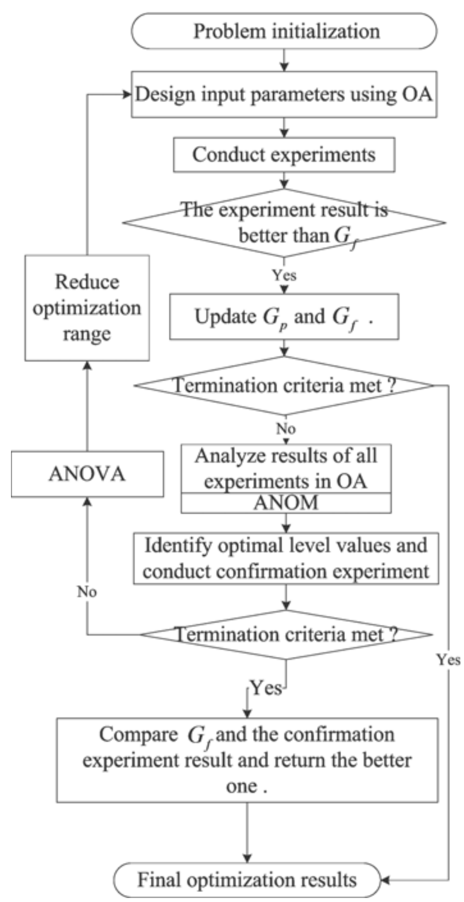
2. THE IMPROVED TAGUCHI'S METHOD

Compared with TM [1], a dynamic reduced rate instead of a constant reduced rate is adopted in ITM. Moreover, OA's experiments in each iteration except the confirmation experiment are checked if any experiment result has met the termination criteria. Two extra terms, denoted by G_p and G_f, will be updated to the values of parameters and objective function if the result of current OA' s experiment is better than the last experiment's. If G_f meets the termination criteria, then return G_p as the optimal solution. The pseudo code and flowchart of IRM are shown in Fig. 1. The detailed procedures of the proposed method are summarized as follows.

1) Problem initialization, which includes the selection of a proper OA, the bounds of optimization parameters, the design of a suitable fitness function, and the initialization of G_p and G_f.

Define objective function $f(\mathbf{x}), \mathbf{x} = (x_1, \dots, x_d)^T$ and select a proper OA.
Set the values of $R_{\min}, R_{\max}$ and k.
Initialize G_p and G_f.
while (criterion is not met or $LD_i / LD \geq k$) **do**
  if it is the first iteration
    $x_i\big|_2^1 = (\text{ub}_i - \text{lb}_i)/(\text{level}+1),\ x_i\big|_1^1 = x_i\big|_2^1 - LD_i^1,\ x_i\big|_3^1 = x_i\big|_2^1 + LD_i^1, (1 \leq i \leq d)$
  else
    $x_i\big|_2^{t+1}$ is the optimal level value of ith parameter in the tth iteration.
    $x_i\big|_1^{t+1} = x_i\big|_2^{t+1} - LD_i^{t+1},\ x_i\big|_3^{t+1} = x_i\big|_2^{t+1} + LD_i^{t+1}, (1 \leq i \leq d)$
  end if
  Design input parameters using OA.
  for $n = 1$ to the number of experiment in the selected OA
    Calculate the value of the *nth* experiment $f(\mathbf{x}_n^{t+1})$.
    if $f(\mathbf{x}_n^{t+1})$ is better than G_f
      $G_f = f(\mathbf{x}_n^{t+1}),\ G_p = \mathbf{x}_n^{t+1}$.
    if G_f has met the termination criterion. Return G_p and G_f as final optimization results.
    end if
  end for
    Calculate σ^2 by ANOVA
    Identify optimal level values by ANOM.
    Conduct confirmation experiment.
    Check the termination criteria. Reduce the optimization range if the termination criterion is not met.
end while
Compare the result of confirmation experiment in the last iteration and G_f, return the better one as the final optimization results.

(a)



(b)

Figure 1. (a) The pseudo code and (b) the flow chart of the proposed algorithm.

2) OA input parameter allocation. When a proper OA is selected, the corresponding numerical values for each level of a parameter should be determined in order to conduct the trial.

3) Conduct experiments and analyze results. After determining the input parameter, all OA's experiments have been conducted, and the fitness function of each experiment can be obtained. In this step, the experiment result is compared with the value of G_f. G_p and G_f are updated to the parameters and result of current experiment if the experiment result is better than G_f's. If the results of all OA's experiments do not meet the termination criteria, all values are converted to signal-to-noise (S/N) ratio. The analysis of mean's (ANOM) response table is built and used to identify the optimum values of each parameter by studying the main effects of each level.

4) Identify optimal level values and conduct confirmation experiment. The optimal level of each parameter is identified by the largest S/N ratio value. When the optimal levels of parameters are identified, a confirmation experiment is performed using the optimal level values, and the fitness function value is regarded as the fitness value of current iteration.

5) Check the termination criteria. The optimization is terminated when the fitness function meets the requirements or when the level difference (LD) rate, the value of $\mathrm{LD}_i/\mathrm{LD}_1$, gets lower than a converged value denoted by k which can be manually set between 0.001 and 0.01.

6) Reduce the optimization range. When the current iteration does not meet the termination criteria, the next iteration, in which the optimal level values of the current iteration are used as central values for the next iteration and the ranges of parameters reduced, is required. The RR multiplied with the LD_i of current iteration is the LD_{i+1} of the next iteration. LD_i is calculated by

$$\mathrm{LD}_i = r \times \mathrm{LD}_{i-1}. \tag{1}$$

For TM, r, which is manually set between 0.5 and 1, is a constant, and the LD rate is an exponential function that $\mathrm{LD}_i = (r)^{i-1} \times \mathrm{LD}_1$. A dynamic RR (DRR) which is linked to a parameter's percentage contribution toward performance variance obtained from the analysis of variance (ANOVA) was proposed in [10],

$$r = [r_{\max} - r_{\min}]\frac{\sigma^2}{100} + r_{\min} \tag{2}$$

where $r_{\max}$ and $r_{\min}$ are the maximum and minimum RR values, and σ^2 is the percentage contribution toward performance variance obtained from ANOVA. It is easy to figure that static RR is obtained when $r_{\max}$ equals $r_{\min}$, and an exponential reduced function is obtained. When the values of $r_{\max}$ and $r_{\min}$ are different, the minimum RR will never be less than $r_{\min}$, and the higher the variance contribution is, the higher the RR is obtained. The detailed procedure of ANOVA can be found in Chapter 6 of [7].

7) For a minimization problem with no determined value to end the optimization procedure, after the value of $\mathrm{LD}_i/\mathrm{LD}_1$ becomes smaller than k, the result of confirmation experiment in the final iteration is compared with G_f, and the best value is regarded as the final result.

3. NUMERICAL RESULTS

3.1. Linear Array Synthesis

Antenna array synthesis is an important area in EM and antenna engineering. In this part, a linear array of $2N$ isotropic elements will be optimized by ITM. Two kinds of linear antenna arrays are used here: null control in specified directions and SLL suppression. Fig. 2 shows a linear antenna array with $2N$ elements placed along the z axis, which is symmetric along the x axis. The array factor of a linear array with $2N$ elements can be written as:

$$\mathrm{AF}(\theta) = 2\sum_{n=1}^{N} a(n)e^{j\varphi(n)}\cos\left[\beta d(n)\cos\theta\right] \tag{3}$$

where β is the wavenumber; $a(n)$, $\varphi(n)$, and $d(n)$ are the excitation magnitude, phase, and location of the nth element, respectively.

The following fitness function [1] is used for this problem.

$$\mathrm{Fitness} = 2\int_{0^\circ}^{90^\circ}\left[\mathrm{AF}(\theta) - \mathrm{AF}_d(\theta)\right]\left[\frac{1+\mathrm{sgn}(\mathrm{AF}(\theta) - \mathrm{AF}_d(\theta))}{2}\right]d\theta \tag{4}$$

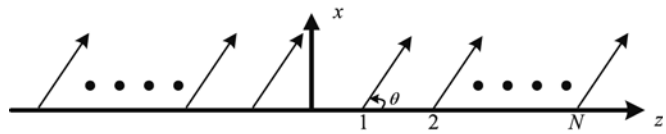


Figure 2. Geometry of a $2N$-element linear array.

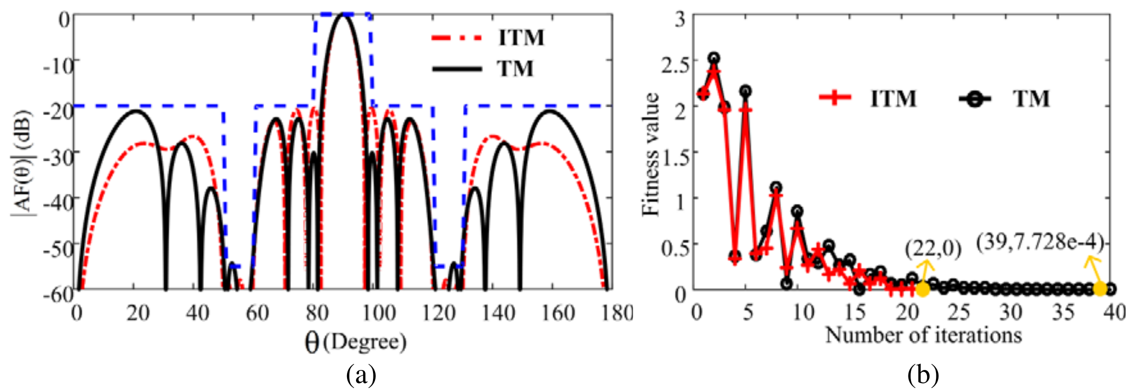


Figure 3. Null controlled patterns (a) and fitness value (b) comparisons between ITM and TM.

where $\mathrm{AF}_d(\theta)$ is the desired array factor, and sgn is a sign function whose value is either 1 or -1.

For the null control synthesis problem, 20 antenna elements are equally spaced along the z axis, and the phase of each element is equal to zero. A wide null is desired to exist between 50° and 60°, having a magnitude of less than -55 dB. Moreover, the SLL should be lower than -20 dB. Owing to the symmetry, only 10 excitation magnitudes are optimized. Thus, an OA (27, 13, 3, 2), from which 10 columns are selected, is adopted in this problem. The magnitude of each element will be optimized in the range of [0, 1], and the termination criteria are the fitness value equal to zero or $k < 0.002$. The performances of ITM and TM are presented in Fig. 3. For ITM, the minimum and maximum RRs are set to 0.82 and 0.95, respectively. The optimization process is terminated at the 20th iteration, and the fitness value is equal to 0. The optimized magnitude values of ten elements are [0.503, 0.603, 0.46, 0.501, 0.357, 0.372, 0.394, 0.353, 0.221, 0.114]. For TM, r and k are set to 0.85 and 0.002, respectively. The optimization process is terminated at the 40th iteration, and the fitness value is less than 0.001. The optimized magnitude values of ten elements are [0.526, 0.556, 0.517, 0.456, 0.309, 0.34, 0.241, 0.343, 0.14, 0.141]. In addition, the same problem is optimized using classic PSO method, and 200 independent runs have been performed. The results are depicted in Fig. 5(a) that the mean number of experiments conducted using PSO is 4239. 1092 experiments are needed for TM to achieve the same goal. However, only 616 experiments are required by ITM.

For SLL suppression problem, ten linear unequally spaced elements with a uniform amplitude excitation ($a(n) = 1$) and no phase difference ($\varphi(n) = 0$) are adopted. The locations of the elements are optimized within a given size of 5λ. The goal of this problem is to suppress the SLL under -19.1 dB between 0° and 78°. An OA (18, 5, 3, 2) is adopted in this problem. The initial level-2 values (element locations) of the parameters are set to [0.25λ, 0.75λ, 1.25λ, 1.75λ, $2{,}25\lambda$], and the level difference of the first iteration is set to quarter wavelength. The fitness values and optimized results of TM and ITM are presented in Fig. 4. For ITM, the minimum and maximum RRs are set to 0.65 and 0.95, respectively. Moreover, the optimized locations of elements are [0.207λ, 0.639λ, 1.092λ, 1.653λ, 2.317λ]. For TM, r is set to 0.75, and the best locations are [0.206λ, 0.652λ, 1.113λ, 1.687λ, 2.356λ]. Almost half of the number of experiments conducted are reduced by the ITM in this case. An improved PSO method [8] involves an OA which is used to initialize the positions of particle and classic PSO that are used for comparisons. Both PSO initialized by OA (OA-PSO) and classic PSO are separately conducted for 200 independent runs, and the mean values of the number of experiments are shown in Fig. 5(b). It is clear that ITM has fast convergence speed and 50%, 85%, and 90% reductions of the total number

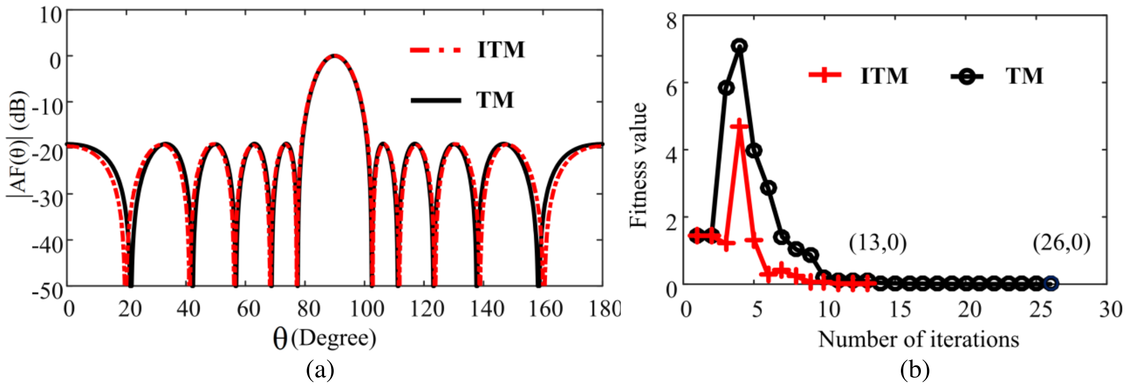


Figure 4. SLL suppression patterns (a) and fitness value (b) comparisons between ITM and TM.

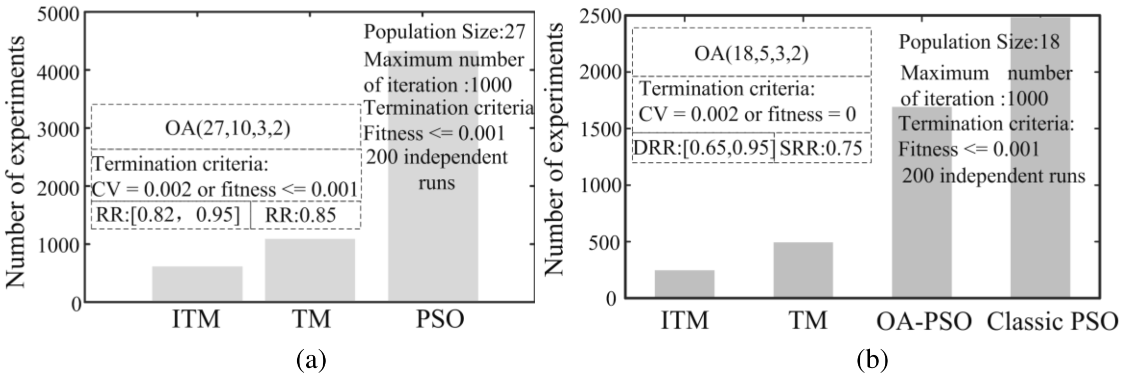


Figure 5. The comparisons of the number of experiments needed to achieve the best result between ITM, TM, OA-PSO and PSO. (a) Null controlled in specified directions case and (b) SLL suppression case.

of experiments compared with TM, OA-PSO, and classic PSO, respectively. These results show the efficiency of ITM in dealing with linear antenna array synthesis problem.

3.2. E-Shaped Patch Antenna Design

An E-shaped patch antenna with an air substrate of 5.5 mm thickness is used as an example to demonstrate the validity of ITM for antenna optimization with configuration restrictions. The same optimization problem is tackled in [9, 10], where wind driven optimization (WDO) and self-adaptive DE are utilized. The size of ground plane is $60 \times 60\text{-mm}^2$, and the patch geometric center is superposition with the center of ground plane. Two identical parallel slots ($Ws \times Ls$) are incorporated into the rectangular patch, and a coaxial feed structure is used for feeding. Moreover, the center length between the slot and patch is Ps. Furthermore, the patch length and width are L and W, respectively. In this design, there are six parameters to be optimized, and the design goal is to minimize the S_{11} magnitude in two frequencies 5 GHz and 5.5 GHz.

The following fitness function is used in this problem.

$$F(\mathbf{x}) = \text{maximize}\left[\min\left\{|S_{11}(\mathbf{x})|_{f=5\,\text{GHz}},\ |S_{11}(\mathbf{x})|_{f=5.5\,\text{GHz}}\right\}\right] \tag{5}$$

where $\mathbf{x}$ is the vector of design parameters, and $|\bullet|$ denotes the absolute value of S_{11} in decibel scale at specified frequency.

The first 6 columns are selected from OA (18, 7, 3, 2) to obtain a proper OA (18, 6, 3, 2) which offers six columns for W, L, Px, Ls, Ws, and Ps. The dynamic RR is chosen to vary from 0.75 to 0.85

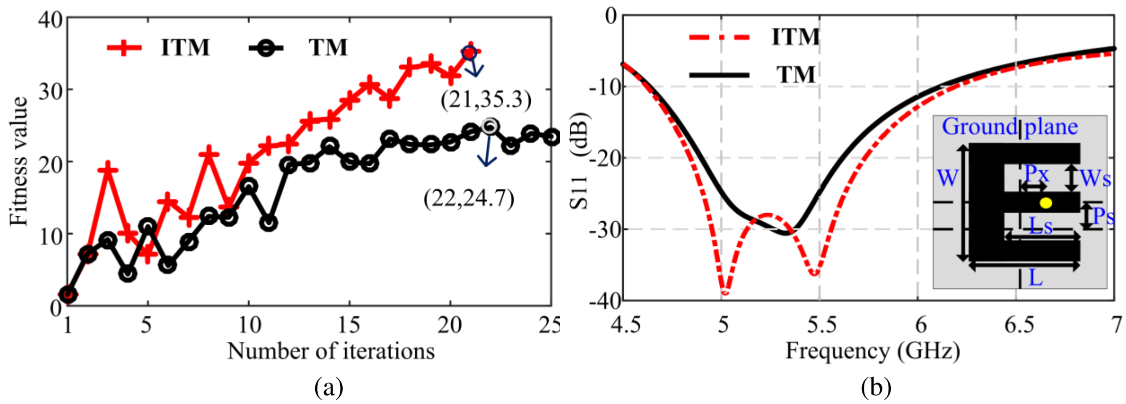


Figure 6. Convergence curves (a) and Reflection coefficients (b) of the optimized E-shaped antenna.

for ITM, and the value of r is 0.8 for TM. In addition, the design bounds and restrictions are listed in Table 1 to maintain the E-shape of the antenna.

The fitness values obtained by two different kinds of RR are shown in Fig. 6(a). For the experiment which breaks restrictions, the fitness value is set to 0. For the ITM, the termination criterion is met at the 21th iteration, and the maximum value of 35.3 dB is obtained. However, the global maximum value, 36 dB, is obtained by the 15th OA's experiment in the 19th iteration. For TM, the termination criterion is met at the 25th iteration, and the maximum fitness value is 24.7 dB at the 22th iteration. As we know, with the increment of the iteration, the level difference decreases, and the fitness values change slowly. The return losses of the best configuration found by the ITM and TM are presented in Fig. 6(b). The impendence matching bandwidth obtained by ITM is between 4.63 and 6.19 GHz, while the design of TM has similar impendence matching bandwidth between 4.63 and 6.11 GHz. Moreover, the best design parameters of ITM and TM are listed in Table 1. Table 2 shows that a lower S_{11} value of -36 dB is obtained, and almost 60% to 80% reductions of the total number of experiments are achieved using ITM compared with other meta-heuristic optimization methods.

Table 1. Dimensions of the optimized E-patch antenna (mm).

	L	*W*	*Px*	*Ls*	*Ws*	*Ps*
Design bounds	[10,30]	[10,50]	[-15,15]	[0.1,30]	[0.5,20]	[0.25,20]
Restrictions	0.5<*Ws*<(*W*/2), 0.5<*Ls*<*L*, (*Ws*/2)<*Ps*<(*W*/2)-(*Ws*/2), |*Px*|<(*L*/2)					
TM	21.17	41.03	4.52	19.44	6.67	5.79
Proposed	20.75	48.68	6.05	16.97	4.4	5.96

Table 2. Performances of WDO, IWPSO, CFPSO, DE, SADE, TM and ITM for E-shaped patch antenna case.

	WDO [9]	IWPSO [10]	DE [10]	SADE [10]	TM [1]	This work
Max{S11}	-31	-24.1	-30.5	-34	-24.7	-36
Number of Experiments	2000				418	399

3.3. UWB Monopole Design

A UWB monopole is investigated here to demonstrate the efficiency of the proposed method. The same optimization problem is tackled in [11] by using gravitational search algorithm (GSA). However, the detailed parameters of GSA are not given. The basic configuration and detailed parameters are depicted in Fig. 7. A 50 Ω microstrip with the size of $L_f \times W_f$ is used to feed the antenna. The antenna is fabricated on an FR-4 substrate with the height of 1.5 mm. The irregular radiator shape is represented as a group of polylines, and the coordinates of each polyline are shown in Fig. 7(a) too. Due to the symmetry along the y-axis, only 10 parameters (5 points) are optimized.

The objective function is designed as follows.

$$\text{Minimize } F = \frac{1}{n}\sum_{i=1}^{n} Q(f_i) \tag{6}$$

$$\text{s.t. } Q(f_i) = \begin{cases} -10 & S_{11}(f_i) \leq -10 \\ S_{11}(f_i) & \text{otherwise} \end{cases}$$

where f_i denotes the ith sampling frequencies within the given operating band 3.1–10.6 GHz. $S_{11}(\cdot)$ is the reflection coefficient, and n is the number of sampling frequencies. The stopping criteria are $F = -10$ or $k \leq 0.01$.

The optimization range of each parameter is shown in Fig. 7(c), and overall 10 parameters are optimized in this case. Hence, OA (27, 10, 3, 2) is selected for this problem. Both $r_{\min}$ and $r_{\max}$ are set to 0.8, and a fixed RR is obtained for ITM. To investigate the performance of checking the OA's experiment results, r is set to 0.8 for the TM, so the RF is the same with ITM. The convergence curves and the final reflection coefficients obtained by using TM and ITM are shown in Fig. 8. It can be observed that the best solution is obtained by the 5th experiment in the 6th iteration, and 145

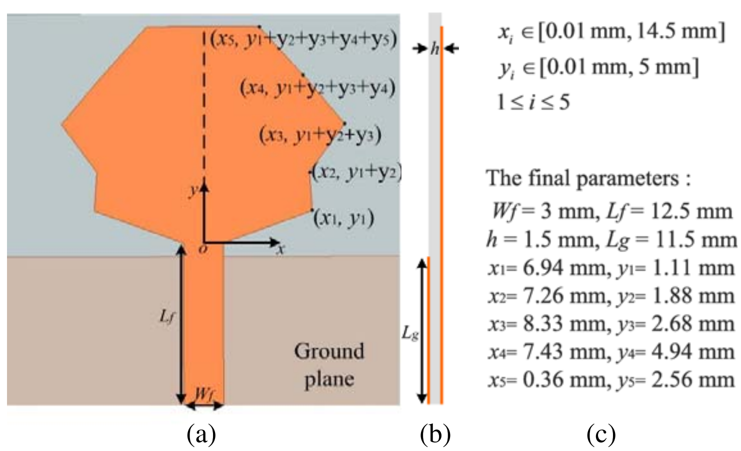


Figure 7. Geometry of the monopole design. (a) front view (b) side view and (c) optimization range and final optimized value of each parameter.

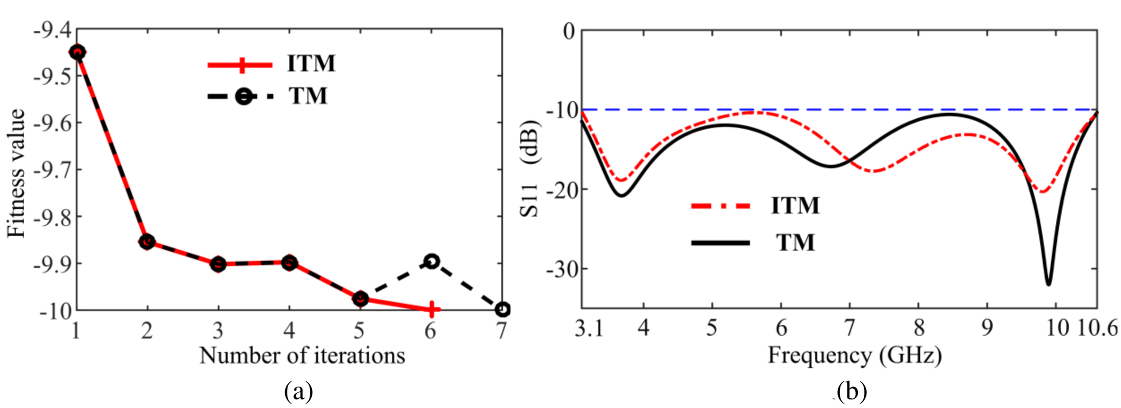


Figure 8. Convergence curves (a) and reflection coefficients (b) of the UWB monopole.

experiments ((27+1)×5+5) in total are conducted by the ITM. However, 196 experiments ((27+1)×7) are conducted by the TM. It comes out that more than 51 experiments are conducted unnecessarily. The final parameters obtained by the ITM are shown in Fig. 7(c). From this experiment, the results show that it is necessary to check whether the result of OA's experiment meets the termination criterion to improve the performance of TM.

4. CONCLUSION

In this paper, an improved Taguchi's method is adopted for EM and antenna optimization. The ITM has been used for the synthesis of linear antenna arrays and the designs of an E-shaped patch antenna and a UWB monopole. Moreover, the ITM performances are compared with performances of TM and other classic meta-heuristic methods. As expected, the ITM convergence rate is the best in these methods, which demonstrates the validity and efficiency of the ITM for these EM optimization problems.

REFERENCES

1. Weng, W. C., F. Yang, and A. Elsherbeni, "Linear antenna array synthesis using Taguchi's method: A novel optimization technique in electromagnetics," *IEEE Trans. Antennas Propag.*, Vol. 55, 723–730, 2007.
2. Weng, W. C., F. Yang, and A. Elsherbini, *Electromagnetics and Antenna Optimization Using Taguchi's Method*, Morgan & Claypool, San Rafael, CA, 2007.
3. Dib, N. I., S. K. Goudos, and H. Muhsen, "Application of Taguchi's optimization method and self-adaptive differential evolution to the synthesis of linear antenna arrays," *Progress In Electromagnetics Research*, Vol. 102, 159–180, 2010.
4. Ghayoula, E., A. Bouallegue, N. Fadlallah, R. Ghayoula, and A. Smida, "Concentric ring array synthesis using Taguchi algorithm for MIMO applications," *2017 International Conference on Control Automation and Diagnosis (ICCAD)*, 373–378, Hammamet, Tunisia, Aug. 2017,
5. Weng, W. C., F. Yang, and A. Z. Elsherbeni, "Electromagnetic optimization using Taguchi's method: A case study of band pass filter design," *IEEE International Symposium on Antennas and Propagation*, Honolulu, HI, Jun. 2007.
6. Sorgdrager A. J., R. Wang, and A. J. Grobler, "Multiobjective design of a line-start PM motor using the taguchi method," *IEEE Transactions on Industry Applications*, Vol. 54, No. 5, 4167–4176, Sept. 2018.
7. Roy, R., *A Primer on the Taguchi Method*, Van Nostrand Reinhold, New York, USA, 1990.
8. Bayraktar, Z., P. L. Werner, and D. H. Werner, "Miniaturization of stochastic linear phased arrays via orthogonal design initialization and a hybrid particle swarm optimizer," *2006 IEEE Antennas and Propagation Society International Symposium*, 3531–3534, Albuquerque, NM, 2006.
9. Bayraktar, Z., M. Komurcu, J. A. Bossard, and D. H. Werner, " The wind driven optimization technique and its application in electromagnetics," *IEEE Trans. Antennas Propag.*, Vol. 61, No. 5, 2745–2757, May 2013.
10. Goudos, S. K., K. Siakavara, T. Samaras, E. E. Vafiadis, and J. N. Sahalos, "Self-adaptive differential evolution applied to real-valued antenna and microwave design problems," *IEEE Trans. Antennas Propag.*, Vol. 59, No. 4, 1286–1298, Apr. 2011.
11. Mahmoud, K. R., "UWB antenna using gravitational search algorithm," *Journal of Engineering Sciences*, Vol. 41, No. 5, Faculty of Engineering, Assiut University, Sept. 2013.